

The Optimal Current Injection

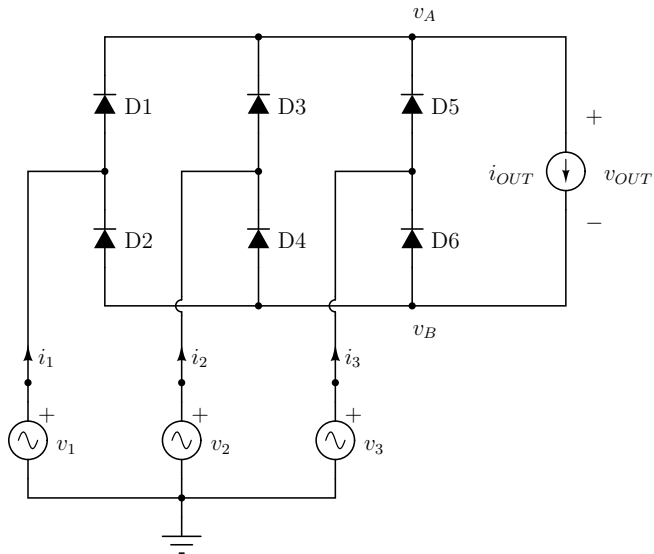
aim ...

is there a way to improve the THD further?

at this point we can get $THD = \sqrt{\frac{32\pi^2}{315} - 1} \approx 5.12\%$

any better?

let's get back to the beginning ...



we have ...

$$m_1 = \cos(\omega_0 t)$$

$$m_2 = \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$m_3 = \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

and we would like to have ...

$$j_1 = \cos(\omega_0 t)$$

$$j_2 = \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$j_3 = \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

a small note: normalized amplitude is 1; if real amplitude is I_m , the normalization is

$$j_X \triangleq \frac{i_X}{I_m}$$

what we can get?

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} d_1 - d_2 \\ d_3 - d_4 \\ d_5 - d_6 \end{bmatrix} [i_{OUT}]$$

linear dependence:

$$j_1 + j_2 + j_3 = 0$$

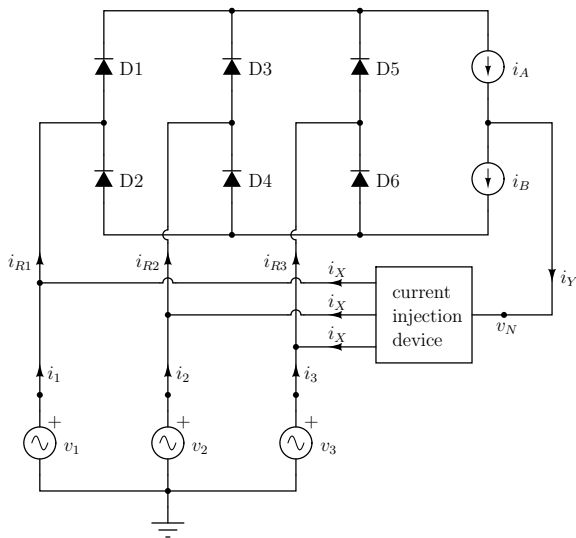
two degrees of freedom, $j_3 = -j_1 - j_2$

some other properties:

$$d_1 + d_3 + d_5 = 1 \quad \text{and} \quad d_2 + d_4 + d_6 = 1$$

no way to get rid of the gaps

let's introduce current injection in an open-minded way



some equations ...

$$i_{R1} = d_1 i_A - d_2 i_B$$

$$i_{R2} = d_3 i_A - d_4 i_B$$

$$i_{R3} = d_5 i_A - d_6 i_B$$

$$i_1 = i_{R1} - i_X$$

$$i_2 = i_{R2} - i_X$$

$$i_3 = i_{R3} - i_X$$

and some more ...

$$i_Y = i_A - i_B$$

$$i_X = \frac{1}{3} i_Y = \frac{1}{3} (i_A - i_B)$$

$$i_1 = d_1 i_A - d_2 i_B - i_X = \left(d_1 - \frac{1}{3}\right) i_A + \left(\frac{1}{3} - d_2\right) i_B$$

$$i_2 = d_3 i_A - d_4 i_B - i_X = \left(d_3 - \frac{1}{3}\right) i_A + \left(\frac{1}{3} - d_4\right) i_B$$

$$i_3 = d_5 i_A - d_6 i_B - i_X = \left(d_5 - \frac{1}{3}\right) i_A + \left(\frac{1}{3} - d_6\right) i_B$$

and in a matrix form, normalized ...

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} d_1 - \frac{1}{3} & \frac{1}{3} - d_2 \\ d_3 - \frac{1}{3} & \frac{1}{3} - d_4 \\ d_5 - \frac{1}{3} & \frac{1}{3} - d_6 \end{bmatrix} \begin{bmatrix} j_A \\ j_B \end{bmatrix}$$

linear dependence, $j_1 + j_2 + j_3 = 0$

equations consistent, third row is linearly dependent ...

reduction to 2×2

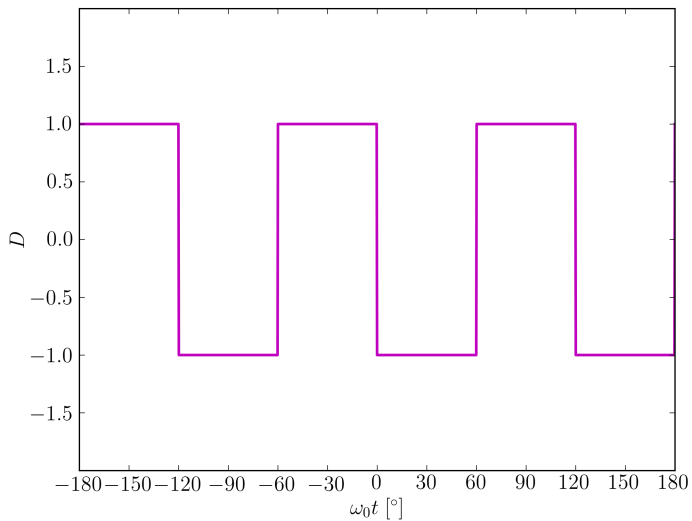
$$\begin{bmatrix} j_1 \\ j_2 \end{bmatrix} = \begin{bmatrix} d_1 - \frac{1}{3} & \frac{1}{3} - d_2 \\ d_3 - \frac{1}{3} & \frac{1}{3} - d_4 \end{bmatrix} \begin{bmatrix} j_A \\ j_B \end{bmatrix}$$

is it possible to solve for j_A and j_B for “any” choice of j_1 and j_2 ?

$$D \triangleq \begin{vmatrix} d_1 - \frac{1}{3} & \frac{1}{3} - d_2 \\ d_3 - \frac{1}{3} & \frac{1}{3} - d_4 \end{vmatrix}$$

$$D = \frac{1}{3} (d_1 - d_2 + (3 d_2 - 1) d_3 + (1 - 3 d_1) d_4)$$

$$D(\omega_0 t)$$



let's solve it ...

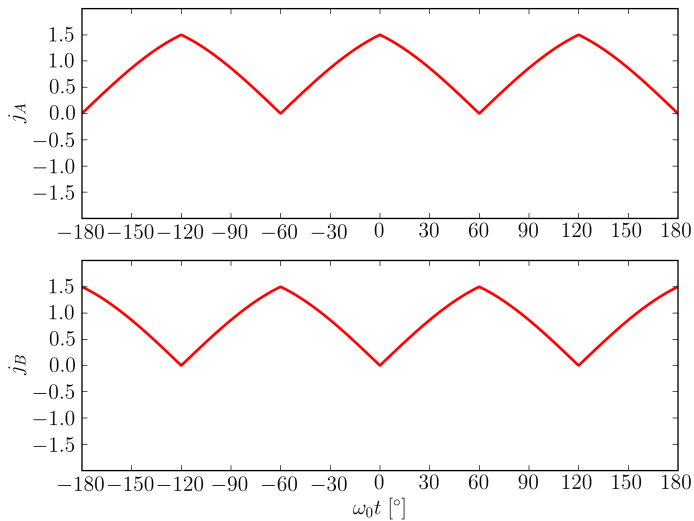
wxMaxima ...

$$j_A = -\frac{(3d_2 - 1)j_2 + (1 - 3d_4)j_1}{(3d_1 - 1)d_4 + (1 - 3d_2)d_3 + d_2 - d_1}$$

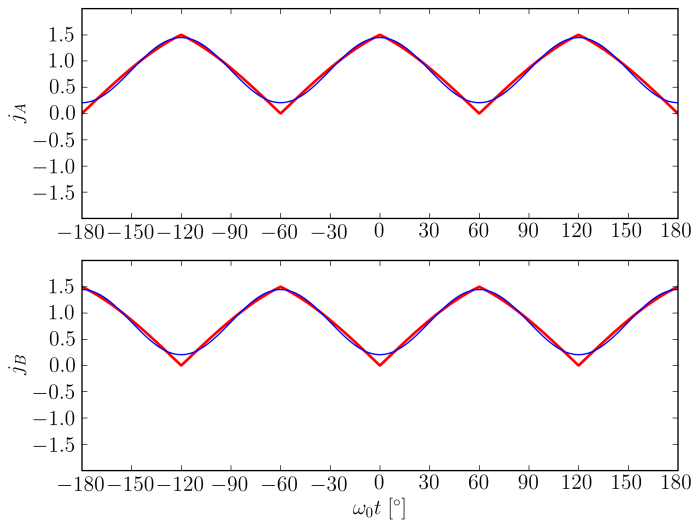
$$j_B = -\frac{(3d_1 - 1)j_2 + (1 - 3d_3)j_1}{(3d_1 - 1)d_4 + (1 - 3d_2)d_3 + d_2 - d_1}$$

j_A and j_B as segment-to-segment linear combinations of j_1 and j_2

let's show it: j_A and j_B ; on the DCM boundary



j_A and j_B , comparison to 3rd harmonic current injection



j_A and j_B , spectra, analytical

\cos was on purpose, to get rid of \Im wherever possible

$$j_A = J_{A,0} + \sum_{k=1}^{\infty} J_{A,k} \cos(3k\omega_0 t)$$

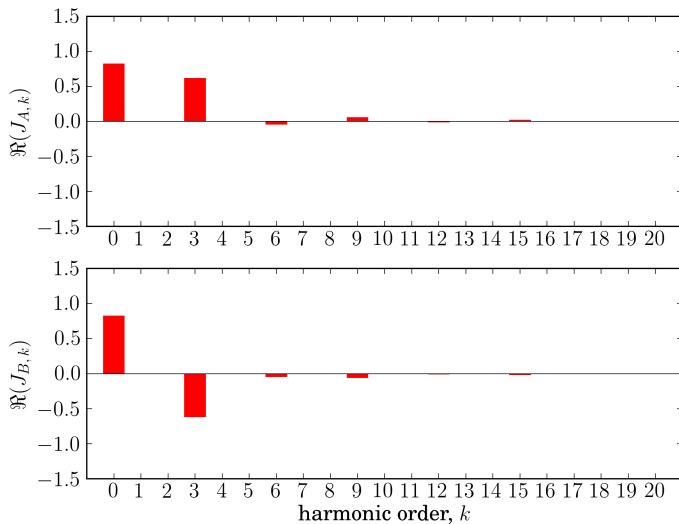
$$j_B = J_{B,0} + \sum_{k=1}^{\infty} J_{B,k} \cos(3k\omega_0 t)$$

