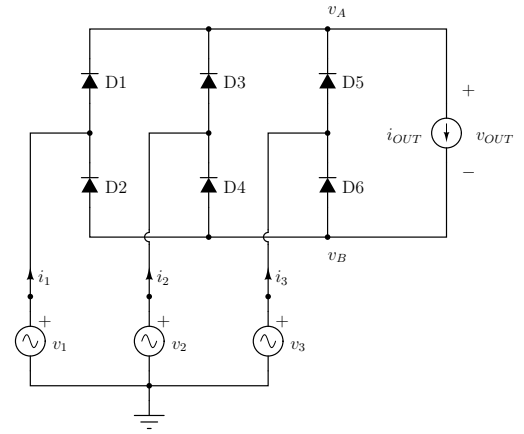


Introduction

— three-phase diode bridge rectifier —

what is this all about?



input voltages

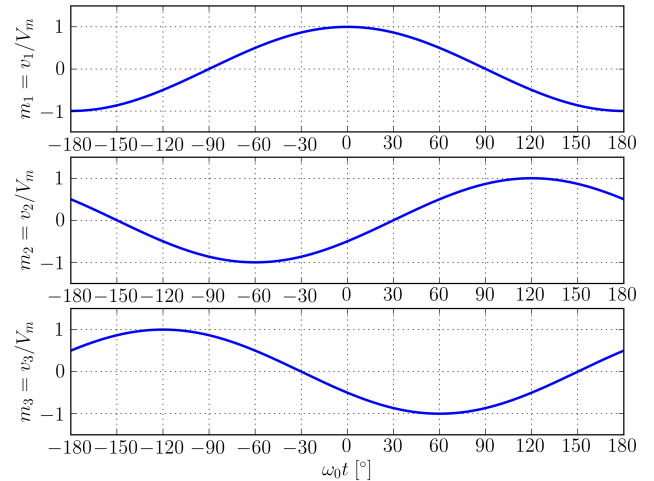
$$v_1 = V_m \cos(\omega_0 t)$$

$$v_2 = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$v_k = V_m \cos\left(\omega_0 t - (k-1) \frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$

input voltages, waveforms



normalization of voltages

$$m_X \triangleq \frac{v_X}{V_m}$$

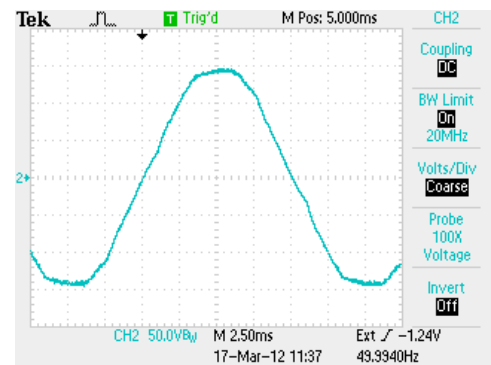
$$m_1 = \cos(\omega_0 t)$$

$$m_2 = \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$m_3 = \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

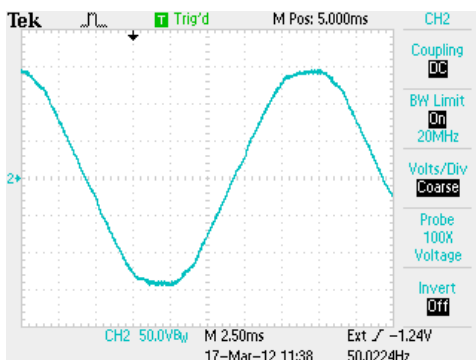
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1) \frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



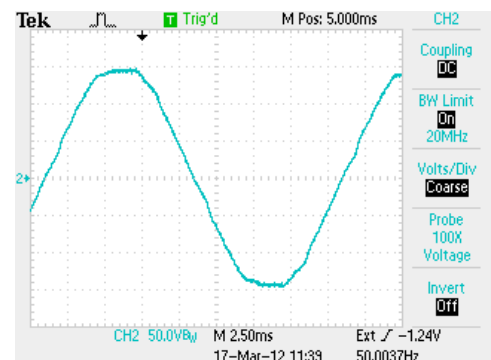
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1) \frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



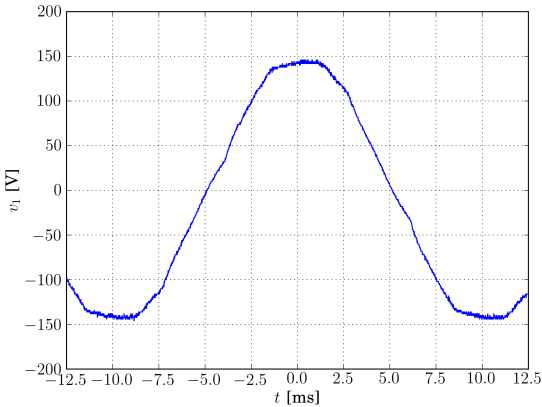
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1) \frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



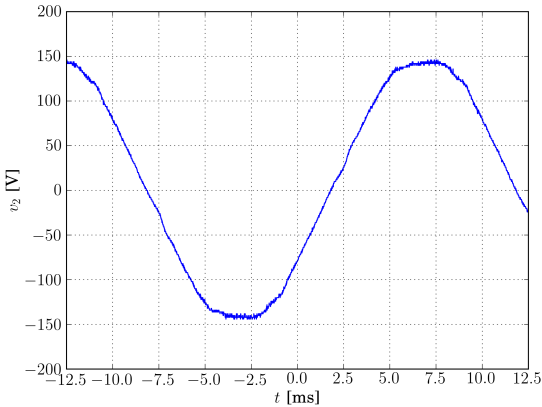
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



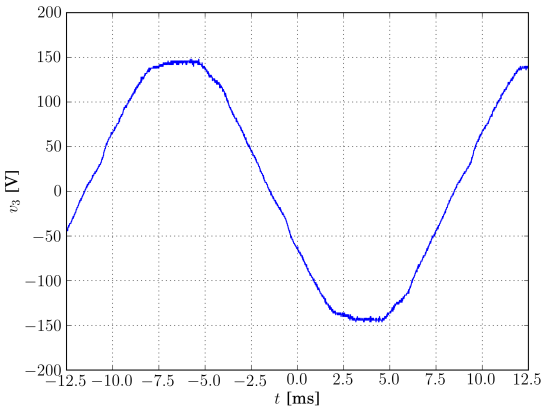
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$

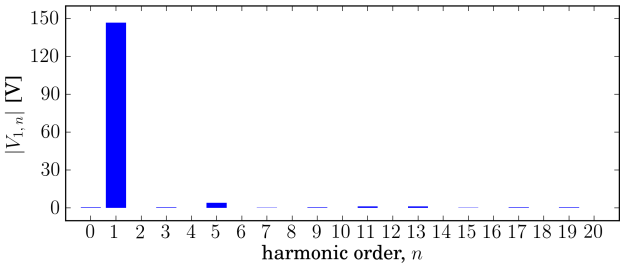


voltages?

