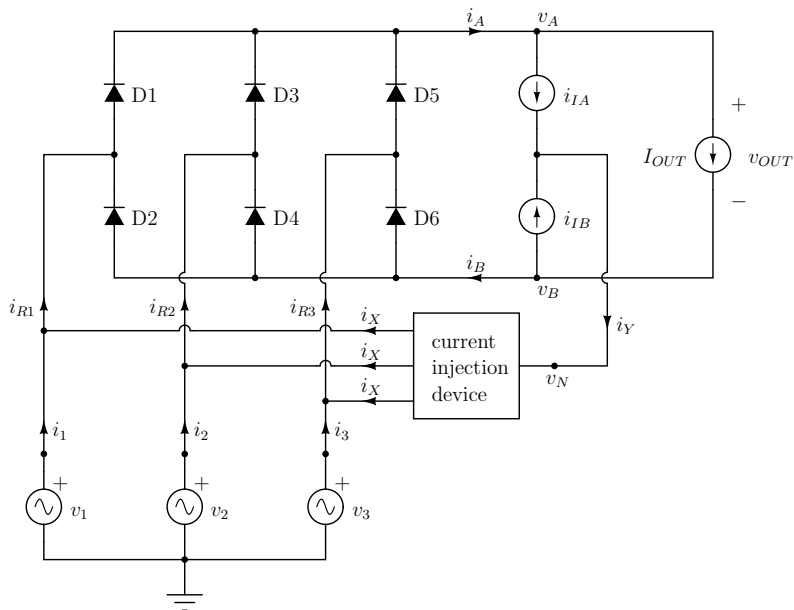


The Third Harmonic Current Injection

how to improve the THD?

1. patch the gaps in the input currents
2. do some shaping

what is this all about?



current injection device

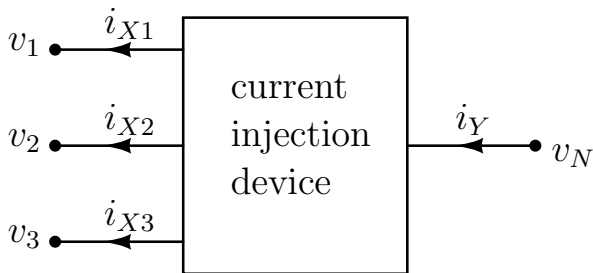
a magnetic device that provides:

$$i_X = \frac{1}{3} i_Y$$

$$v_N = \frac{1}{3} (v_1 + v_2 + v_3) = 0$$

- ▶ patching even where not needed ...
- ▶ more to be said about the device ...
- ▶ hard to put parallel events into a sequence ...
- ▶ but, let's take a closer look ...

let's complicate a little bit



some circuit theory ...

8 variables introduced

4 element equations needed

the remaining 4 will be covered by KCL and KVL

$$i_{X1} = \frac{1}{3} i_Y$$

$$i_{X2} = \frac{1}{3} i_Y$$

$$i_{X3} = \frac{1}{3} i_Y$$

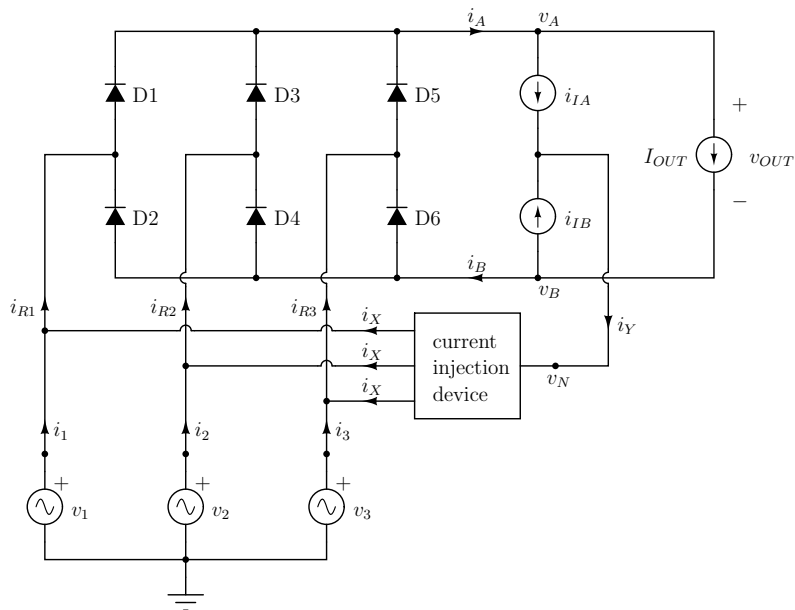
$$v_N = \frac{1}{3} (v_1 + v_2 + v_3)$$

the device is **resistive**, please remember

$$v_N i_Y - v_1 i_1 - v_2 i_2 - v_3 i_3 = 0$$

the device is **non-dissipative**, please remember this

back to the circuit ...



some equations, KCL, ...

$$i_k = i_{Rk} - i_X, \quad k \in \{1, 2, 3\}$$

$$i_X = \frac{1}{3} i_Y$$

$$i_{IA} = i_{IB} = \frac{1}{2} i_Y$$

$$i_A = I_{OUT} + i_{IA} = I_{OUT} + \frac{1}{2} i_Y$$

$$i_B = I_{OUT} - i_{IB} = I_{OUT} - \frac{1}{2} i_Y$$

some equations normalized

$$j_X \triangleq \frac{i_X}{I_{OUT}}$$

$$j_k = j_{Rk} - j_X, \quad k \in \{1, 2, 3\}$$

$$j_X = \frac{1}{3} j_Y$$

$$j_{IA} = j_{IB} = \frac{1}{2} j_Y$$

$$j_A = 1 + j_{IA} = 1 + \frac{1}{2} j_Y$$

$$j_B = 1 - j_{IB} = 1 - \frac{1}{2} j_Y$$

j_Y limitations

for the continuous conduction mode (CCM):

$$j_A > 0 \quad \Rightarrow \quad 1 + \frac{1}{2} j_Y > 0$$

$$j_B > 0 \quad \Rightarrow \quad 1 - \frac{1}{2} j_Y > 0$$

$$-2 < j_Y < 2$$

or

$$-2 I_{OUT} < i_Y < 2 I_{OUT}$$

how to get j_{Rk} , $k \in \{1, 2, 3\}$?