$$J_{A,0} = J_{B,0} = \frac{3\sqrt{3}}{2\pi}$$

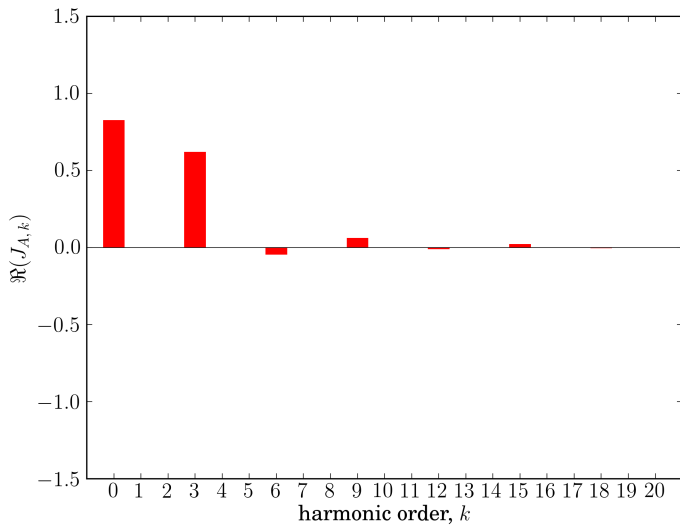
$$J_{A,k} = \frac{3\sqrt{3}}{\pi} \frac{1 - 2(-1)^k}{9k^2 - 1} \quad \text{for } k \in \mathbb{N}$$

$$J_{B,k} = \frac{3\sqrt{3}}{\pi} \frac{(-1)^k - 2}{9k^2 - 1} \quad \text{for } k \in \mathbb{N}$$

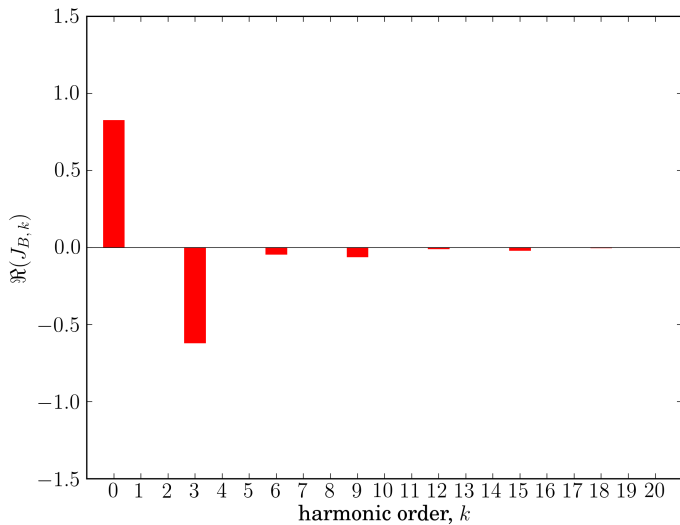
j_A and j_B , spectra, real part



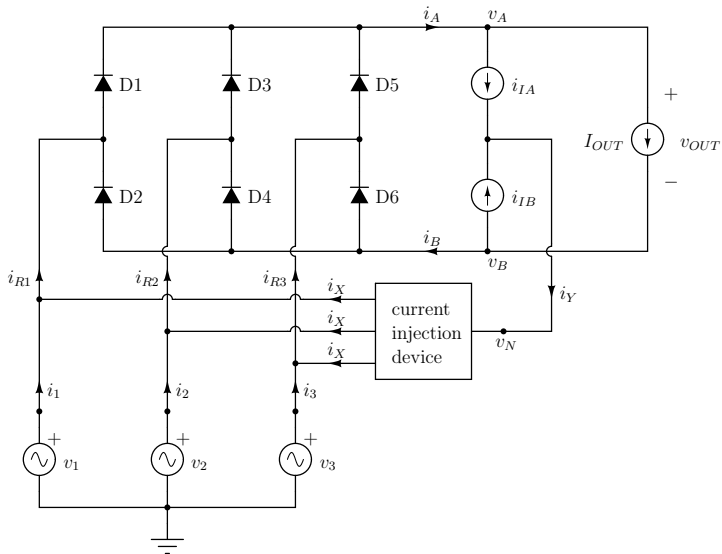
j_A , spectrum, real part



j_B , spectrum, real part



separation: I_{OUT}



separation: injection part

$$J_{OUT} = J_{A,0} = J_{B,0} = \frac{3\sqrt{3}}{2\pi}$$

J_{OUT} is not 1 any more!!! new normalization of currents:

$$j_X \triangleq \frac{i_X}{I_m}$$

$$j_{IA} = j_A - J_{OUT}$$

$$j_{IB} = J_{OUT} - j_B$$

$$j_Y = j_{IA} + j_{IB}$$

$$P_{OUT}, P_{IN}, \eta$$

since we are already here, around J_{OUT}, \dots

$$M_{OUT} = \frac{3\sqrt{3}}{\pi}$$

$$P_{OUT} = M_{OUT} J_{OUT} = \frac{27}{2\pi^2}$$

$$P_{IN} = \frac{3}{2}$$

$$P_{INJ} = P_{IN} - P_{OUT} = \frac{3\pi^2 - 27}{2\pi^2}$$

$$\boxed{\eta = \frac{P_{OUT}}{P_{IN}} = \frac{9}{\pi^2} \approx 91.19\%}$$

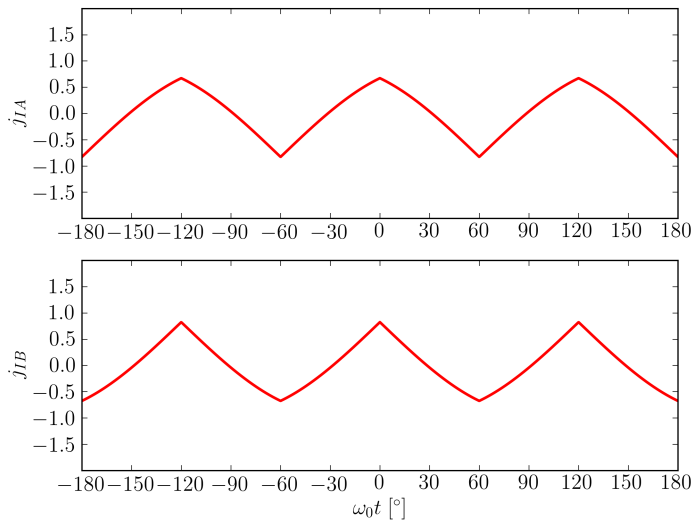
comparison to the 3rd harmonic current injection

$$\Delta\eta = \frac{32}{35} - \frac{9}{\pi^2}$$

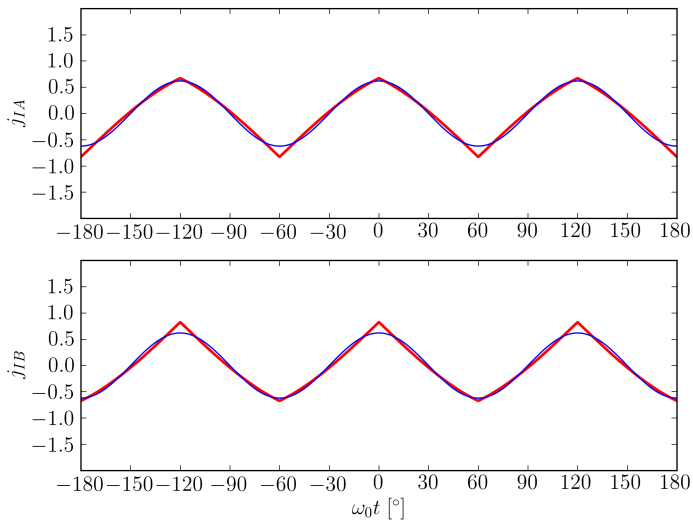
$$\Delta\eta \approx -0.24\%$$

negligible decrease; this is not a decision-making argument

back to j_{IA} and j_{IB} , waveforms



j_{IA} and j_{IB} , comparison to 3rd harmonic current injection



j_{IA} and j_{IB} , spectra, analytical

$$j_{IA} = \sum_{k=1}^{\infty} J_{IA,k} \cos(3k\omega_0 t)$$

$$j_{IB} = \sum_{k=1}^{\infty} J_{IB,k} \cos(3k\omega_0 t)$$

$$J_{IA,k} = \frac{3\sqrt{3}}{\pi} \frac{1 - 2(-1)^k}{9k^2 - 1} \quad \text{for } k \in \mathbb{N}$$

$$J_{IB,k} = \frac{3\sqrt{3}}{\pi} \frac{2 - (-1)^k}{9k^2 - 1} \quad \text{for } k \in \mathbb{N}$$

j_{IA} and j_{IB} , spectra, parity of k

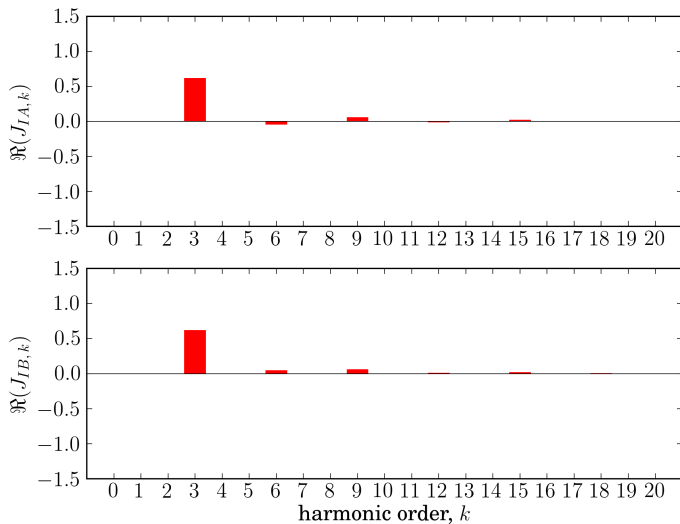
for k odd, $k = 2n - 1$, $n \in \mathbb{N}$

$$J_{IA,k} = J_{IB,k} = \frac{9\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