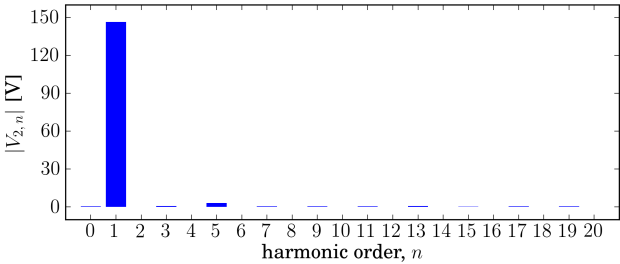
$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



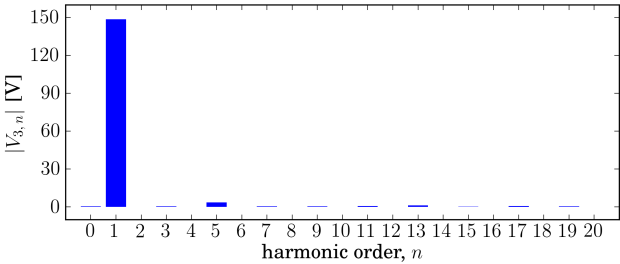
v1, spectrum



v2, spectrum



v3, spectrum



voltages, quantitative characterization

k	$V_{k\,RMS}$	$THD(v_k)$
1	103.83 V	3.34 %
2	103.70 V	2.77 %
3	105.12 V	3.06 %

THD

And what is THD?

$$THD \triangleq \frac{\sqrt{\sum_{k=2}^{\infty} I_{k\,RMS}^2}}{I_{1\,RMS}}$$

Parseval's identity:

$$I_{RMS}^2 = \sum_{k=1}^{\infty} I_{k\,RMS}^2 \quad \text{assumed} \quad I_0 = 0$$

results in

$$THD \triangleq \frac{\sqrt{I_{RMS}^2 - I_{1\,RMS}^2}}{I_{1\,RMS}}$$

simple, but important
computational issues, finite sums ...

normalization of currents and time

$$j_X \triangleq \frac{i_X}{I_{OUT}}$$

unless otherwise noted

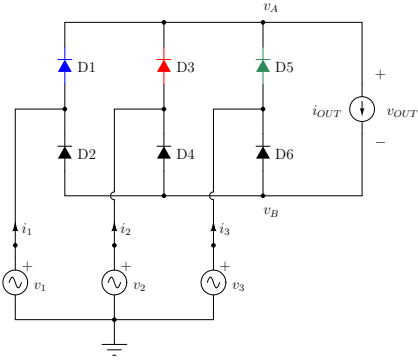
$$\varphi \triangleq \omega_0 t$$

- good:

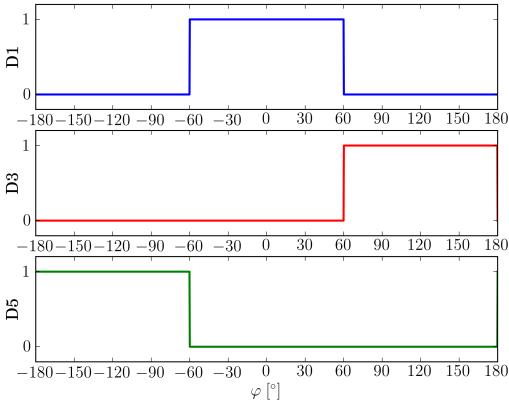
physical dimensions lost, reduced number of variables, results are generalized, core of the problem focused
- bad:

physical dimensions lost, perfect double-check tool is lost

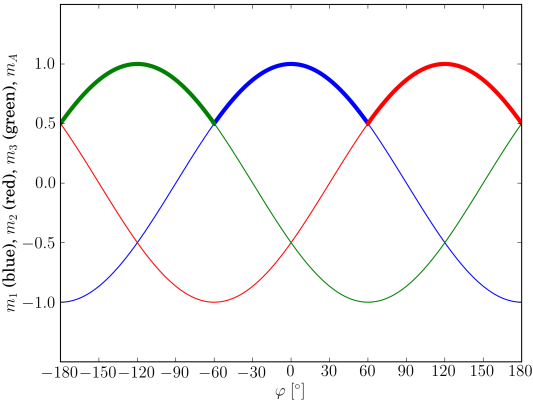
how does it work? part 1: theory



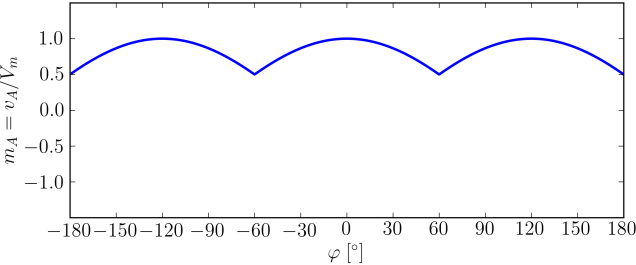
one of the three: D1, D3, D5



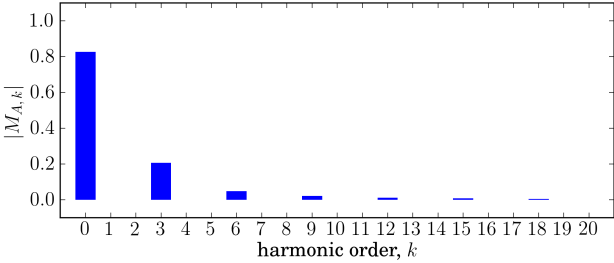
vA



vA, analytical



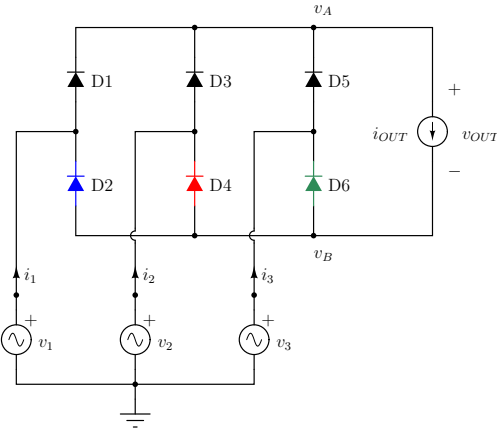
vA, spectrum



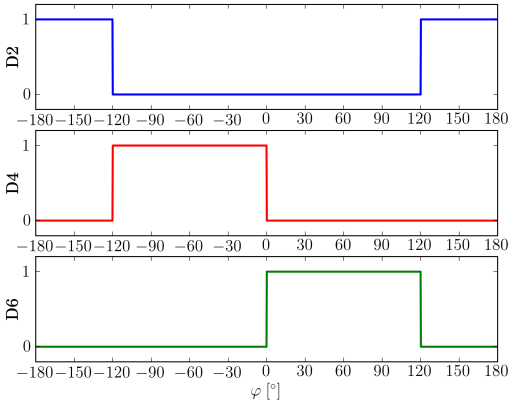
$$m_A = \max(m_1, m_2, m_3)$$

$$m_A = \frac{3\sqrt{3}}{2\pi} \left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{9k^2 - 1} \cos(3k\omega_0 t) \right)$$

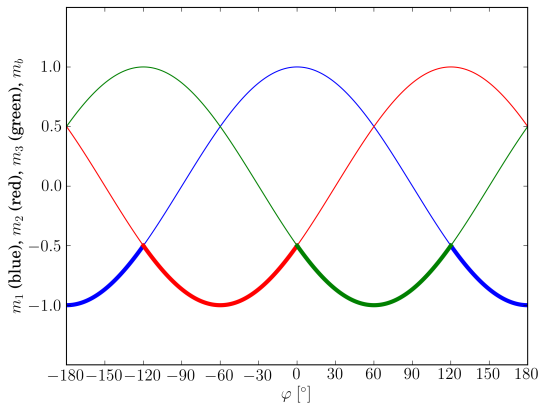
what about vB?



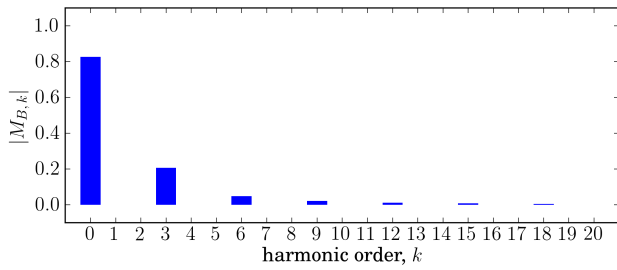
one of the three, again: D2, D4, D6



v_B

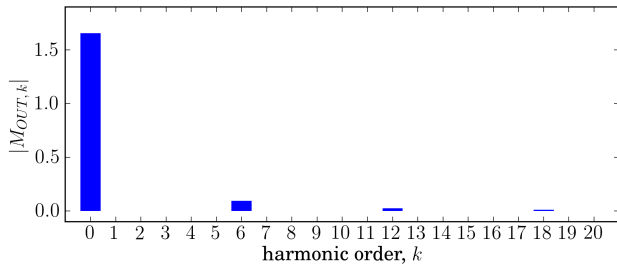


v_B , spectrum



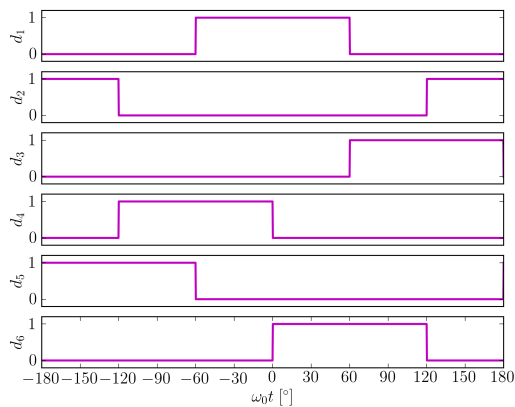
$$m_B = \frac{3\sqrt{3}}{2\pi} \left(-1 + 2 \sum_{k=1}^{\infty} \frac{1}{9k^2 - 1} \cos(3k\omega_0 t) \right)$$

v_{OUT} , spectrum

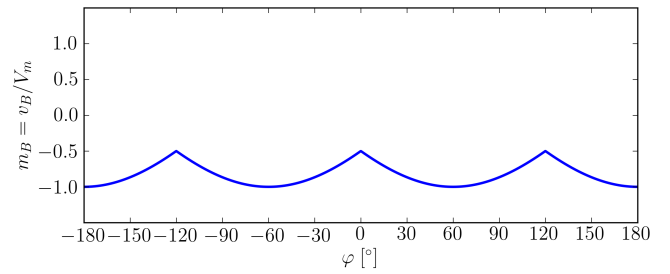


$$m_{OUT} = \frac{3\sqrt{3}}{\pi} \left(1 - 2 \sum_{k=1}^{\infty} \frac{1}{36k^2 - 1} \cos(6k\omega_0 t) \right)$$