$$j_{R1} = d_1 j_A - d_2 j_B$$

$$j_{R2} = d_3 j_A - d_4 j_B$$

$$j_{R3} = d_5 j_A - d_6 j_B$$

or, in general terms

$$j_{Rk} = d_{2k-1} j_A - d_{2k} j_B$$

for

$$k \in \{1, 2, 3\}$$

the input currents

plugging in expressions for i_{Rk} and i_X

$$j_k = d_{2k-1} \left(1 + \frac{1}{2} j_Y \right) - d_{2k} \left(1 - \frac{1}{2} j_Y \right) - \frac{1}{3} j_Y$$

there are three cases to be considered:

$$j_k = \begin{cases} 1 + \frac{1}{6} j_Y & \text{for } d_{2k-1} = 1 \text{ and } d_{2k} = 0 \\ -1 + \frac{1}{6} j_Y & \text{for } d_{2k-1} = 0 \text{ and } d_{2k} = 1 \\ -\frac{1}{3} j_Y & \text{for } d_{2k-1} = 0 \text{ and } d_{2k} = 0 \end{cases}$$

some symmetry

since

$$d_1(\omega_0 t) = d_3\left(\omega_0 t + \frac{2\pi}{3}\right) = d_5\left(\omega_0 t + \frac{4\pi}{3}\right)$$

$$d_2(\omega_0 t) = d_4\left(\omega_0 t + \frac{2\pi}{3}\right) = d_6\left(\omega_0 t + \frac{4\pi}{3}\right)$$

to have

$$j_1(\omega_0 t) = j_2\left(\omega_0 t + \frac{2\pi}{3}\right) = j_3\left(\omega_0 t + \frac{4\pi}{3}\right)$$

we need

$$j_Y(\omega_0 t) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right) = j_Y\left(\omega_0 t + \frac{4\pi}{3}\right)$$

more about j_Y

to get

$$j_Y(\omega_0 t) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right) = j_Y\left(\omega_0 t + \frac{4\pi}{3}\right)$$

we need

$$j_Y(\omega_0 t) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right)$$

meaning that j_Y is periodic with $\frac{1}{3} T_0$, i.e. the fundamental frequency of i_Y is $3 f_0$!

thus, **the third-harmonic current injection** is the simplest case

let's assume i_Y

$$i_Y = k I_{OUT} \cos(3\omega_0 t - \phi)$$

$$\boxed{j_Y = k \cos(3\omega_0 t - \phi)}$$

range:

1. for the continuous conduction mode $-2 < k < 2$
2. let us assume $-\pi < \phi < \pi$

which couple (k, ϕ) minimizes the THD ?

optimization over k and ϕ

$$j_1 = \begin{cases} 1 + \frac{1}{6} j_Y & \text{for } d_1 = 1 \text{ and } d_2 = 0 \\ -1 + \frac{1}{6} j_Y & \text{for } d_1 = 0 \text{ and } d_2 = 1 \\ -\frac{1}{3} j_Y & \text{for } d_1 = 0 \text{ and } d_2 = 0 \end{cases}$$

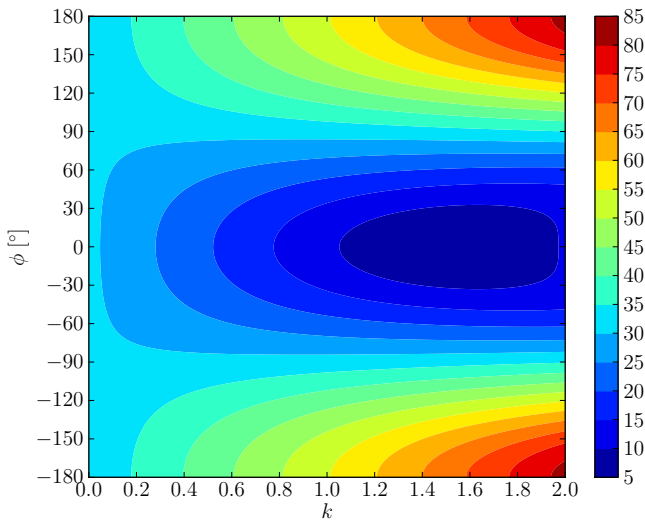
and

$$j_Y = k \cos(3\omega_0 t - \phi)$$

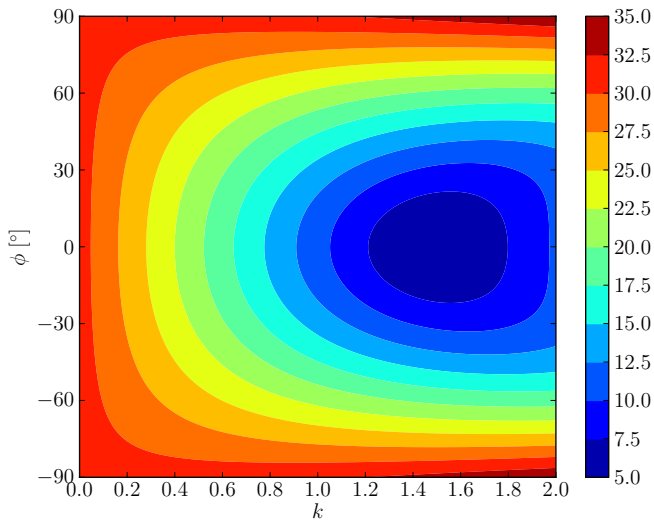
let's go to work!

goal: minimize the THD

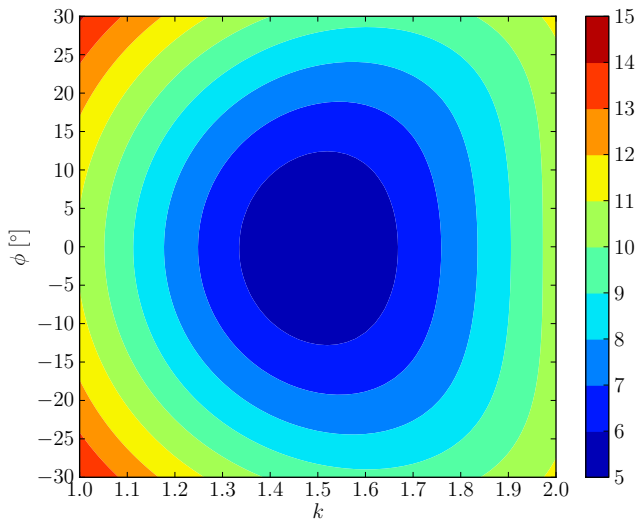
numerical optimization, 1, THD [%]



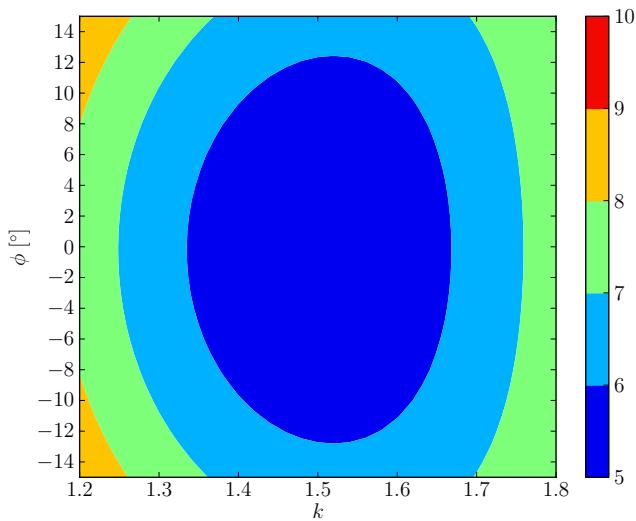
numerical optimization, 2, THD [%]