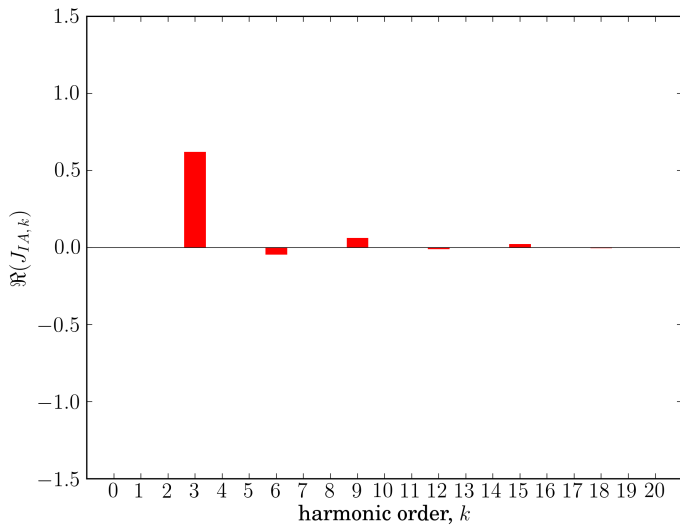
for k even, $k = 2n$, $n \in \mathbb{N}$

$$J_{IA,k} = -J_{IB,k} = -\frac{3\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

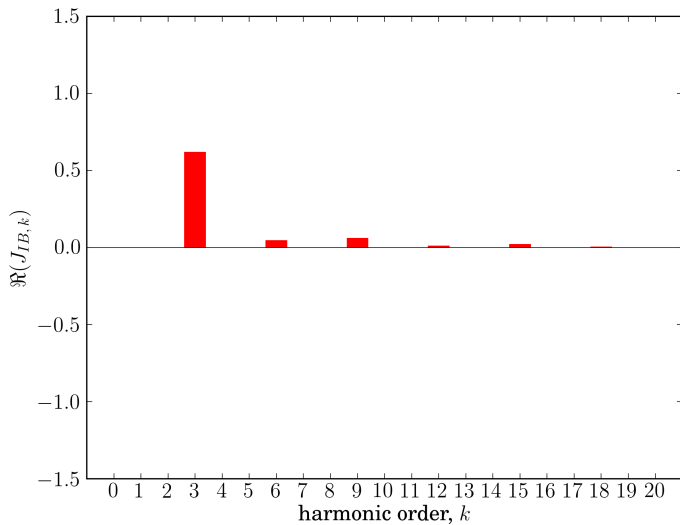
j_{IA} and j_{IB} , spectra, real part



j_{IA} , spectrum, real part



j_{IB} , spectrum, real part



separation, again: j_{odd} and j_{even}

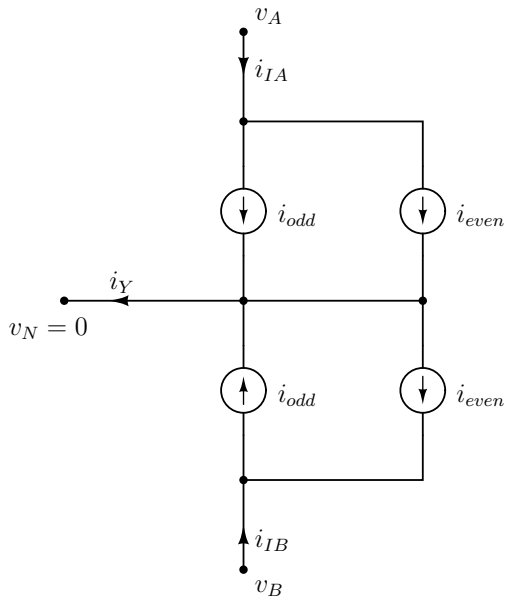
$$j_{IA} = j_{odd} + j_{even}$$

$$j_{IB} = j_{odd} - j_{even}$$

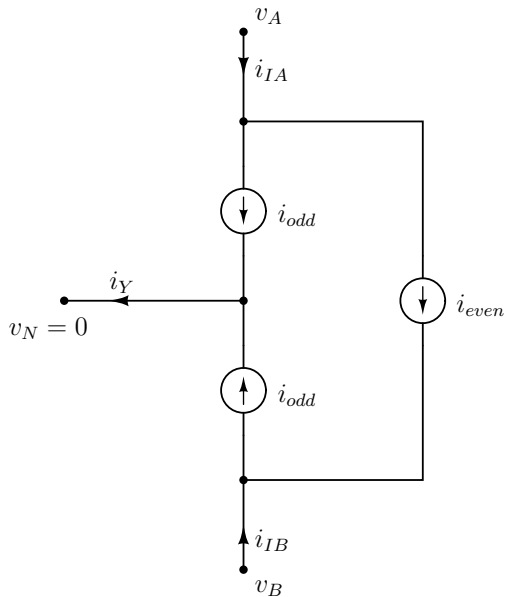
$$j_{odd} \triangleq \frac{9\sqrt{3}}{\pi} \sum_{k=1,3,5\dots}^{\infty} \frac{1}{9k^2 - 1} \cos(3k\omega_0 t)$$

$$j_{even} \triangleq -\frac{3\sqrt{3}}{\pi} \sum_{k=2,4,6\dots}^{\infty} \frac{1}{9k^2 - 1} \cos(3k\omega_0 t)$$

physical interpretation and visualization



reduction ...



detour ...

- ▶ “position and hold”
- ▶ we’ve gone too far
- ▶ put the parallel in sequence, remember the problem?
- ▶ we have to walk another line
- ▶ that historically was in parallel to this one
- ▶ but now, we have to put this presentation in sequence
- ▶ remember the “reduction”, we gonna need it

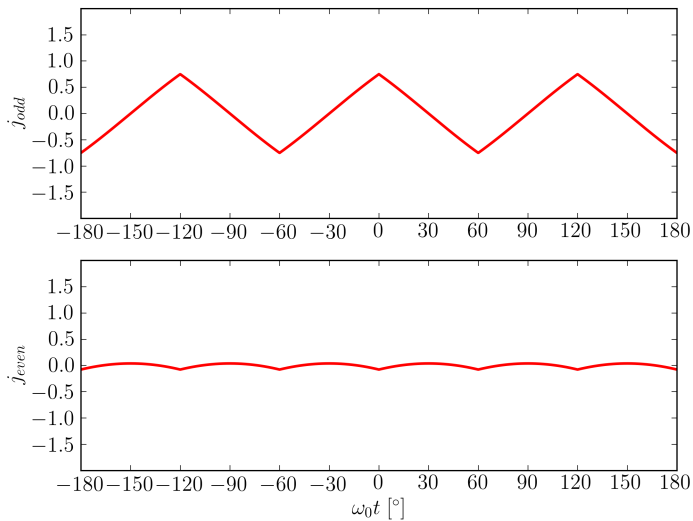
how to draw j_{odd} and j_{even} ?

forget about Fourier, linear algebra does the job:

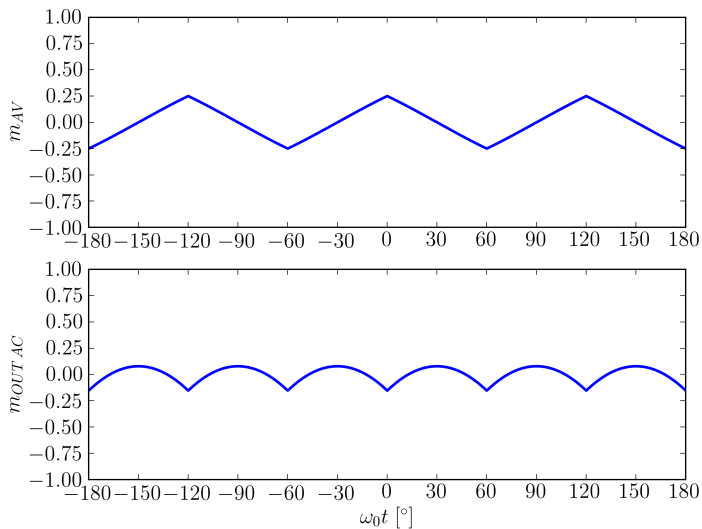
$$j_{odd} = \frac{1}{2} (j_{IA} + j_{IB})$$

$$j_{even} = \frac{1}{2} (j_{IA} - j_{IB})$$

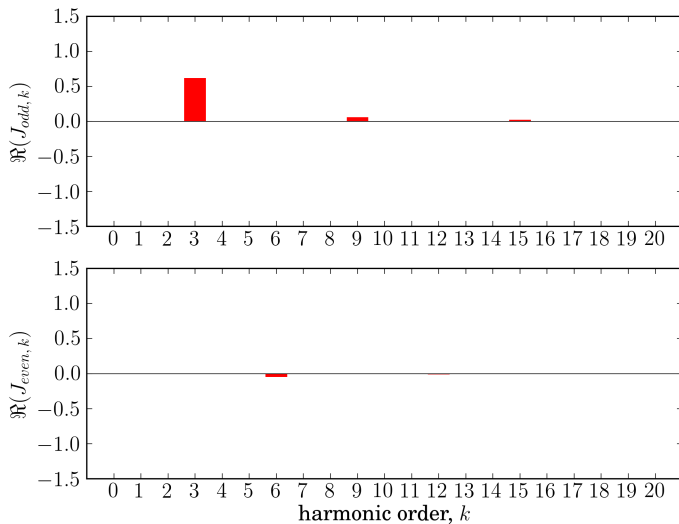
j_{odd} and j_{even} , waveforms



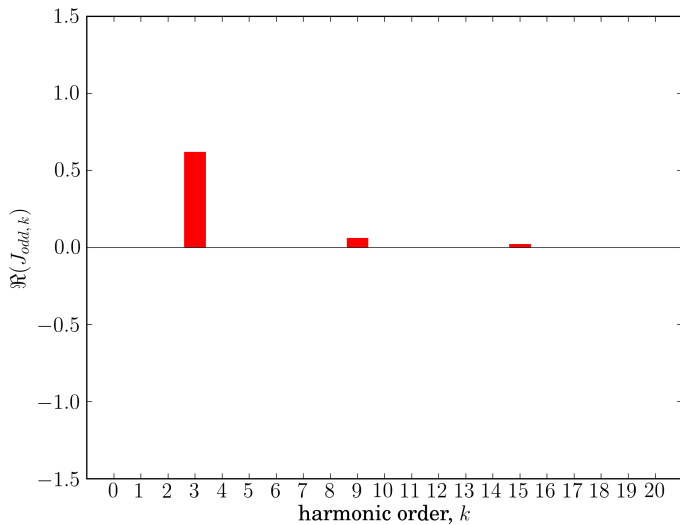
can't resist, just a hint, m_{AV} , $m_{OUT AC}$