states of the diodes

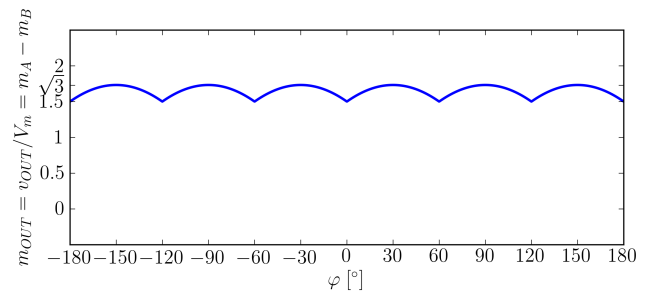


v_B , analytical



$$m_B = \min(m_1, m_2, m_3)$$

the output voltage, v_{OUT}



$$m_{OUT} = m_A - m_B = \max(m_1, m_2, m_3) - \min(m_1, m_2, m_3)$$

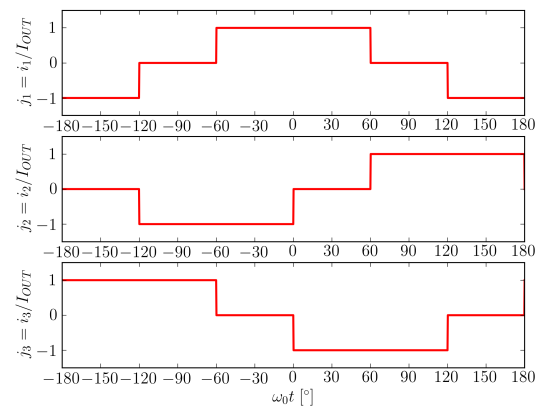
currents?

$$i_1(t) = (d_1(t) - d_2(t)) I_{OUT}$$

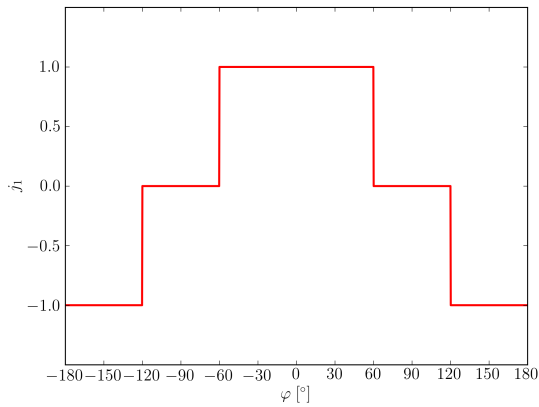
$$i_2(t) = (d_3(t) - d_4(t)) I_{OUT}$$

$$i_3(t) = (d_5(t) - d_6(t)) I_{OUT}$$

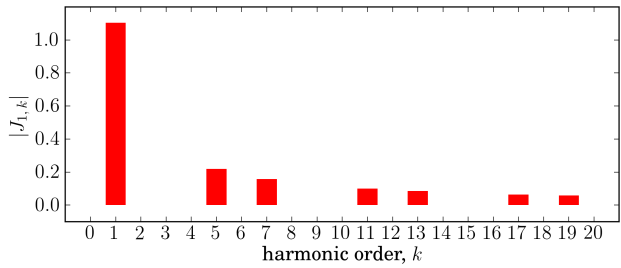
the input currents



consider i_1



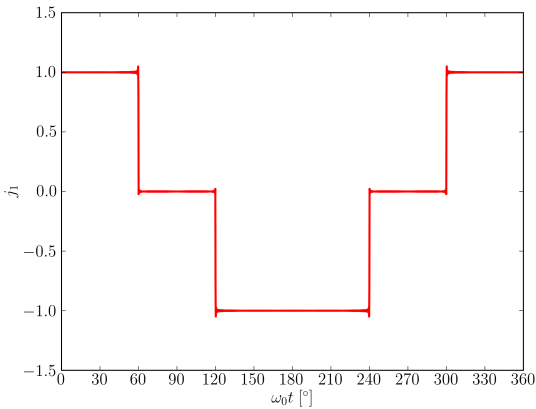
spectra of the input currents



spectra of the input currents, analytical

$$j_1(t) = \sum_{k=1}^{+\infty} J_{1C,k} \cos(k\omega_0 t)$$
$$J_{1C,k} = \frac{2\sqrt{3}}{\pi} \begin{cases} -\frac{1}{k}, & k = 6n - 1 \\ \frac{1}{k}, & k = 6n + 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } n \in \mathbb{N}_0, k > 0$$

numerical verification, Gibbs phenomenon



double-check:

$$P_{IN} = \frac{3}{2} \times 1 \times \frac{2\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi} = P_{OUT}$$

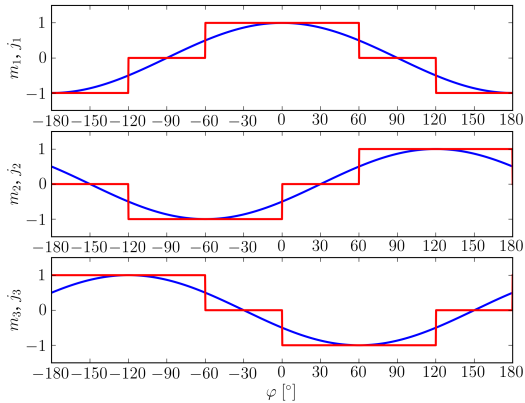
obtained using wxMaxima

THD of the input currents

$$I_{k\,RMS} = \sqrt{\frac{2}{3}} I_{OUT}$$
$$I_{k\,RMS,1} = \frac{\sqrt{6}}{\pi} I_{OUT}$$
$$THD \triangleq \frac{\sqrt{I_{k\,RMS}^2 - I_{k\,RMS,1}^2}}{I_{k\,RMS,1}}$$
$$THD = \sqrt{\frac{\pi^2}{9}} - 1 \approx 31.08\%$$

Parseval's identity based formula turned out to be useful

voltages and currents



some more parameters

$$X_{RMS} \triangleq \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (x(\omega_0 t))^2 d(\omega_0 t)}, \quad x \in \{i, v\}$$

already used for the THD

$$S \triangleq I_{RMS} V_{RMS}$$
$$P \triangleq \frac{1}{2\pi} \int_0^{2\pi} v(\omega_0 t) i(\omega_0 t) d(\omega_0 t)$$
$$PF \triangleq \frac{P}{S}$$
$$DPF \triangleq \cos \phi_1$$

and ϕ_1 is ...

and if the voltages are sinusoidal ...