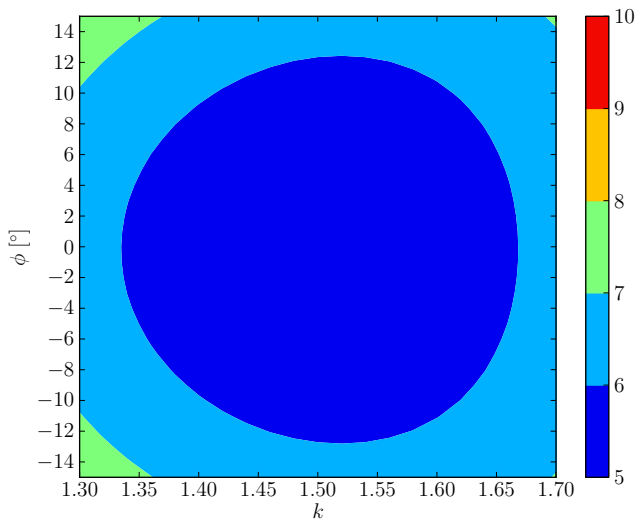
numerical optimization, 3, THD [%]



numerical optimization, 4, THD [%]



numerical optimization, 5, THD [%]



numerical optimization, conclusions

- ▶ improvement possible!
- ▶ $5\% < THD < 6\%$ achievable
- ▶ $k \approx 1.5$, $\phi \approx 0$
- ▶ it is promising to continue
- ▶ analytical optimization?
- ▶ circuit?
- ▶ **let's continue ...**

analytical optimization, start

$$j_1 = \begin{cases} 1 + \frac{1}{6} j_Y & \text{for } d_1 = 1 \text{ and } d_2 = 0 \\ -1 + \frac{1}{6} j_Y & \text{for } d_1 = 0 \text{ and } d_2 = 1 \\ -\frac{1}{3} j_Y & \text{for } d_1 = 0 \text{ and } d_2 = 0 \end{cases}$$

and

$$j_Y = k \cos(3\omega_0 t - \phi)$$

analytical optimization, rms and the fundamental

after lots of work (fortunately, wxMaxima's):

$$J_{RMS} = \frac{\sqrt{k^2 + 24}}{6}$$

$$J_{1Cm} = \frac{3 k \cos \phi + 48}{8 \sqrt{3} \pi}$$

$$J_{1Sm} = \frac{3 \sqrt{3} k \sin \phi}{8 \pi}$$

$$J_{1RMS} = \sqrt{\frac{J_{1Cm}^2 + J_{1Sm}^2}{2}}$$

$$J_{1RMS} = \frac{\sqrt{768 + 27 k^2 + 96 k \cos \phi - 24 k^2 (\cos \phi)^2}}{8 \sqrt{2} \pi}$$

analytical optimization, make it simple

$$THD = \sqrt{\left(\frac{J_{RMS}}{J_{1RMS}}\right)^2 - 1}$$

make it simple: to minimize THD is to minimize

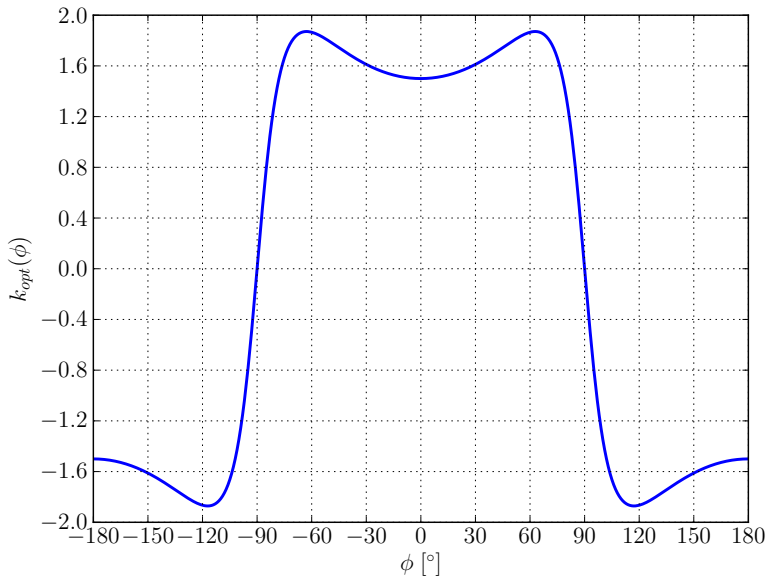
$$y(k, \phi) \triangleq \left(\frac{J_{RMS}}{J_{1RMS}}\right)^2$$

$$y(k, \phi) = \frac{2\pi^2 (16k^2 + 384\pi^2)}{6912 + 243k^2 + 864k \cos \phi - 216k^2 (\cos \phi)^2}$$

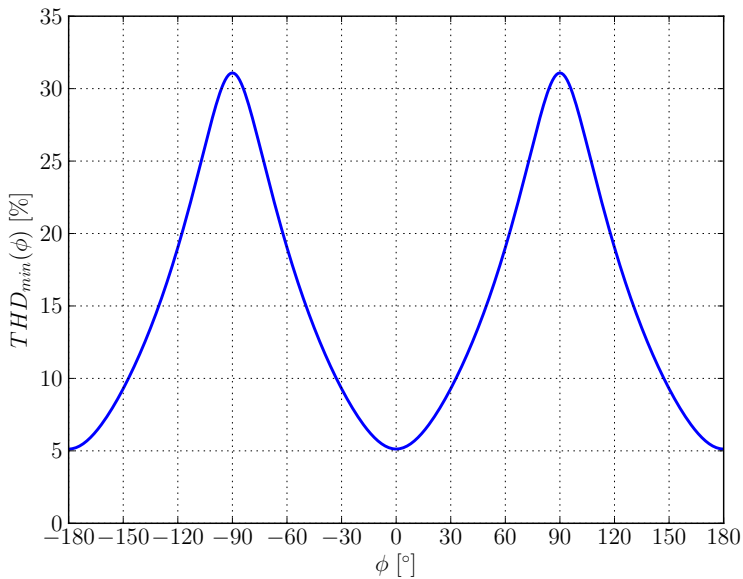
analytical optimization, let's fix ϕ

$$\frac{\partial y(k, \phi)}{\partial k} = 0$$

$$k_{opt}(\phi) = \frac{\sqrt{576 (\cos \phi)^4 + 624 (\cos \phi)^2 + 25} - 24 (\cos \phi)^2 - 5}{4 \cos \phi}$$