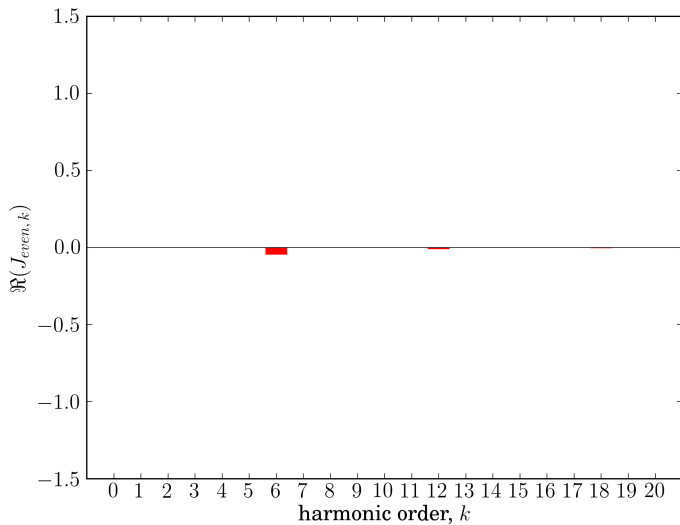
j_{odd} and j_{even} , spectra, real part



j_{odd} , spectrum, real part



j_{even} , spectrum, real part



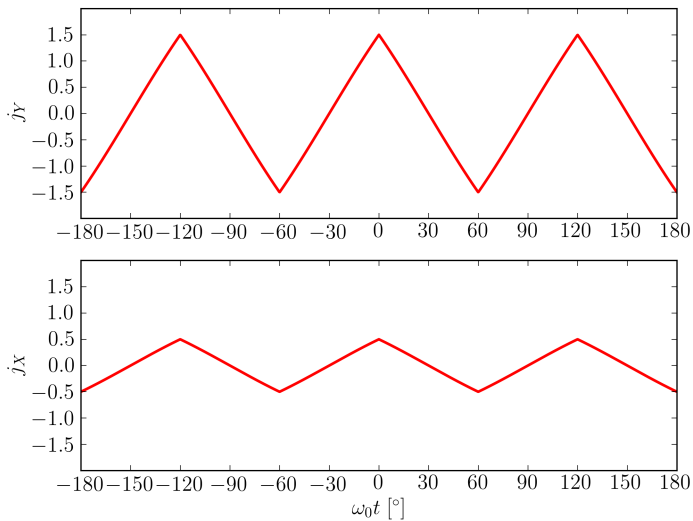
j_Y and j_X

$$j_Y = j_{IA} + j_{IB} = 2 j_{odd}$$

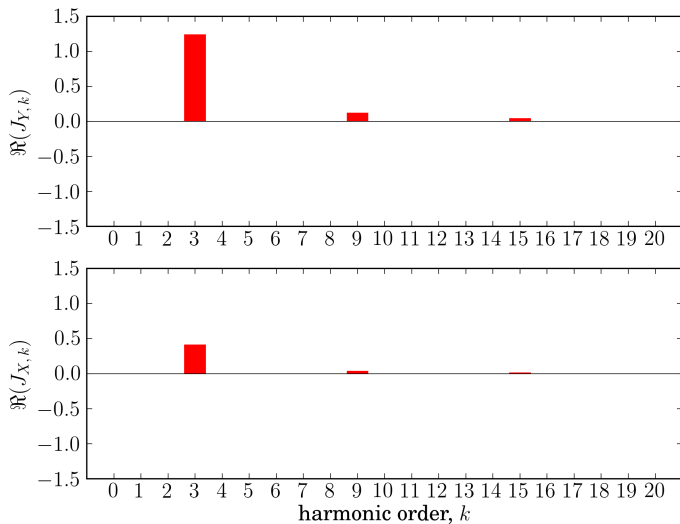
$$j_X = \frac{1}{3} j_Y = \frac{2}{3} j_{odd}$$