$$S = V_{RMS} I_{RMS}$$
$$P = V_{RMS} I_{1,RMS} \cos \phi_1$$
$$PF = \frac{P}{S} = \frac{I_{1,RMS}}{I_{RMS}} \cos \phi_1 = \frac{I_{1,RMS}}{I_{RMS}} DPF$$
$$DPF = \cos \phi_1$$
$$THD = \frac{\sqrt{I_{RMS}^2 - I_{1,RMS}^2}}{I_{1,RMS}} = \sqrt{\left(\frac{I_{RMS}}{I_{1,RMS}}\right)^2 - 1}$$

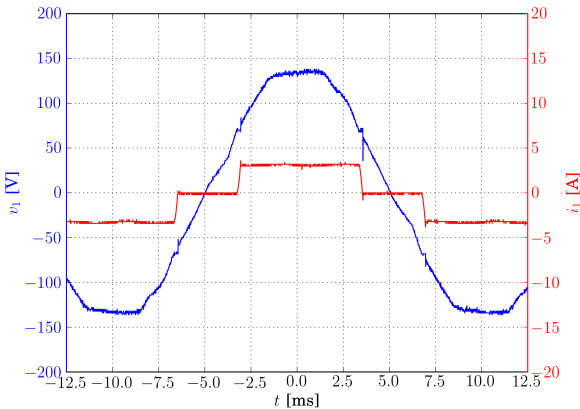
i.e. everything depends on the current waveform and its position

some more parameters, plain rectifier

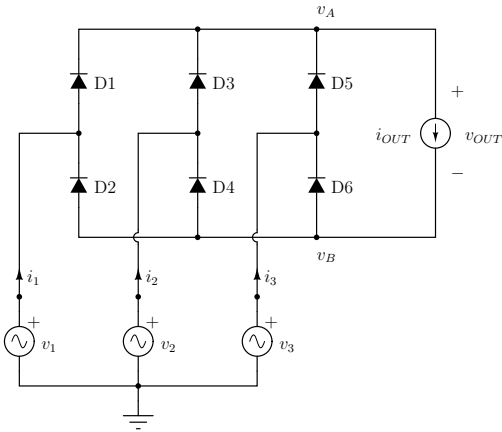
$$I_{k\,RMS} = \sqrt{\frac{2}{3}}\, I_{OUT} \quad V_{k\,RMS} = \frac{1}{\sqrt{2}}\, V_m$$
$$S = 3 \times \sqrt{\frac{2}{3}}\, I_{OUT} \times \frac{1}{\sqrt{2}}\, V_m = \sqrt{3}\, V_m\, I_{OUT}$$
$$P = V_{OUT}\, I_{OUT} = \frac{3\sqrt{3}}{\pi}\, V_m\, I_{OUT}$$
$$PF = \frac{3}{\pi} \approx 95.5\%$$
$$DPF = 1$$

actually, not so bad; *THD* is the problem

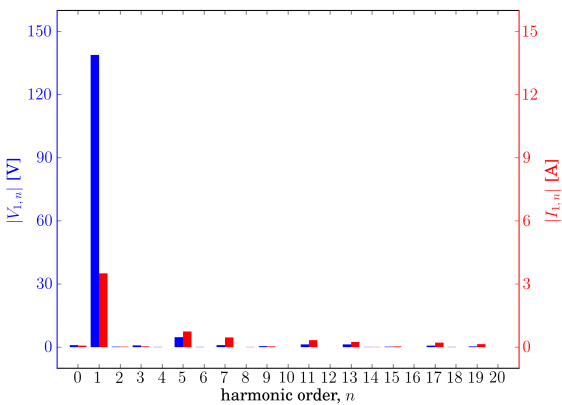
input, at $I_{OUT} = 3\text{ A}$



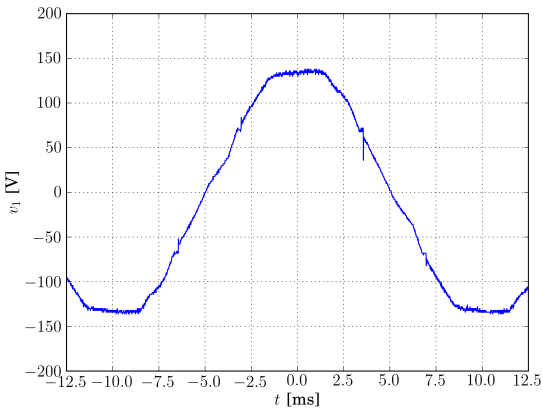
back to the rectifier:
how does it work? part 2: experiment