$k_{opt}(\phi)$ 

$$THD(k_{opt}(\phi), \phi)$$



$$THD_{MIN}, k_{OPT}, \phi_{OPT}$$

$$THD_{MIN} = \min (THD (k_{opt} (\phi) , \phi)) = THD (k_{OPT}, \phi_{OPT})$$

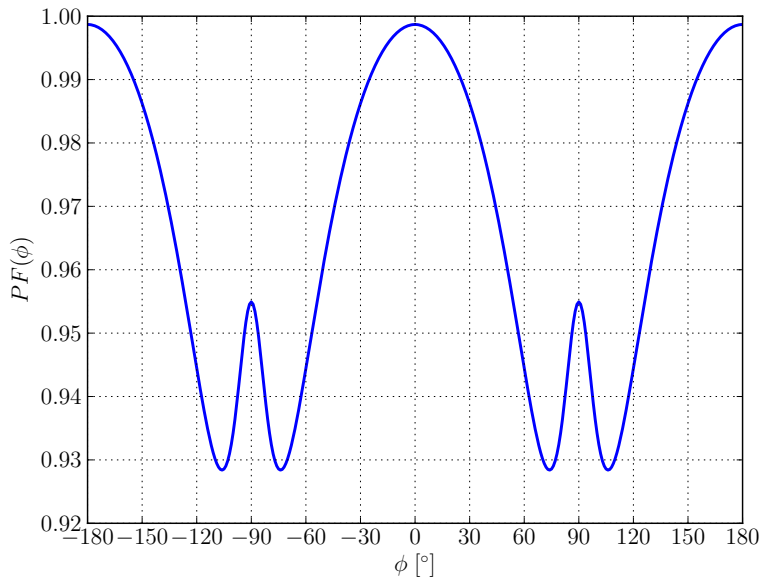
$$\phi_{OPT} = 0$$

$$k_{OPT} = k_{opt}(0) = \frac{3}{2}$$

$$THD_{MIN} = \sqrt{\frac{32 \pi^2}{315}} - 1 \approx 5.12 \%$$

and that's all the third harmonic current injection can do

$$PF(k_{opt}(\phi), \phi)$$



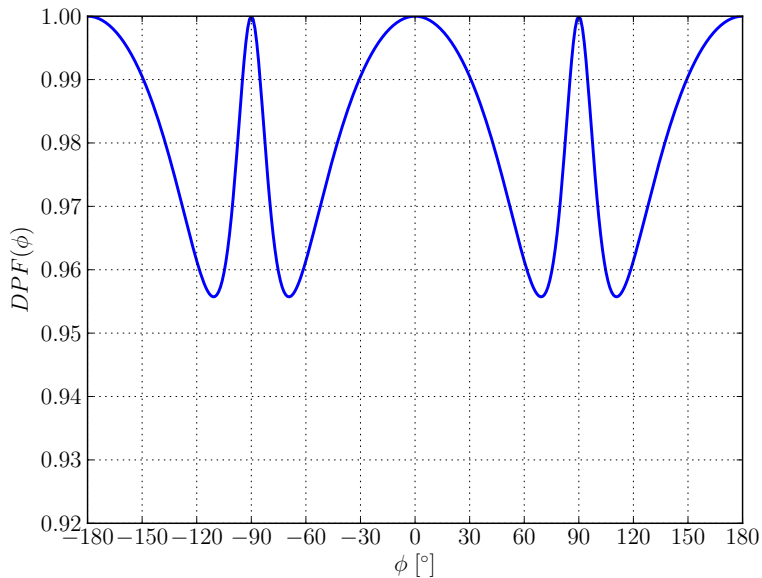
$$PF(k_{OPT}, \phi_{OPT})$$

$$PF(k, \phi) = \frac{\sqrt{6} (3k \cos \phi + 48)}{8\pi \sqrt{k^2 + 24}}$$

$$PF\left(\frac{3}{2}, 0\right) = \frac{3\sqrt{70}}{8\pi} \approx 0.9987$$

we could have optimized $PF(k, \phi) \dots$

$$DPF(k_{opt}(\phi), \phi)$$

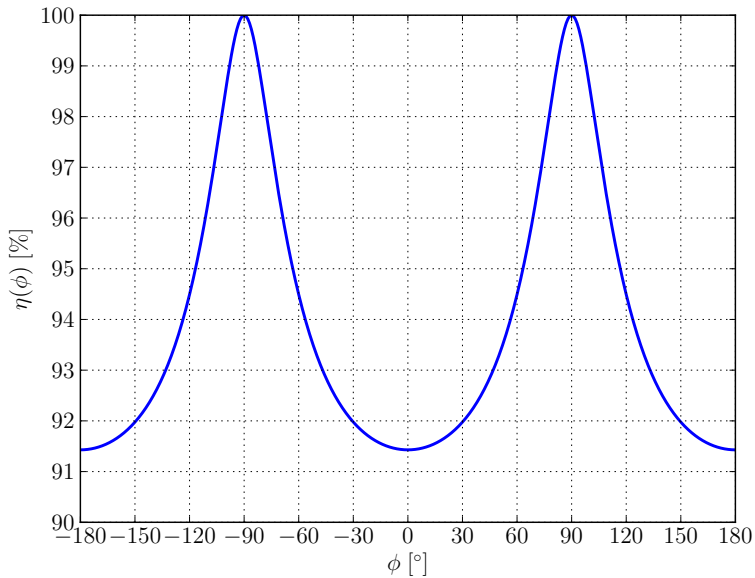


$$DPF(k_{OPT}, \phi_{OPT})$$

$$DPF(k, \phi) = \frac{\sqrt{3}(k \cos \phi + 16)}{\sqrt{768 + 27k^2 + 96k \cos \phi - 24k^2 (\cos \phi)^2}}$$

$$\boxed{DPF\left(\frac{3}{2}, 0\right) = 1}$$

$$\eta(k_{opt}(\phi), \phi)$$



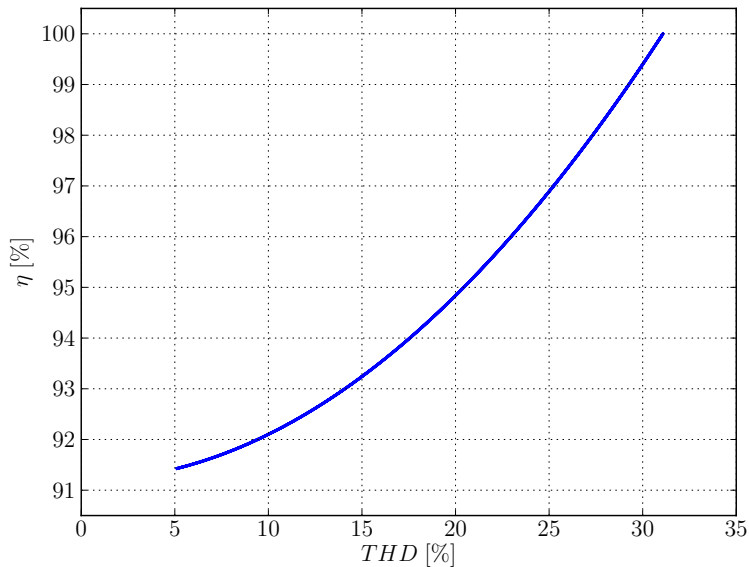
$$\eta(k_{OPT}, \phi_{OPT})$$

$$\eta(k, \phi) = \frac{48}{3k \cos \phi + 48}$$

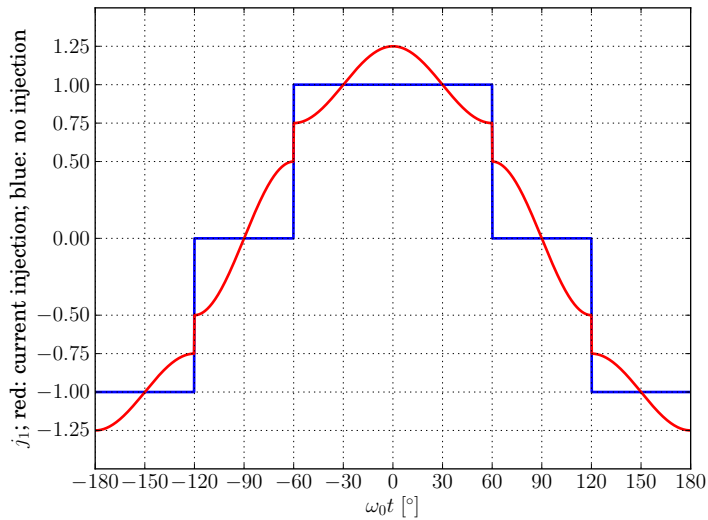
$$\eta\left(\frac{3}{2}, 0\right) = \frac{32}{35} \approx 91.43\%$$

optimizing $\eta(k, \phi)$?