they do not have anything in common with j_{even}

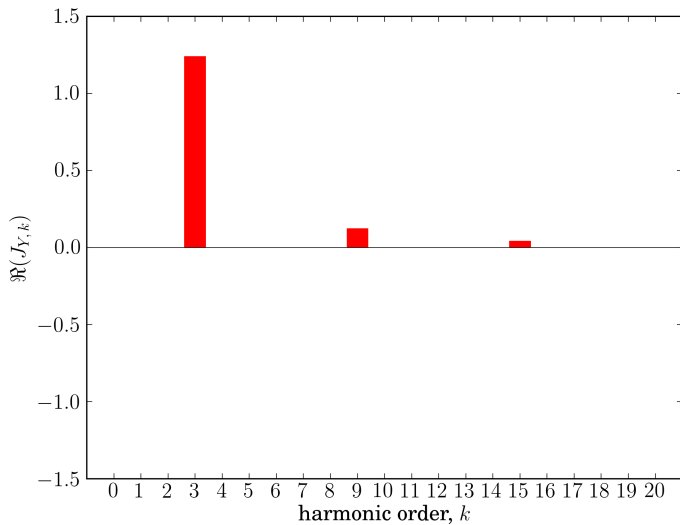
j_Y and j_X , waveforms



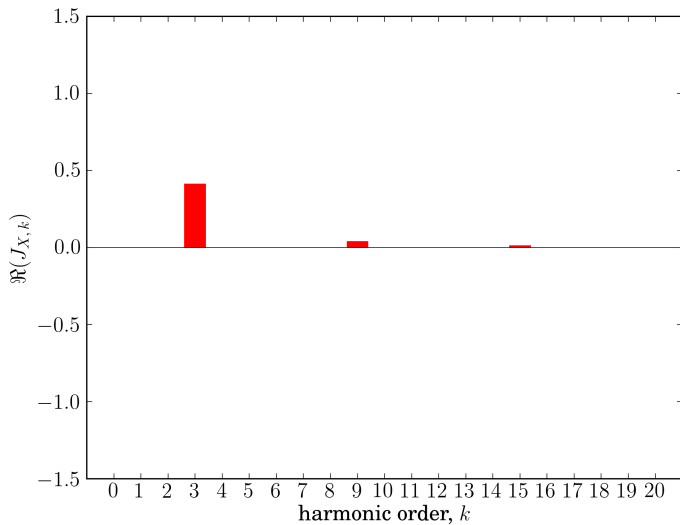
j_Y and j_X , spectra, real part



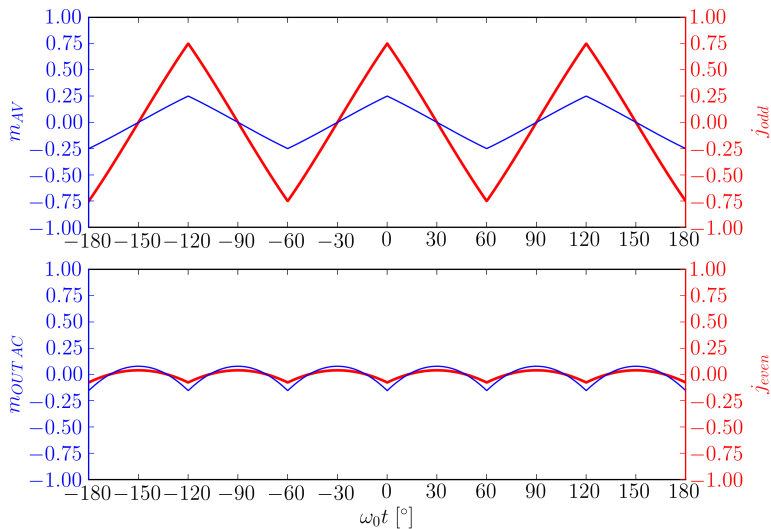
j_Y , spectrum, real part



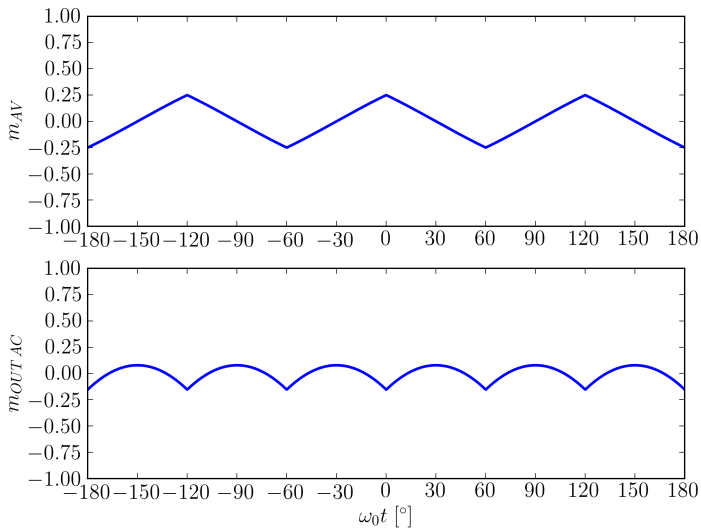
j_X , spectrum, real part



finally, time to relate voltages and currents



m_{AV} and $m_{OUT AC}$, waveforms, detour, again



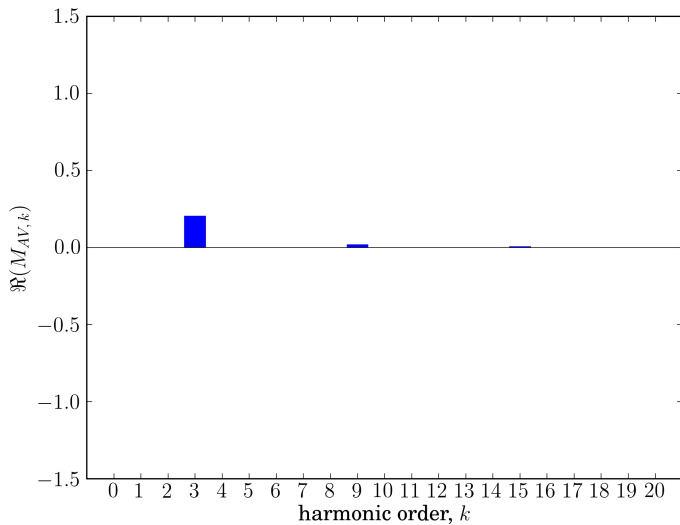
m_{AV} , spectrum, analytical

$$m_{AV} \triangleq \frac{m_A + m_B}{2}$$

$$m_{AV} = \sum_{k=1,3,5\dots}^{\infty} M_{AV,k} \cos(3k\omega_0 t)$$

$$M_{AV,k} = \frac{3\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

m_{AV} , spectrum, real part



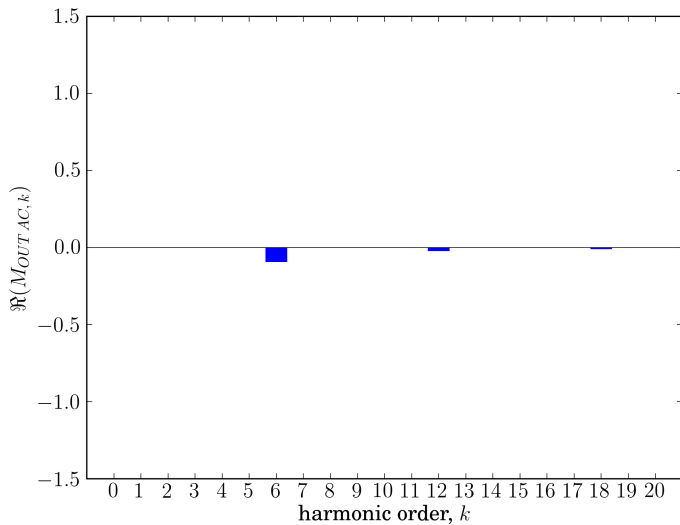
$m_{OUT\ AC}$, spectrum, analytical

$$m_{OUT\ AC} \triangleq m_{OUT} - M_{OUT}$$

$$m_{OUT, AC} = \sum_{k=2, 4, 6 \dots}^{\infty} M_{OUT, k} \cos(3k\omega_0 t)$$

$$M_{OUT, k} = -\frac{6\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

$m_{OUT\ AC}$, spectrum, real part



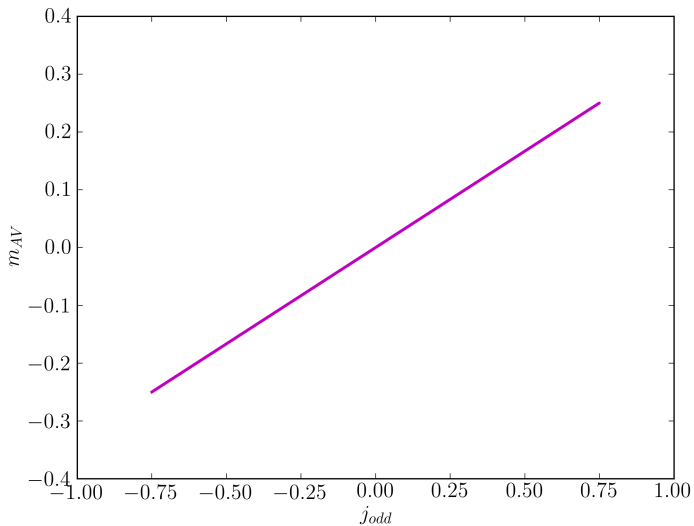
spectral relations, odd

$$J_{odd,k} = \frac{9\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

$$M_{AV,k} = \frac{3\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

$$\boxed{\frac{M_{AV,k}}{J_{odd,k}} = \frac{1}{3}}$$

odd, time domain relation ...



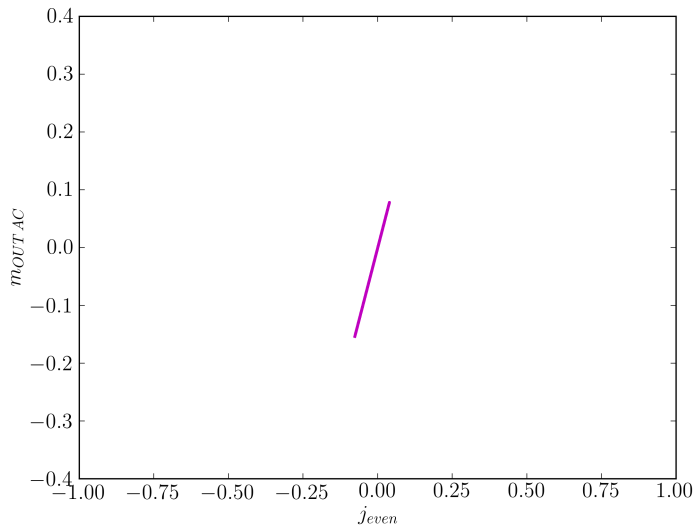
spectral relations, even

$$J_{even,k} = -\frac{3\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

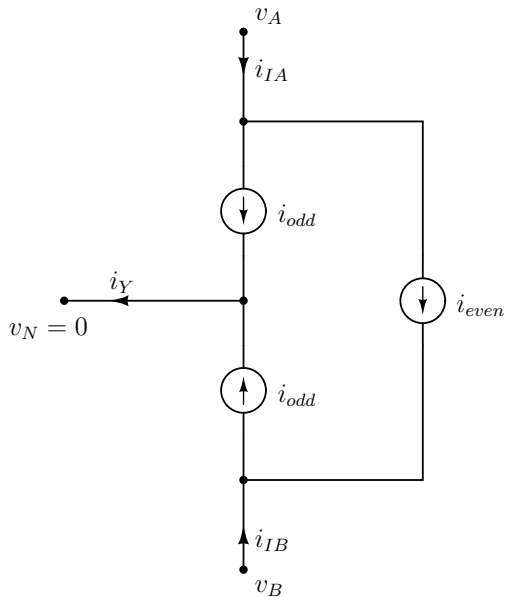
$$M_{OUT,k} = -\frac{6\sqrt{3}}{\pi} \frac{1}{9k^2 - 1}$$

$$\boxed{\frac{M_{OUT,k}}{J_{even,k}} = 2}$$

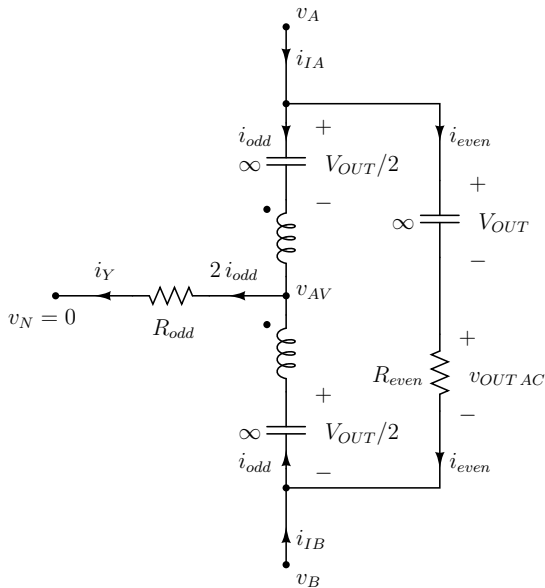
even, time domain relation ...



remember the “reduction”?



time to relate voltages and currents



R_{even}

$$R_{even} = \frac{v_{OUT\ AC}}{i_{even}} = \frac{V_m}{I_m} \frac{m_{OUT\ AC}}{j_{even}} = 2 \frac{V_m}{I_m}$$

let's introduce

$$R_E \triangleq \frac{V_m}{I_m}$$

as the resistance emulated at the rectifier input

$$R_{even} = 2 R_E \quad \text{or} \quad \rho_{even} = 2$$

$$R_{odd}$$

$$R_{odd} = \frac{v_{AV}}{i_Y} = \frac{V_m}{I_m} \frac{m_{AV}}{j_Y} = \frac{V_m}{I_m} \frac{m_{AV}}{2 j_{odd}} = \frac{1}{6} \frac{V_m}{I_m}$$

$$R_{odd} = \frac{1}{6} R_E \quad \text{or} \quad \rho_{odd} = \frac{1}{6}$$

relation to CIN #3, $Q = 0$

CIN #3:

$$R = \frac{\sqrt{3}}{4\pi} \frac{V_m}{I_{OUT}}$$

in the optimal current injection:

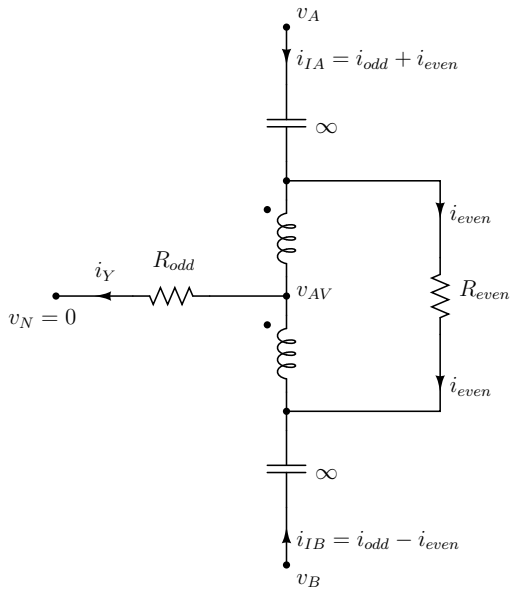
$$I_{OUT} = \frac{3\sqrt{3}}{2\pi} I_m$$

and R would be

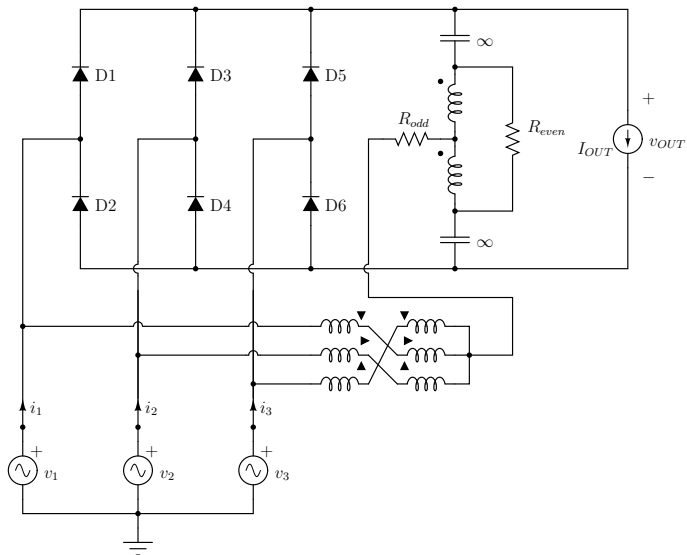
$$R = \frac{\sqrt{3}}{4\pi} \frac{2\pi}{3\sqrt{3}} \frac{V_m}{I_m} = \frac{1}{6} \frac{V_m}{I_m} = R_{odd}$$

except for the i_{even} path, they are the same!

some reductionism, again



and we have the rectifier!



power on R_{odd} ?

not a big deal (with wxMaxima, though):

$$J_{odd, RMS} = \sqrt{\frac{18\pi - 27\sqrt{3}}{16\pi}}$$

$$P_{odd} = \frac{1}{6} (2 J_{odd, RMS})^2 = \frac{6\pi - 9\sqrt{3}}{8\pi}$$

$$\frac{P_{odd}}{P_{OUT}} = \frac{\pi (6\pi - 9\sqrt{3})}{108} \approx 9.49\%$$

$$\frac{P_{odd}}{P_{IN}} = \frac{6\pi - 9\sqrt{3}}{12\pi} \approx 8.65\%$$

power on R_{even} ?

$$J_{even,RMS} = \frac{\sqrt{3\pi (2\pi + 3\sqrt{3}) - 108}}{4\pi}$$

$$P_{even} = 2 (J_{even,RMS})^2 = \frac{3 (2\pi^2 + 3\pi\sqrt{3} - 36)}{8\pi^2}$$

$$\frac{P_{even}}{P_{OUT}} = \frac{2\pi^2 + 3\pi\sqrt{3} - 36}{36} \approx 0.18 \%$$

$$\frac{P_{even}}{P_{IN}} = \frac{2\pi^2 + 3\pi\sqrt{3} - 36}{4\pi^2} \approx 0.16 \%$$

and just a few notes ...