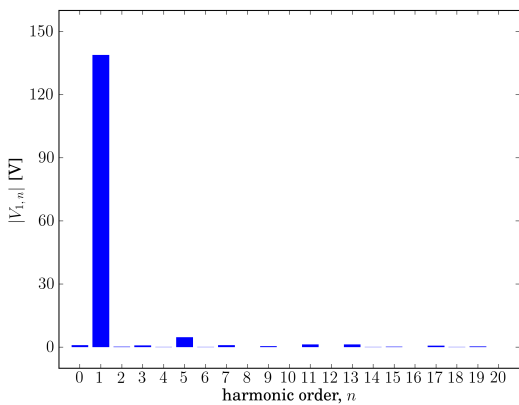
input, at $I_{OUT} = 3\text{ A}$



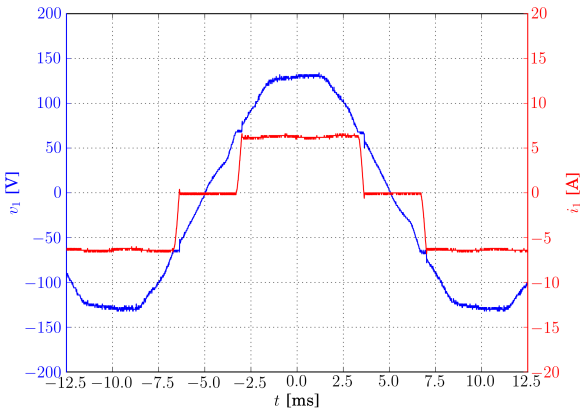
input, at $I_{OUT} = 3\text{ A}$



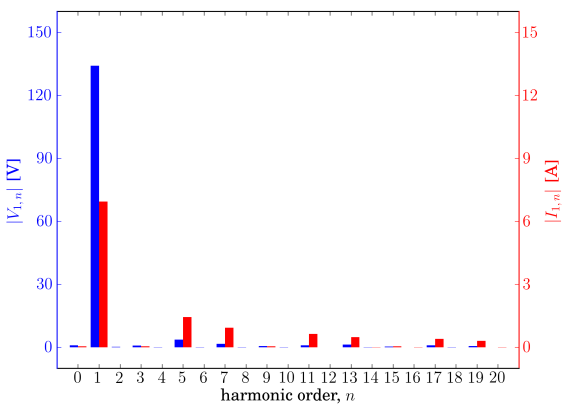
input, at $I_{OUT} = 3\text{ A}$



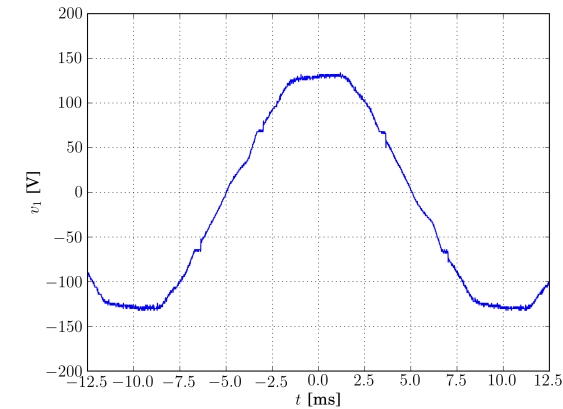
input, at $I_{OUT} = 6\text{ A}$



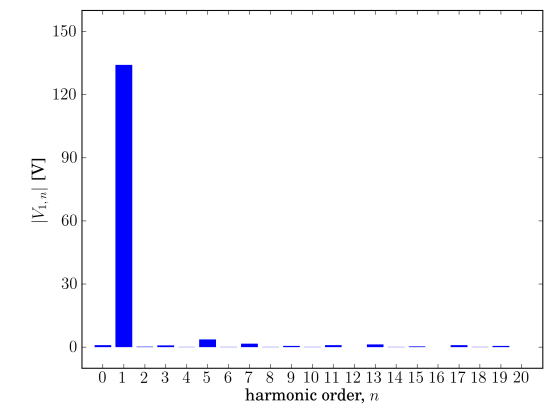
input, at $I_{OUT} = 6\text{ A}$



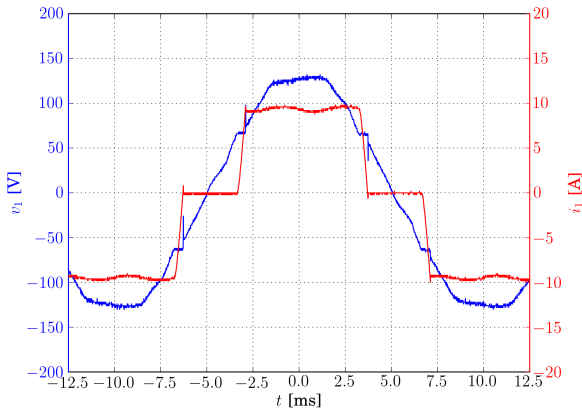
input, at $I_{OUT} = 6\text{ A}$



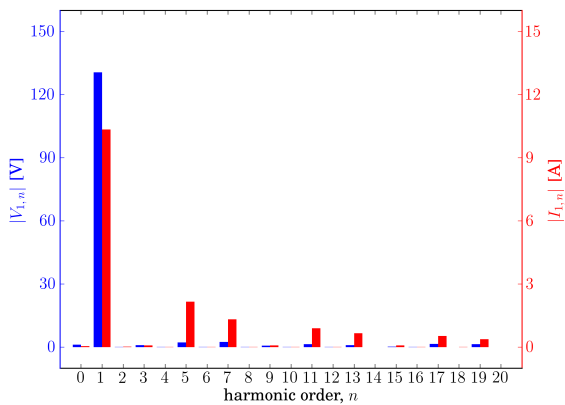
input, at $I_{OUT} = 6\text{ A}$



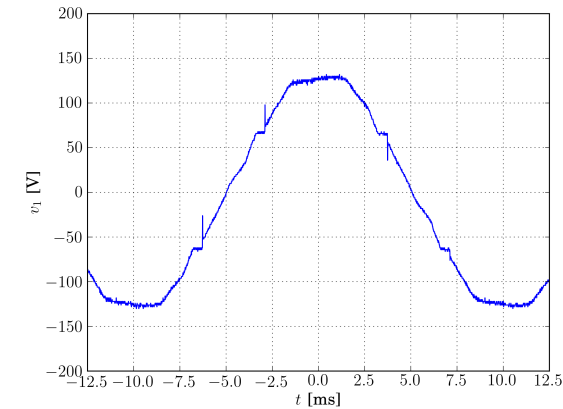
input, at $I_{OUT} = 9\text{ A}$



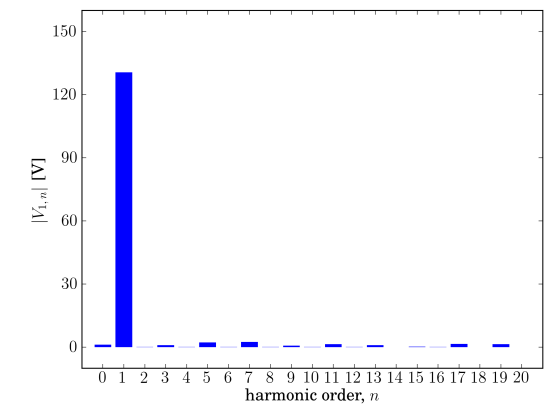
input, at $I_{OUT} = 9\text{ A}$



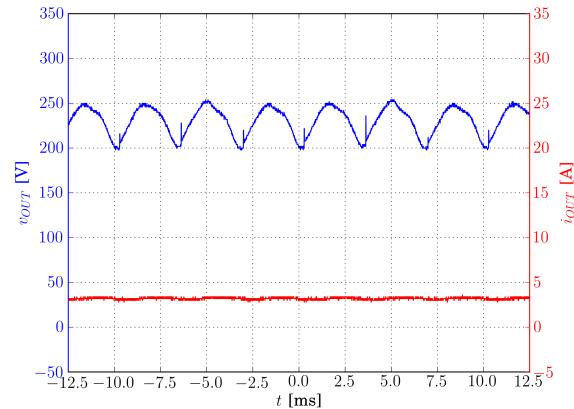
input, at $I_{OUT} = 9\text{ A}$



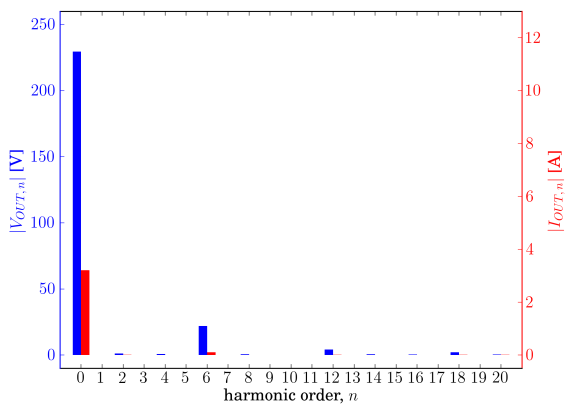
input, at $I_{OUT} = 9\text{ A}$



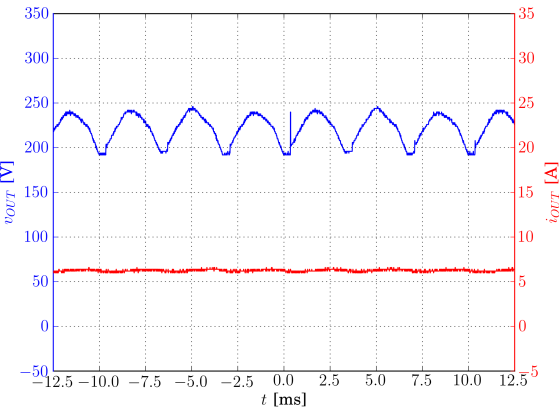
output, at $I_{OUT} = 3\text{ A}$



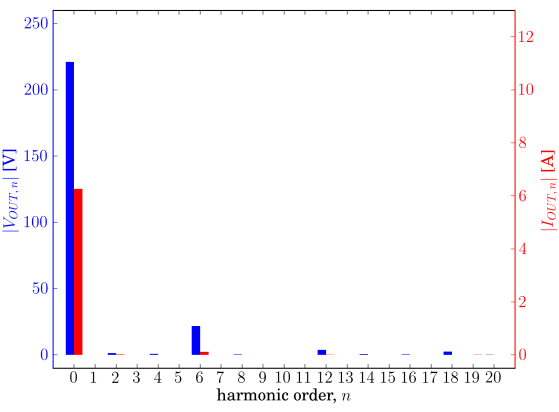