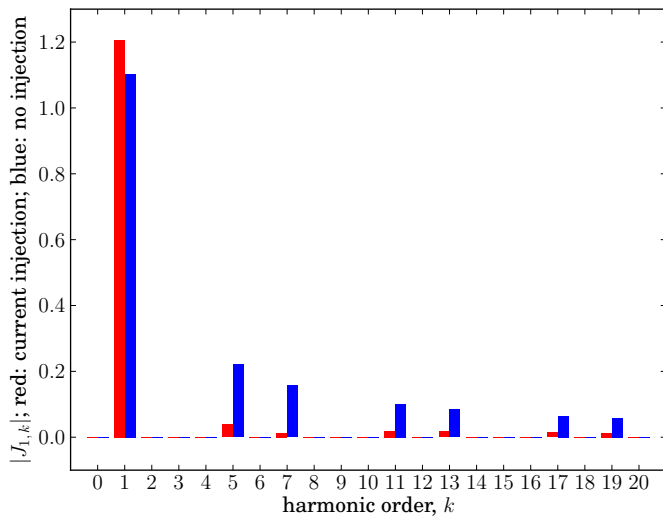
$\eta(THD)$, the tradeoff



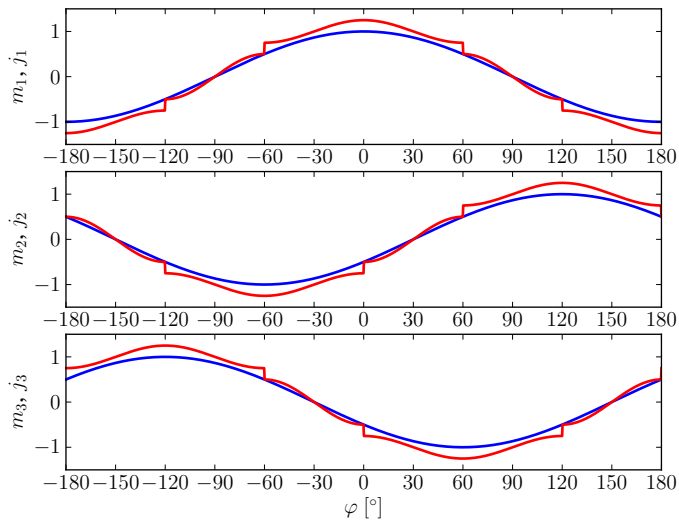
j_1 , with and without injection



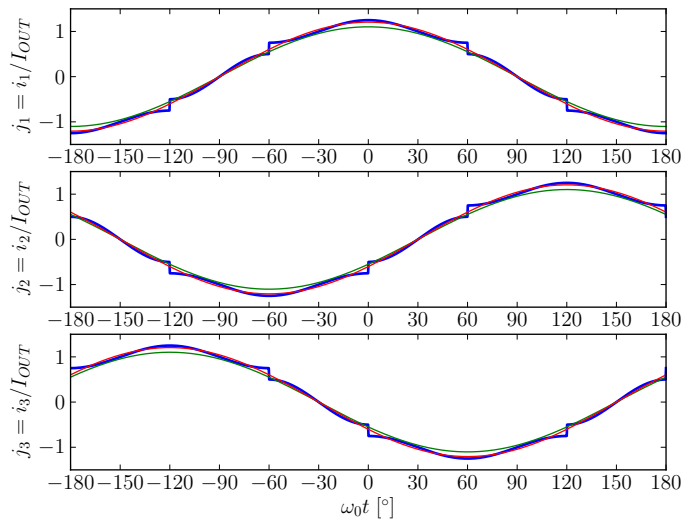
$|J_{1,k}|$, with and without injection



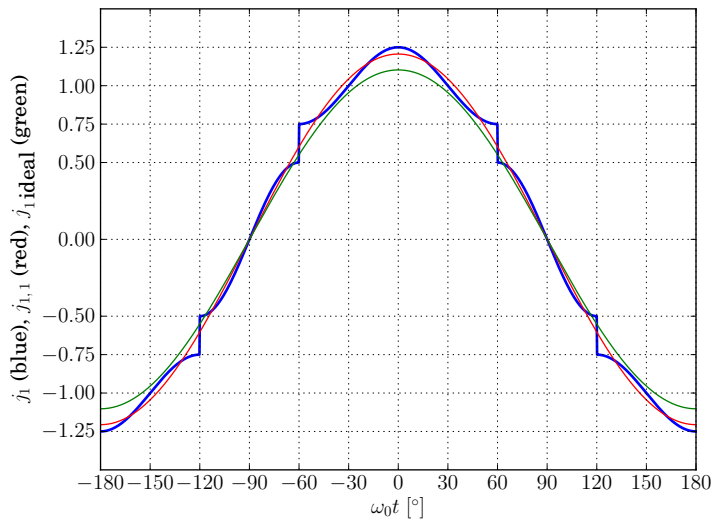
m_k and j_k , $k \in 1, 2, 3$



j_k , with ideal waveforms



j_k , with ideal waveforms, a closer look



increase in the amplitude

$$J_{1m} = \frac{35\sqrt{3}}{16\pi} \approx 1.2060$$

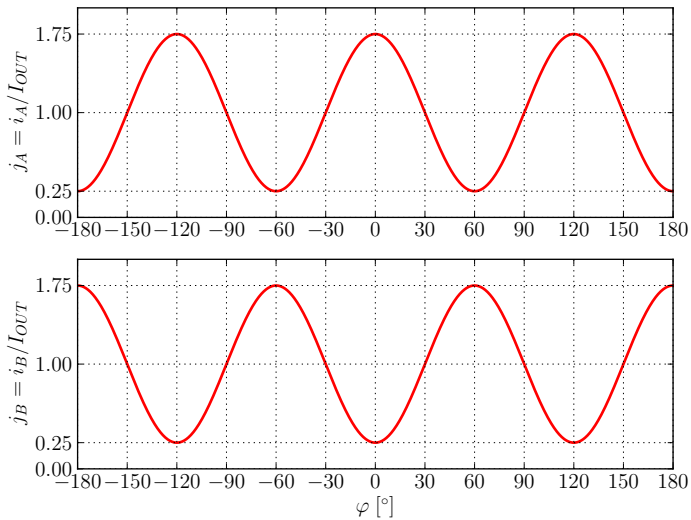
$$J_{1m\,ideal} = \frac{2\sqrt{3}}{\pi} \approx 1.1027$$