- ▶ new normalization of currents introduced, over I_m
- ▶ resonance constraint avoided, tuned circuits not required
- ▶ operation at the DCM boundary, soft switching of diodes
- ▶ notches absent from the input voltages
- ▶ finite capacitance, ...
- ▶ R_{even} can compensate for the load type:
 1. resistor as a load: omit R_{even}
 2. constant power load: half R_{even}
- ▶ omitting R_{even} reduces to CIN #3 with $Q = 0$, $THD \approx 4\%$
- ▶ some minor issues: the resistance distribution parameter, a

published in ...

Predrag Pejović, Žarko Janda

“Three Phase Rectifiers that Apply Optimal Current Injection”

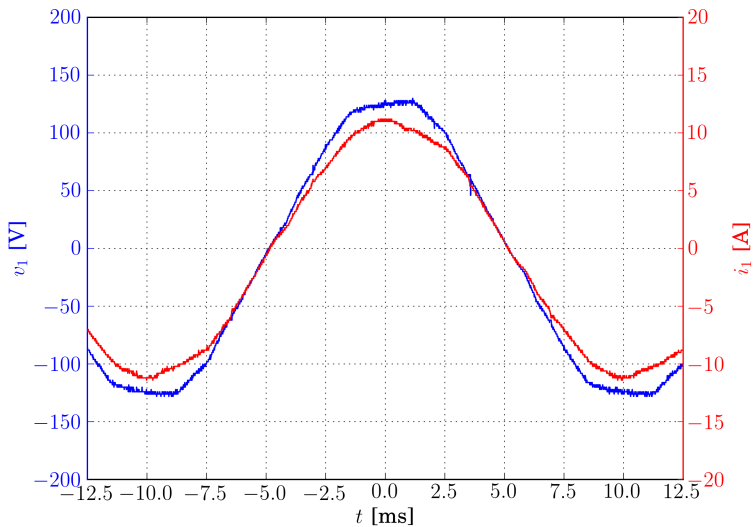
IEEE Transactions on Aerospace and Electronic Systems,
vol. 38, no. 1, pp. 163–173, January 2002

and rejected for IEEE Transactions on Industry Applications ...

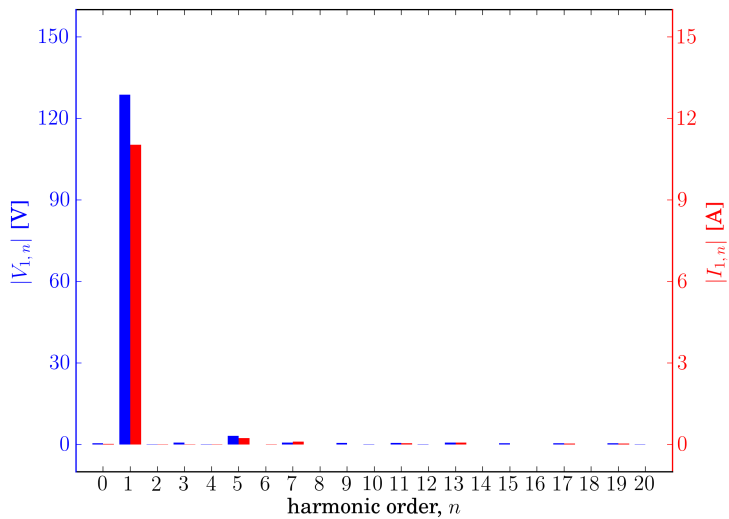
a motivating factor for the choice? fair overpage policies ...

largely unnoticed ...

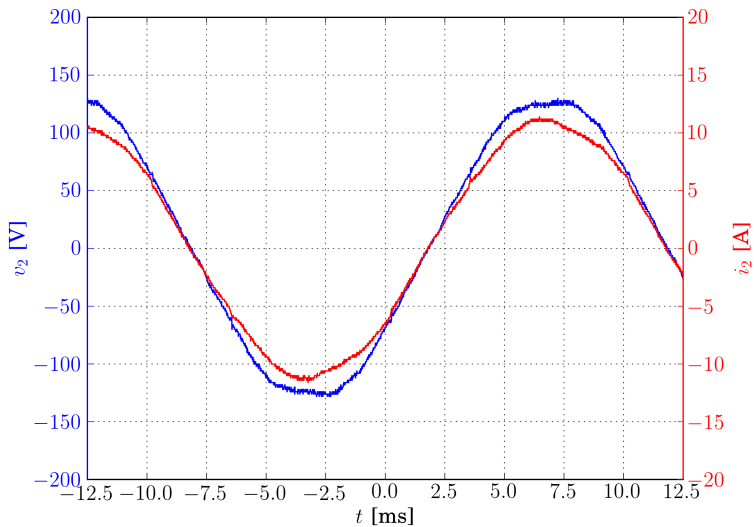
v_1 and i_1 , experimental



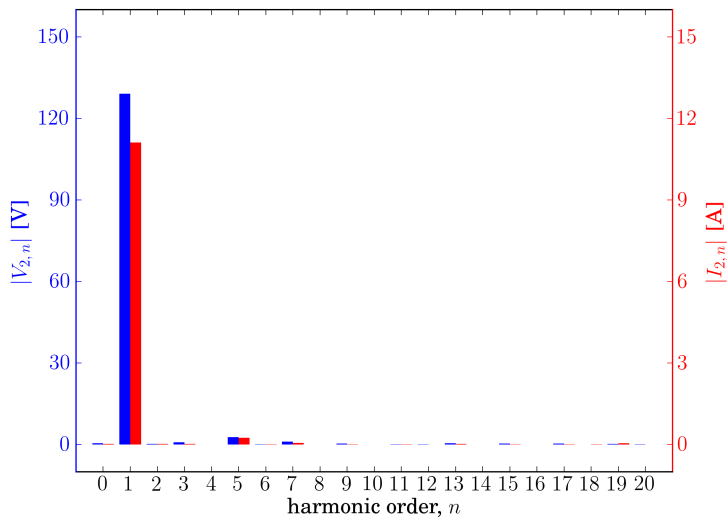
v_1 and i_1 , experimental, spectra



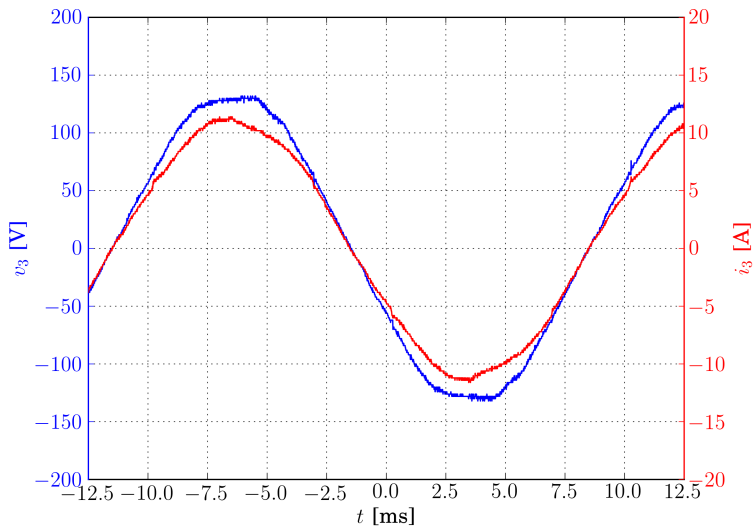
v_2 and i_2 , experimental



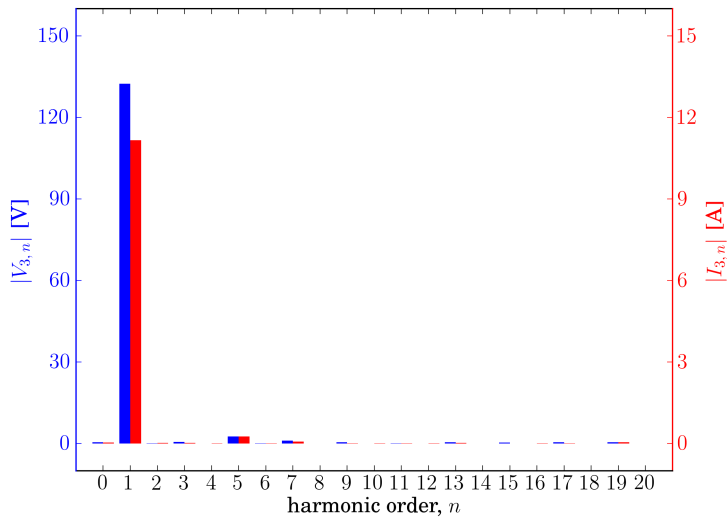
v_2 and i_2 , experimental, spectra



v_3 and i_3 , experimental



v_3 and i_3 , experimental, spectra



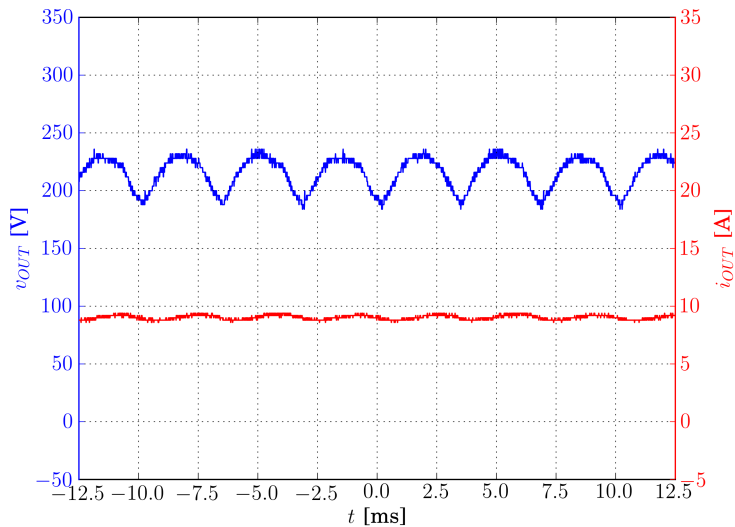
input, experimental results, #1 ...