output, at $I_{OUT} = 3\text{ A}$



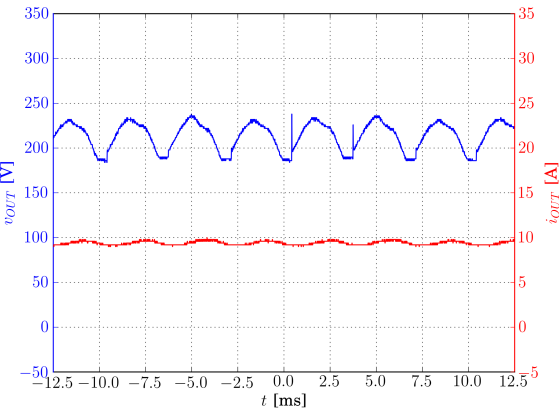
output, at $I_{OUT} = 6\text{ A}$



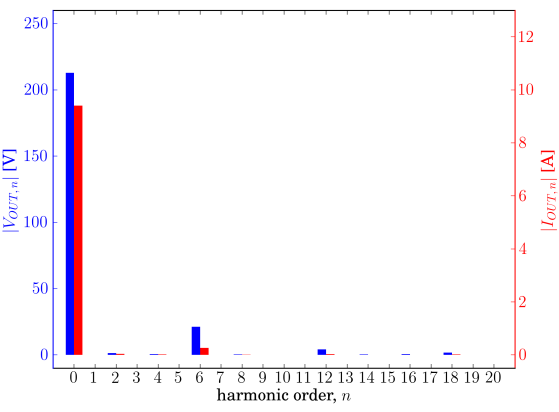
output, at $I_{OUT} = 6\text{ A}$



output, at $I_{OUT} = 9\text{ A}$



output, at $I_{OUT} = 9\text{ A}$



in quantitative terms, input, 1st

I_{OUT}	k	$I_{k\,RMS}$ [A]	$V_{k\,RMS}$ [V]	S_k [VA]	P_k [W]
0 A	1	—	101.29	—	—
	2	—	100.63	—	—
	3	—	102.40	—	—
3 A	1	2.60	98.23	255.01	245.16
	2	2.61	97.73	254.60	244.37
	3	2.63	98.82	259.71	251.00
6 A	1	5.12	94.87	485.41	466.87
	2	5.12	94.34	482.67	464.25
	3	5.13	96.80	496.95	477.08
9 A	1	7.59	92.38	701.53	673.86
	2	7.64	91.95	702.47	675.16
	3	7.66	94.04	720.00	692.30

in quantitative terms, input, 2nd

I_{OUT}	k	PF_k	$THD(i_k)$ [%]	$THD(v_k)$ [%]
0 A	1	—	—	4.33
	2	—	—	3.75
	3	—	—	4.75
3 A	1	0.9614	30.50	4.17
	2	0.9598	29.57	3.86
	3	0.9665	29.97	5.38
6 A	1	0.9618	29.26	3.87
	2	0.9618	28.37	3.66
	3	0.9600	28.31	3.87
9 A	1	0.9605	28.00	4.01
	2	0.9611	27.21	3.92
	3	0.9615	27.06	4.19