$$J_{RMS} = \frac{\sqrt{105}}{12} \approx 0.8539$$

$$J_{1RMS} = \frac{35\sqrt{6}}{32\pi} \approx 0.8528$$

$$J_{1RMS\,ideal} = \frac{\sqrt{6}}{\pi} \approx 0.77970$$

j_A and j_B



j_A and j_B , mean and rms

$$J_A = \overline{j_A} = \langle j_A \rangle = 1$$

$$J_B = \overline{j_B} = \langle j_B \rangle = 1$$

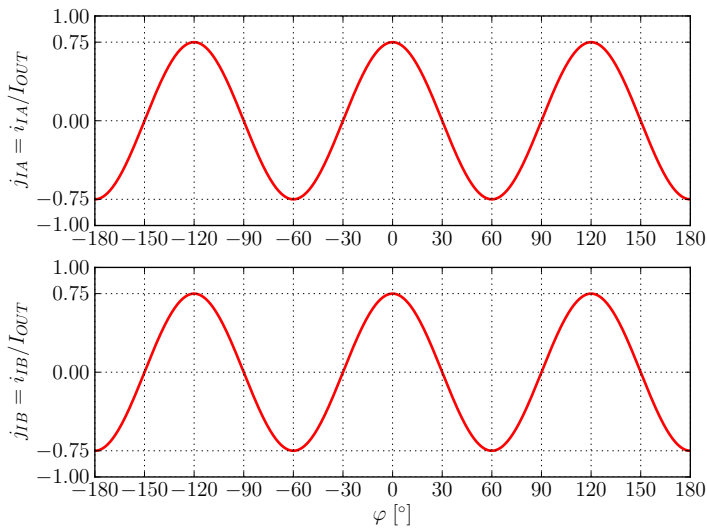
as it was before, but

$$J_{A\,RMS} = \sqrt{\frac{41}{32}} \approx 1.1319 > 1$$

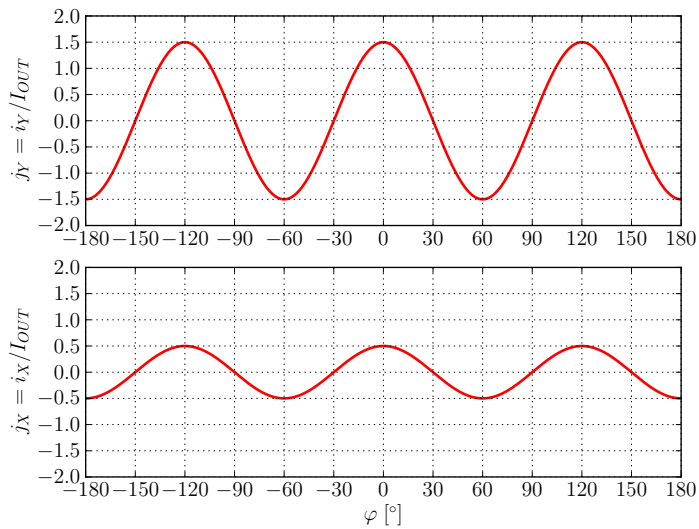
$$J_{B\,RMS} = \sqrt{\frac{41}{32}} \approx 1.1319 > 1$$

$v_D = V_D + R_D i_D$, somewhat increased losses in the diodes

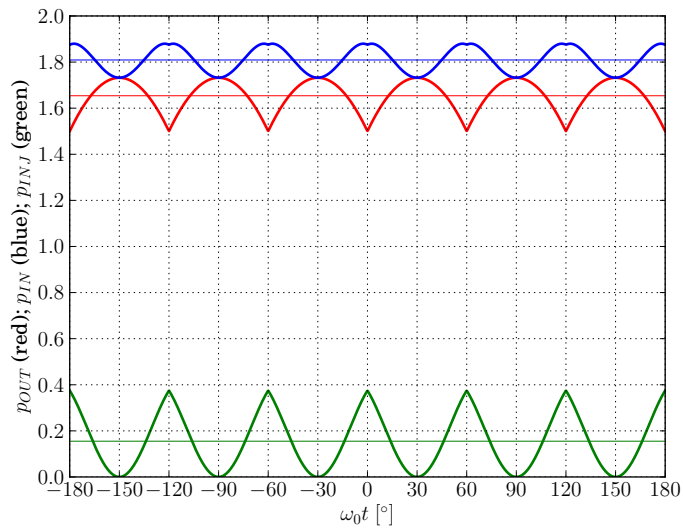
j_{IA} and j_{IB}



j_Y and j_X



the cost ...



achieved

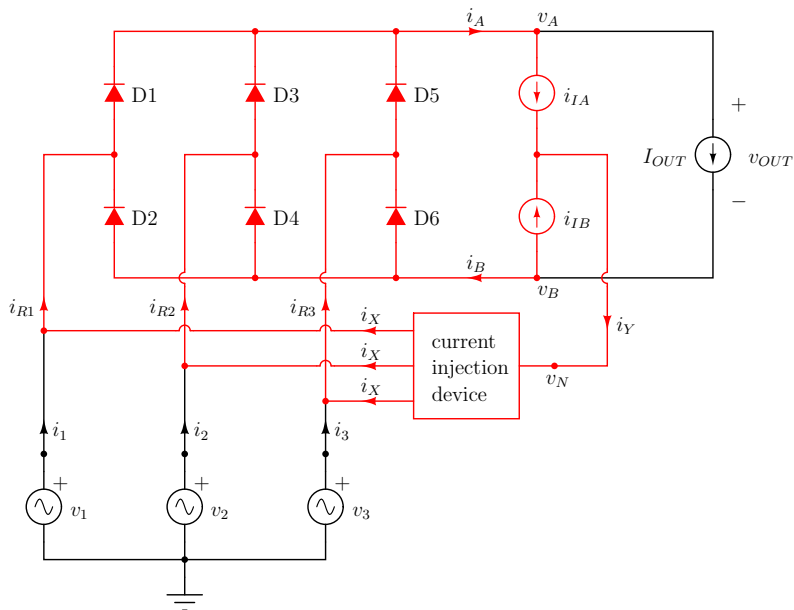
$$THD = \sqrt{\frac{32 \pi^2}{315}} - 1 \approx 5.12 \%$$