k	$I_{k\,RMS}$ [A]	$V_{k\,RMS}$ [V]	S [VA]	P [W]
1	7.80	91.00	709.53	708.76
2	7.86	91.26	717.21	716.62
3	7.89	93.64	738.87	738.24

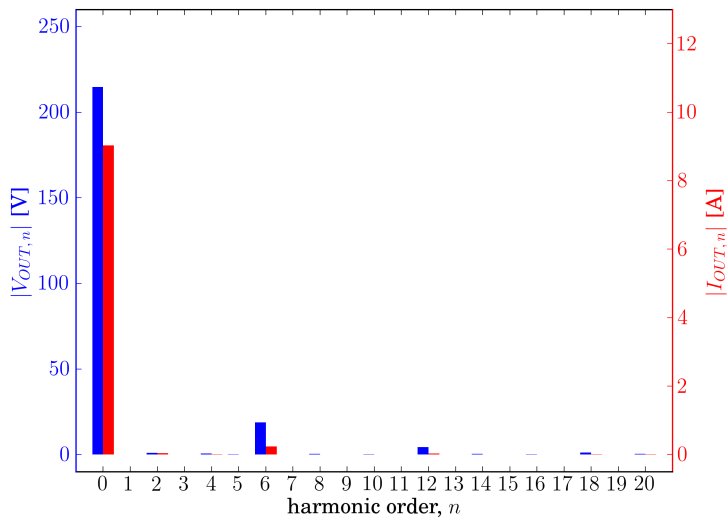
input, experimental results, #2 ...

k	PF	$THD(i_k)$ [%]	$THD(v_k)$ [%]
1	0.9989	2.82	3.14
2	0.9992	2.73	2.74
3	0.9991	2.80	2.61

i_{OUT} and v_{OUT} , experimental



i_{OUT} and v_{OUT} , experimental, spectra



output, experimental results

$$I_{OUT} = 9.04 \text{ A}$$

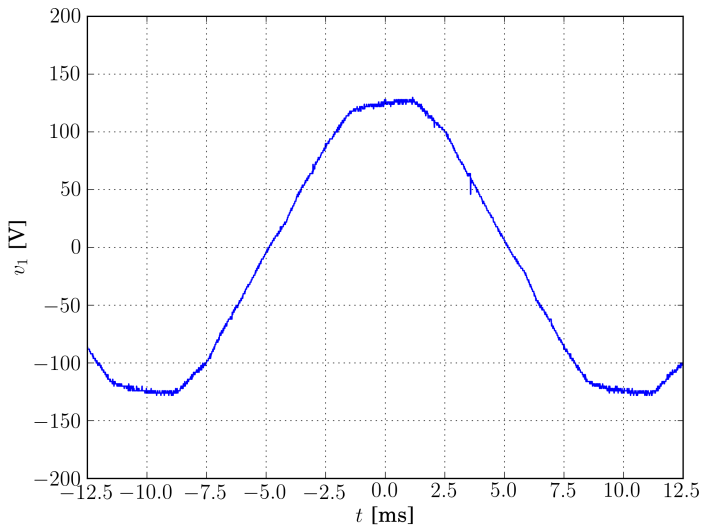
$$V_{OUT} = 214.77 \text{ V}$$

$$P_{OUT} = 1941.50 \text{ W}$$

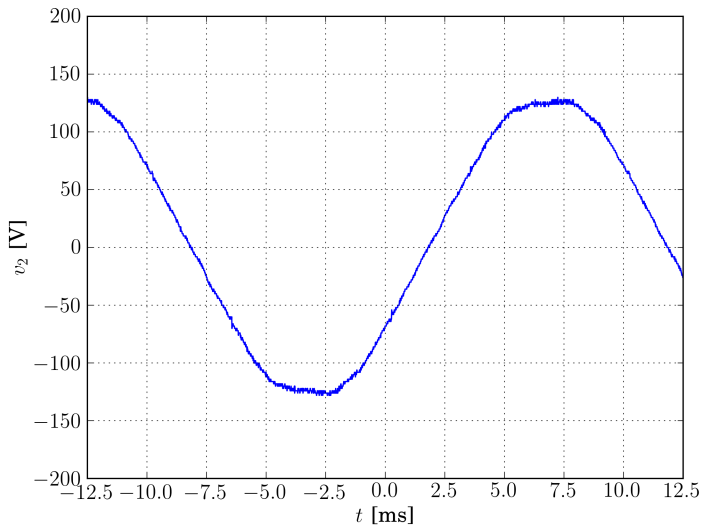
$$\eta = 89.73 \%$$

fits pretty good!!!