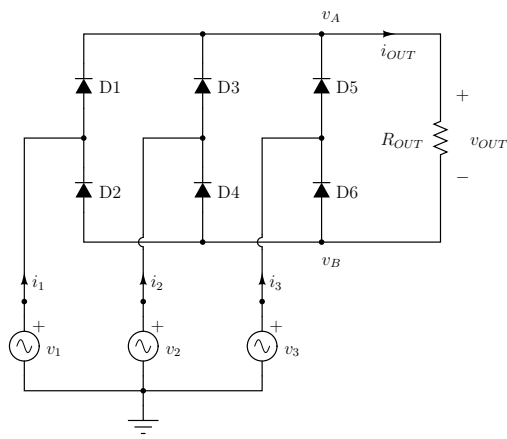
in quantitative terms, output

I_{OUT} [A]	V_{OUT} [V]	P_{OUT} [W]	P_{IN} [W]	η [%]
0.00	239.79	1.07	−0.81	—
3.21	229.51	736.72	740.53	99.49
6.27	221.23	1386.56	1408.20	98.46
9.41	212.91	2004.12	2041.32	98.18

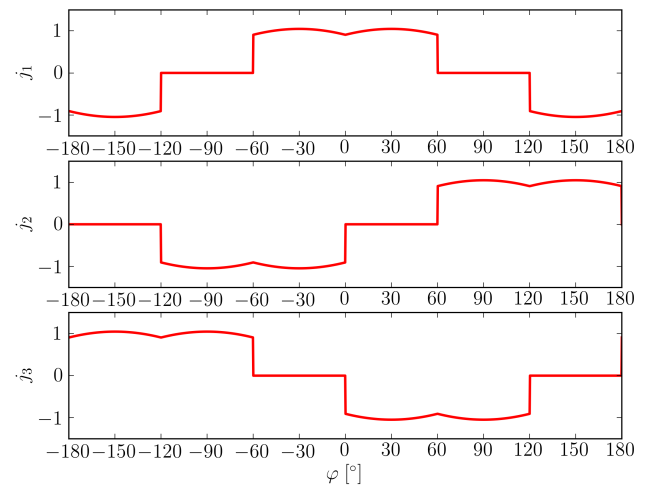
overall impressions

- ▶ pretty good rectifier
- ▶ simple, robust, cheap
- ▶ good symmetry
- ▶ excellent DPF
- ▶ acceptable PF
- ▶ poor THD (but not that poor)
- ▶ up to this point:
 - ▶ diode bridge rectifier analyzed
 - ▶ measurement tools developed
- ▶ is there a way to do something with the THD ?

fruitless effort #1: shaping the output current



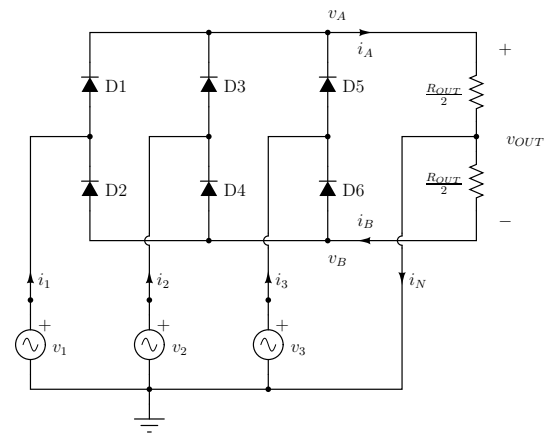
fruitless effort #1: waveforms



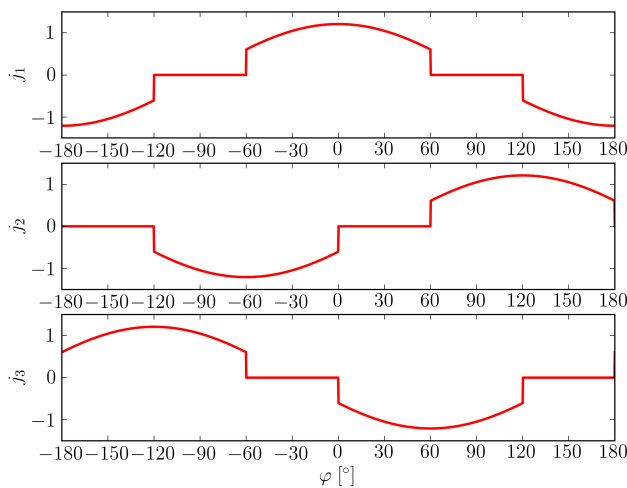
fruitless effort #1: quantitative

- ▶ $THD = 30.79\%$
- ▶ not a big deal of an improvement
- ▶ only one degree of freedom, i_{OUT}
- ▶ shaping i_1 , i_2 , and i_3 is the goal
- ▶ two degrees of freedom needed, since $i_1 + i_2 + i_3 = 0$

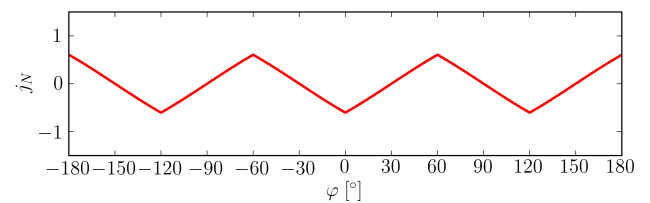
fruitless effort #2: additional degree of freedom



fruitless effort #2: waveforms



fruitless effort #2: neutral current



fruitless effort #2: quantitative

- ▶ $THD = 24.76\%$
- ▶ somewhat better
- ▶ all of i_1 , i_2 , and i_3 cannot be fixed by programming i_A and i_B in this circuit
- ▶ example: $i_1 = i_A$, $i_2 = -i_B$, no way to fix i_3
- ▶ gaps in the input currents in both of the “patches”
- ▶ the additional degree of freedom is taken by i_N
- ▶ which is a disaster of itself
- ▶ we would need another degree of freedom to fix i_N
- ▶ but this is a wrong approach, i_N was not an issue before

conclusions

- ▶ three-phase diode bridge rectifier analyzed
- ▶ quantitative measures of rectifier performance introduced
- ▶ measurement tools developed
- ▶ theoretical predictions related to experiments
- ▶ gaps in the input currents identified as a problem
- ▶ **how to fill in the gaps?**
- ▶ an answer is current injection ...