$$PF = \frac{3\sqrt{70}}{8 \pi} \approx 0.9987$$

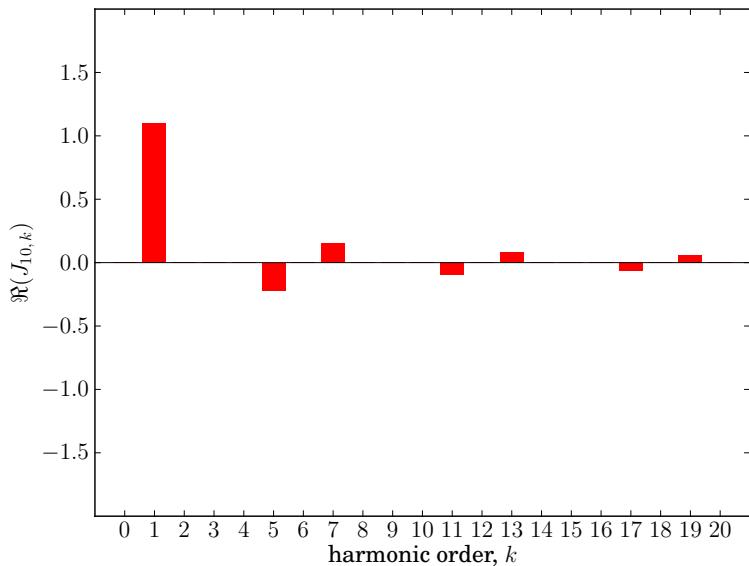
$$DPF = 1$$

$$\eta = \frac{32}{35} \approx 91.43 \%$$

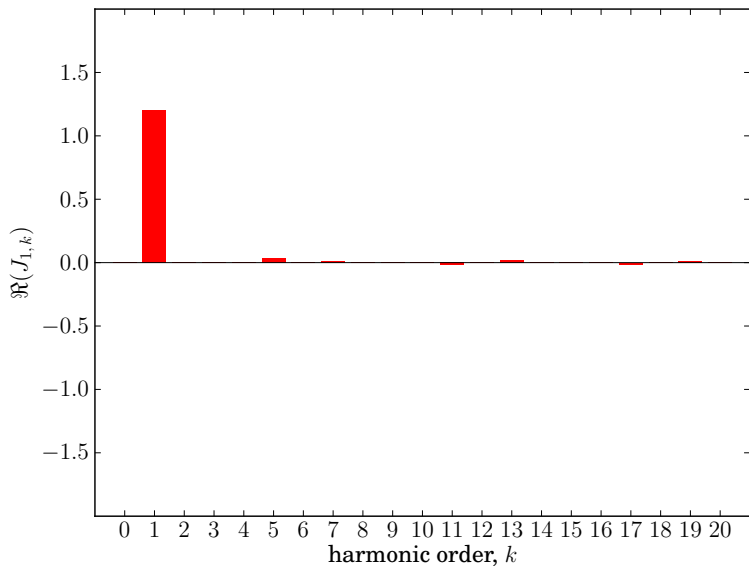
a common misconception ...



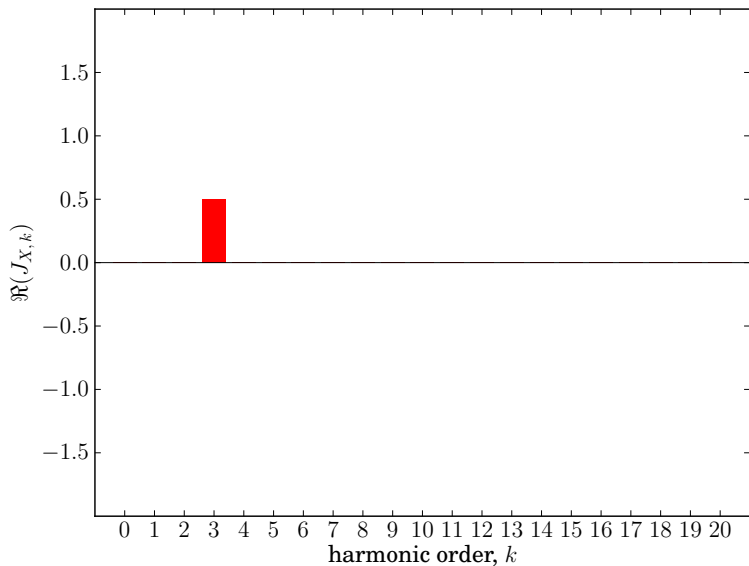
let's walk around, j_{10} , real (cosine) part



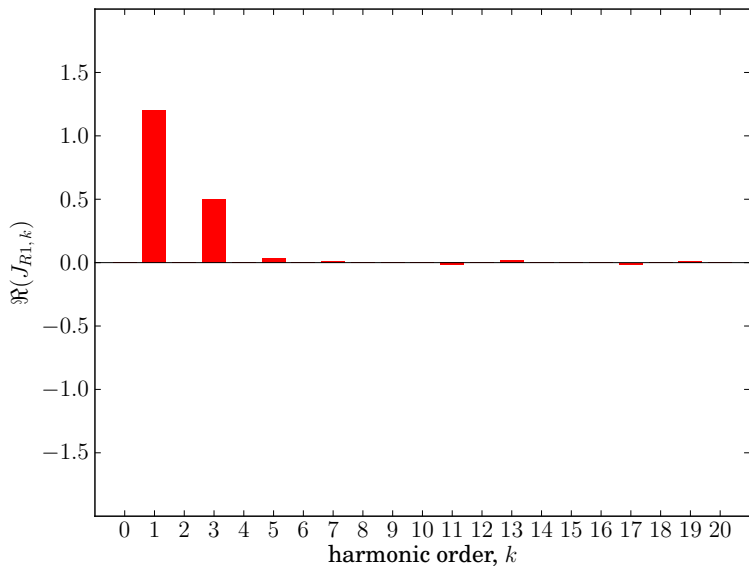
let's walk around, j_1 , real (cosine) part



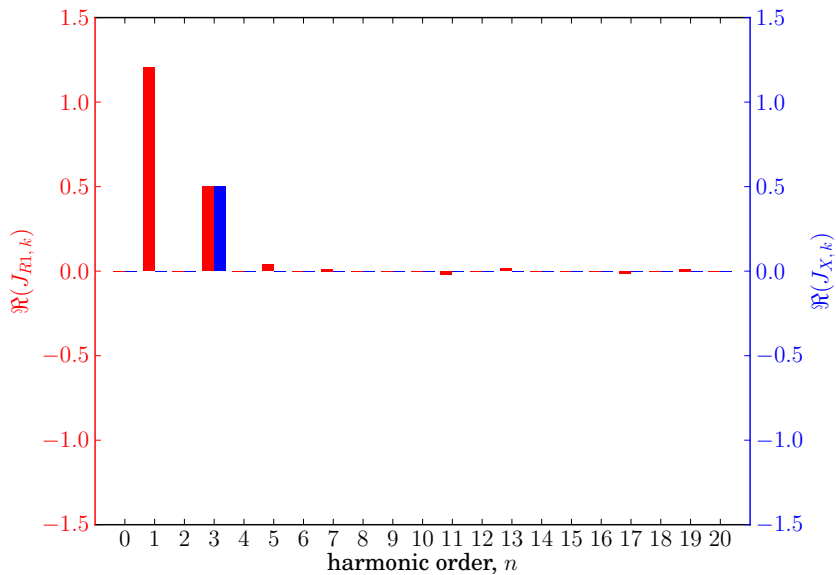
let's walk around, j_X , real (cosine) part