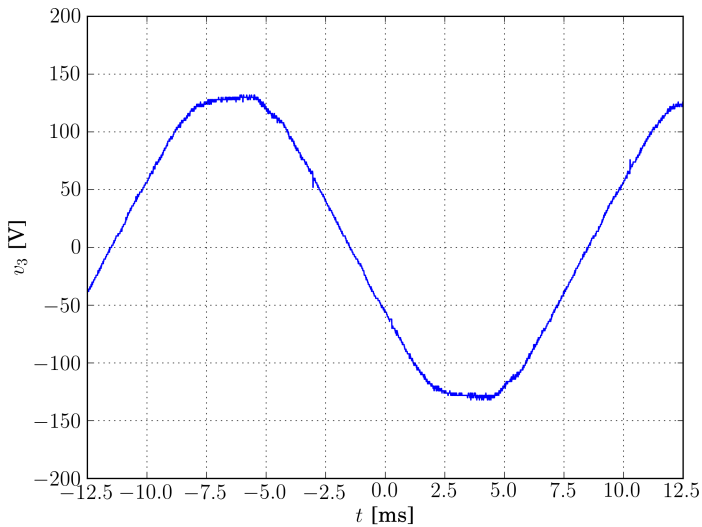
v_1 , experimental



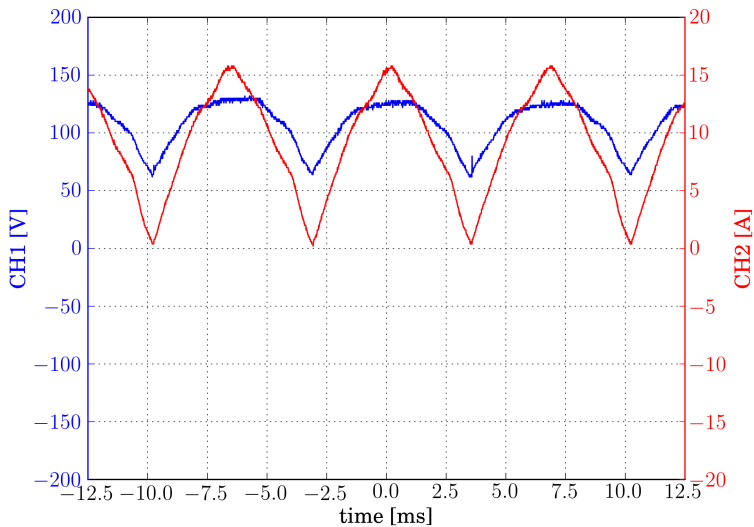
v_2 , experimental



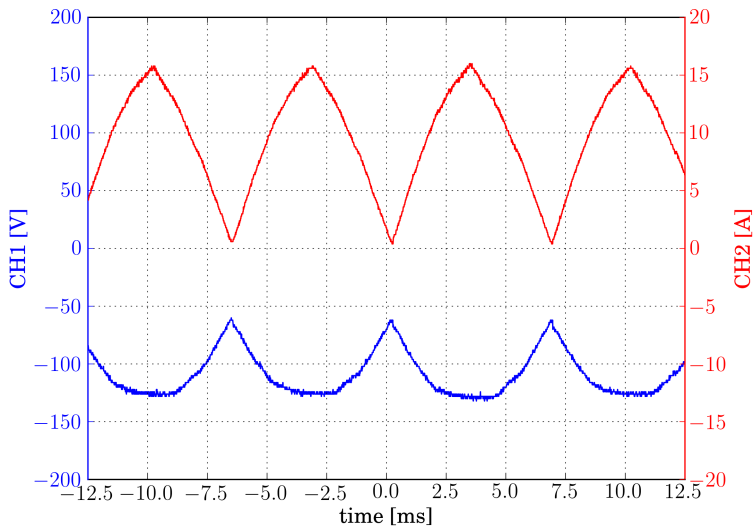
v_3 , experimental



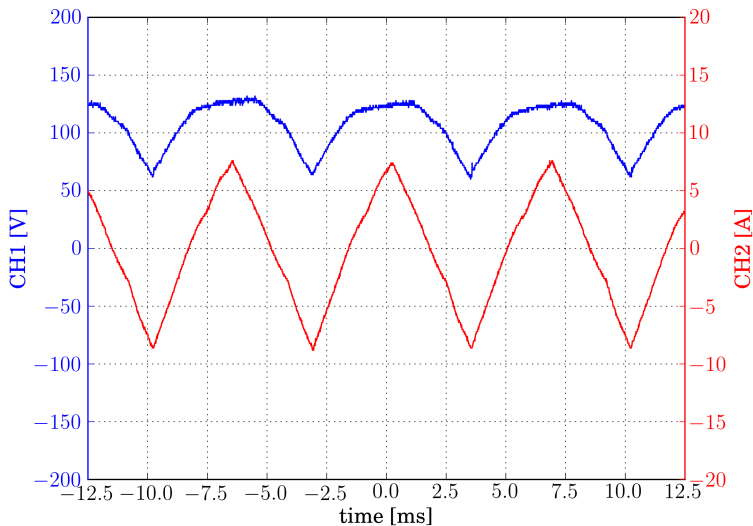
v_A and i_A , experimental



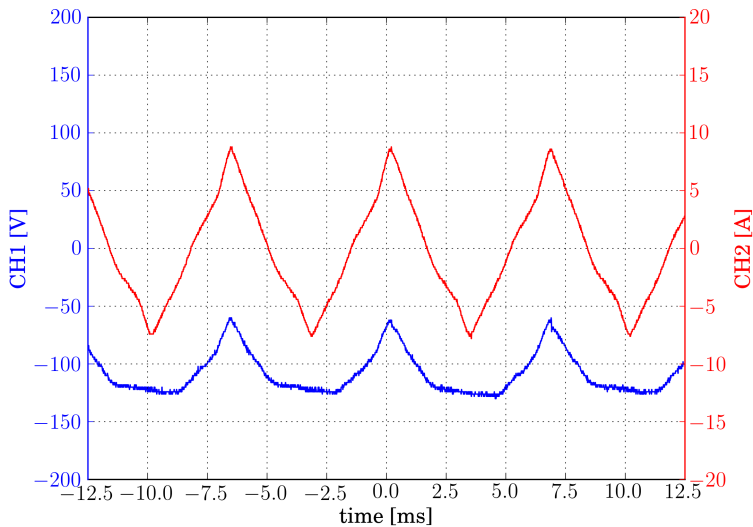
v_B and i_B , experimental



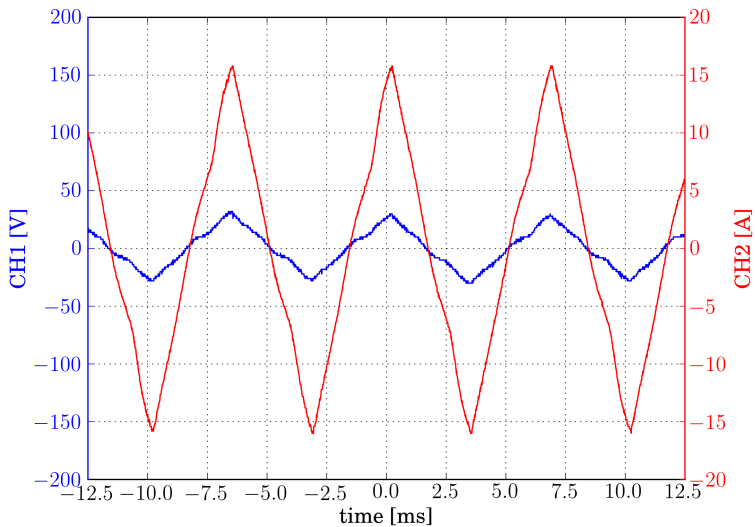
v_A and i_{IA} , experimental



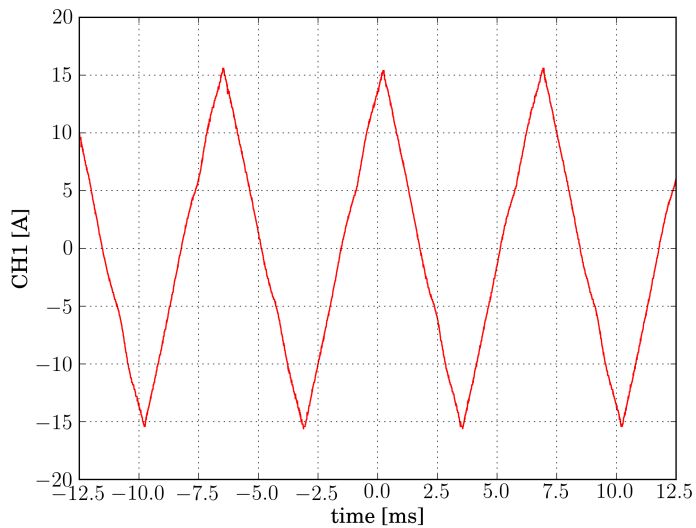
v_B and i_{IB} , experimental



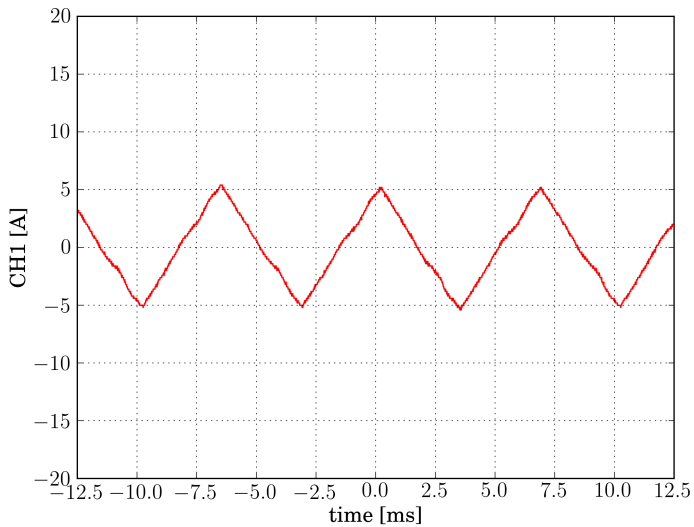
v_{AV} and i_Y , experimental



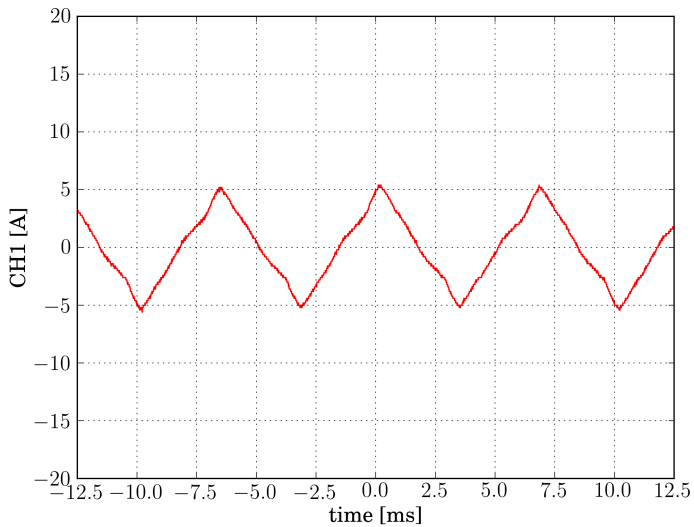
i_Y , experimental



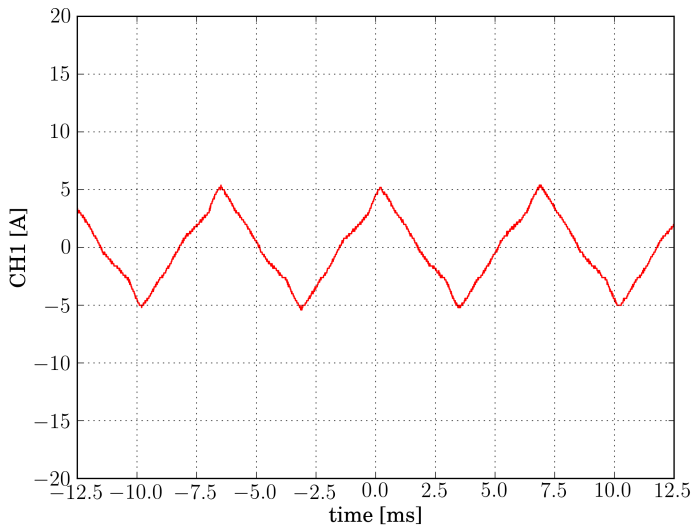
i_{X1} , experimental



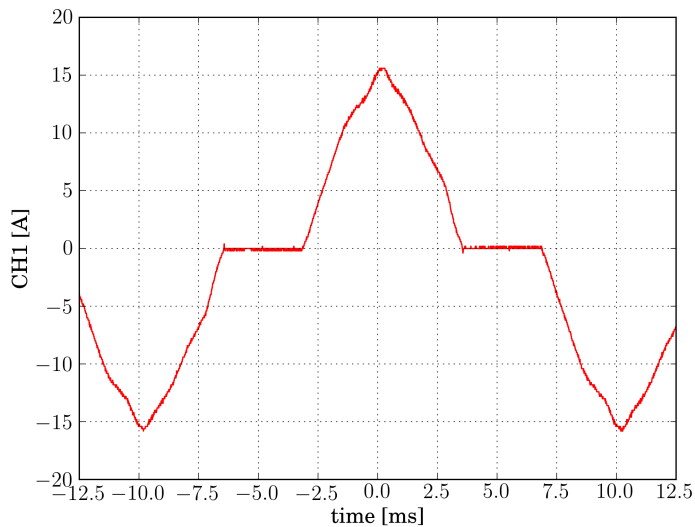
i_{X2} , experimental



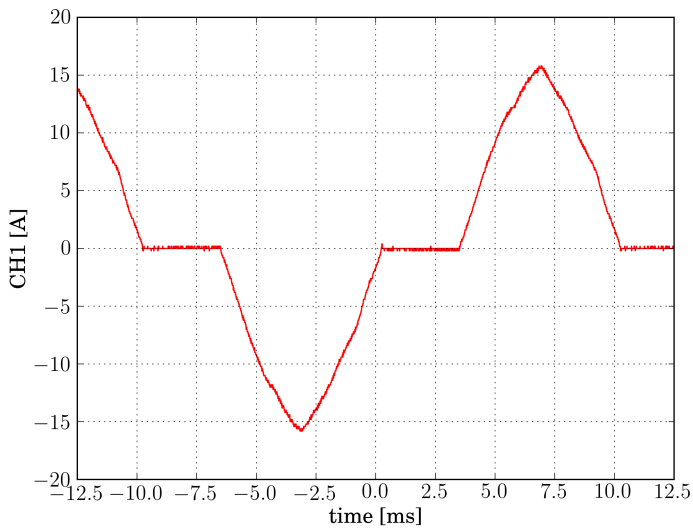
i_{X3} , experimental



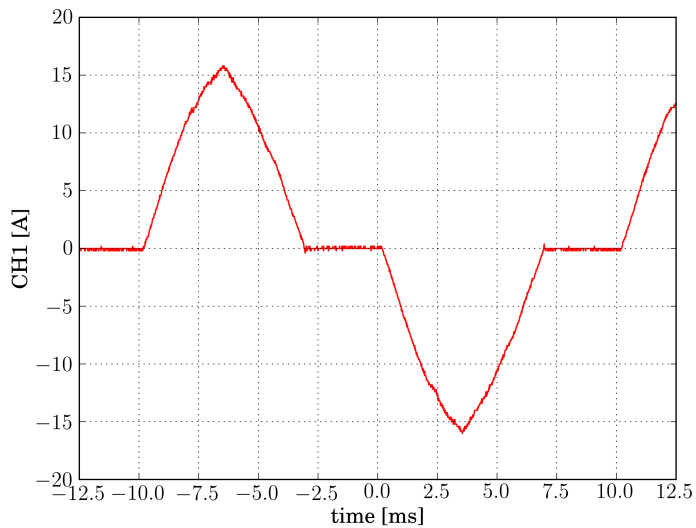
i_{R1} , experimental



i_{R2} , experimental



i_{R3} , experimental



“future work”

1. there is **no way** to improve the THD further.
2. is there a simple way to restore the power taken by the current injection network?
3. let's be “open”: is there a way to get rid of the current injection device?