# An Analysis of Three-Phase Rectifiers with Constant Voltage Loads 

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## The rectifier



## The rectifier, properties

- simple
- inexnensive



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- simple
> inexpensive
- robust


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-AC side parameters?
-AC side compliance with regulations?


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- DC side parameters, dependence of $V_{\text {OUT }}$ on $I_{\text {OUT }}$ ?


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- DC side parameters, dependence of $V_{O U T}$ on $I_{O U T}$ ?


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* old rectifier, well known?
* V Calickan N I Porrear1t 'T.M. Jahms, and J. G. Kassakian.
"Analysis of three-phase rectifiers with constant-voltage loads,"
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- continuous conduction mode (CCM) covered, three diodes conduct
- what about the discontinuous conduction mode(s) (DCM)?


## The rectifier to be analyzed



## Assumptions

```
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## The rectifier model



Normalization

- $v_{k}=V_{m} \cos \left(\omega t-(k-1) \frac{2 \pi}{3}\right)$, for $k \in\{1,2,3\}$ $\Rightarrow m=\frac{v}{T T}$


## Normalization

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- $\frac{d j_{k}}{d \varphi}=m_{k}-m_{X k}$

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- final reduction to 13 combinations (out of 64 )


## Combinations of diode states, listed

| combination | phase leg state |  |  |
| :---: | ---: | ---: | ---: |
|  | 1 | 2 | 3 |
| 0 | 0 | 0 | 0 |
| 1 | +1 | -1 | 0 |
| 2 | +1 | 0 | -1 |
| 3 | -1 | +1 | 0 |
| 4 | 0 | +1 | -1 |
| 5 | -1 | 0 | +1 |
| 6 | 0 | -1 | +1 |
| 7 | +1 | +1 | -1 |
| 8 | +1 | -1 | +1 |
| 9 | -1 | +1 | +1 |
| 10 | +1 | -1 | -1 |
| 11 | -1 | +1 | -1 |
| 12 | -1 | -1 | +1 |

## Circuit description

## - equations over inductor currents <br> - enmations for the outnut terminal voltages

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- equations over inductor currents
- equations for the output terminal voltages - boundary conditions, theoretically 6 of them


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- $j_{k}=j_{l}=j_{n}=0$


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- $j_{k}=j_{l}=j_{n}=0$
- $m_{A}-m_{B}=M_{O U T}$


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- $m_{A}-m_{B}=M_{\text {OUT }}$
- $m_{k l}<M_{\text {OUT }}$


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- $m_{k l}<M_{\text {OUT }}$
- the system order is zero


## Equivalent circuit for two conducting diodes



Equations for two conducting diodes

$$
\begin{aligned}
& \text { state }(k)=+1, \text { state }(l)=-1, \text { state }(n)=0 \\
& \frac{d j_{k}}{d \varphi}=\frac{1}{2}\left(m_{k}-m_{l}-M_{O U T}\right), j_{l}=-j_{k}, j_{n}=0
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- $m_{A}=\frac{1}{2}\left(M_{O U T}-m_{n}\right), m_{B}=\frac{1}{2}\left(M_{O U T}+m_{n}\right)$


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- $j_{k}>0$, to combination 0 if violated


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- $j_{k}>0$, to combination 0 if violated
- $m_{n}<\frac{1}{3} M_{\text {OUT }}$ leg $n$ to state +1 if violated


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- $j_{k}>0$, to combination 0 if violated
- $m_{n}<\frac{1}{3} M_{\text {OUT }}$ leg $n$ to state +1 if violated
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- $j_{k}>0$, to combination 0 if violated
- $m_{n}<\frac{1}{3} M_{\text {OUT }}$ leg $n$ to state +1 if violated
- $m_{n}>-\frac{1}{3} M_{\text {OUT }}$ leg $n$ to state -1 if violated
- the circuit is of the first order

Equivalent circuit for three conducting diodes, two to the positive output terminal


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- $m_{A}=\frac{1}{3} M_{O U T}, m_{B}=-\frac{2}{3} M_{\text {OUT }}$
- possible instantaneous combination transitions, additional incoualitics

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- $m_{A}=\frac{1}{3} M_{\text {OUT }}, m_{B}=-\frac{2}{3} M_{\text {OUT }}$
- $j_{k}>0, j_{l}>0, j_{n}<0$
- possible instantaneous combination transitions, additional inequalities
- the system is of the second order

Equivalent circuit for three conducting diodes, one to the positive output terminal


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\frac{d j_{k}}{d \varphi}=m_{k}-\frac{2}{3} M_{O U T}
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Equations for three conducting diodes, one to the positive output terminal

$$
\begin{aligned}
& -\frac{d j_{k}}{d \varphi}=m_{k}-\frac{2}{3} M_{O U T} \\
& -\frac{d j_{l}}{d \varphi}=m_{l}+\frac{1}{3} M_{O U T}
\end{aligned}
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Modes
$\rightarrow$ mode 0 , only combination 0

- mode 1 combinations with 0 and 2 conducting diodes mode 2 , combinations with 0,2 , and 3 conducting diodes - mode 3 combinatione with 2 and 3 aonduationg diodas



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## Dependence of the operating mode on $M_{O U T}$



Mode 1, waveforms of $i_{1}, v_{1}$, and $v_{X 1}, M_{O U T}=1.7$


Mode 2, waveforms of $i_{1}, v_{1}$, and $v_{X 1}, M_{O U T}=1.6475$


Mode 3, waveforms of $i_{1}, v_{1}$, and $v_{X 1}, M_{O U T}=1.5$


Mode 4 , waveforms of $i_{1}, v_{1}$, and $v_{X 1}, M_{O U T}=1$


## Dependence of $M_{\text {OUT }}$ on $J_{\text {OUT }}$



Dependence of the rectifier power and apparent power on $M_{\text {OUT }}$


Dependence of the rectifier power factor and the displacement power factor on $M_{O U T}$.


## Dependence of $P F_{X}$ and $D P F_{X}$ on $M_{O U T}$



## Dependence of $T H D\left(v_{X k}\right)$ on $M_{O U T}$



## Dependence of $T H D\left(i_{k}\right)$ on $M_{\text {OUT }}$



## Conclusions 1

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－analvsis nerformed on the eanation svstem level． normalization
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- out of $2^{6}=64$ combinations of diode states only 13 might occur
- for all 13 circuit equations are derived
- circuit order might be zero, one, or two, depending on the diode state combination


## Conclusions 2

```
* boundary inequalities are derived, combination transition
    rules
> instantaneous combination transitions
```



## Conclusions 2

- boundary inequalities are derived, combination transition rules
> instantaneous combination transitions - modes of the circuit operation are defined


## Conclusions 2

- boundary inequalities are derived, combination transition rules
- instantaneous combination transitions
- modes of the circuit operation are defined
- simulation, MOUT from 2 to 0 in steps of 0.0005


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- some interest in education