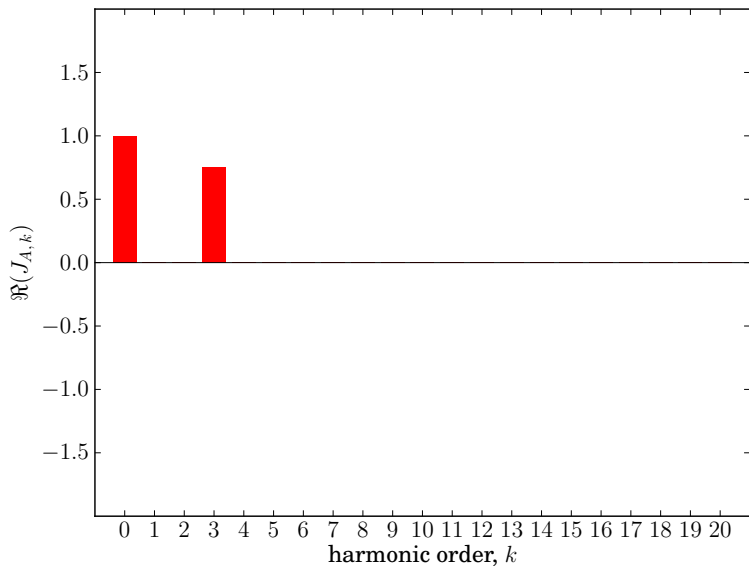
let's walk around, j_{R1} , real (cosine) part



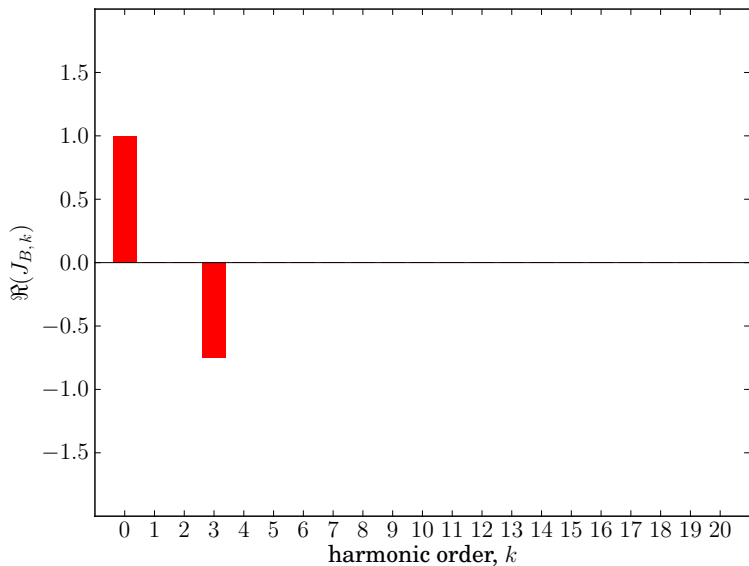
how j_1 is fixed?



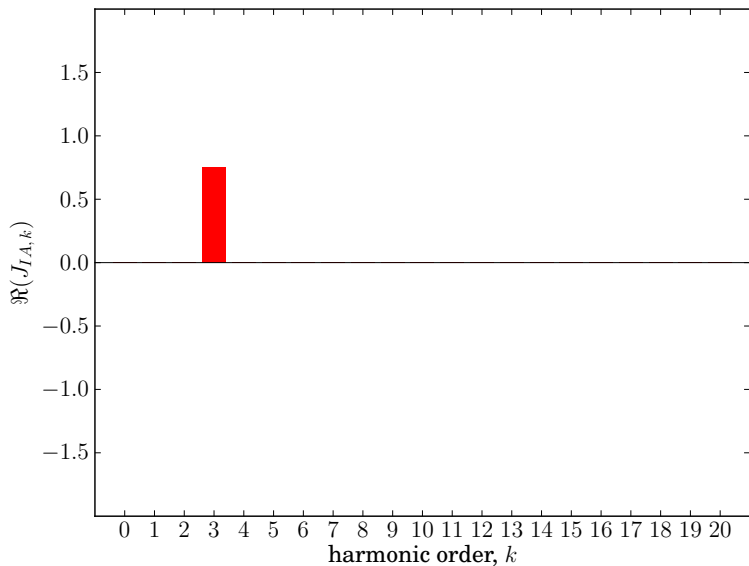
let's walk around, j_A , real (cosine) part



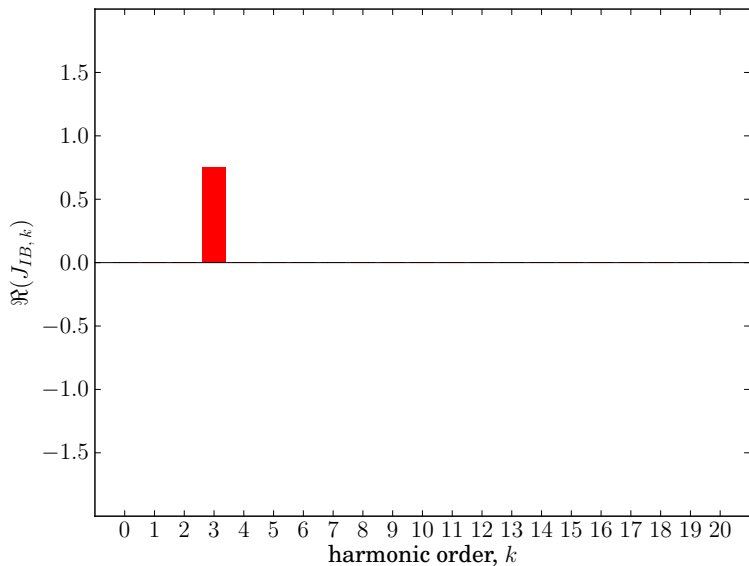
let's walk around, j_B , real (cosine) part



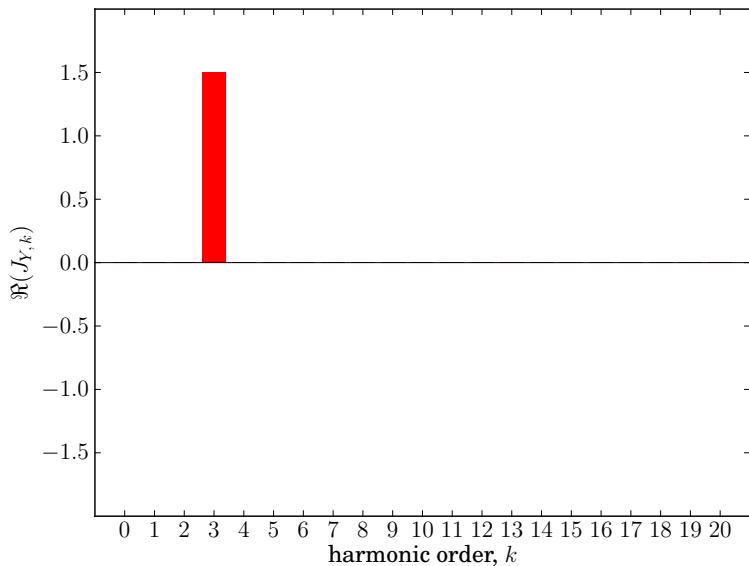
let's walk around, j_{IA} , real (cosine) part



let's walk around, j_{IB} , real (cosine) part



let's walk around, j_Y , real (cosine) part



what the 3rd harmonic current injection is not ...

- ▶ the third harmonic current injection is not a compensation for the third harmonics in the rectifier input currents
- ▶ there were **no** third harmonics in the input currents **before** the injection was applied
- ▶ **neither** there are **any** third harmonics in the input currents **after** the injection was applied
- ▶ the third harmonics circle through and around the diode bridge
- ▶ the diode bridge creates the third harmonics and “consumes” them
- ▶ the diode bridge is a nonlinear system, capable of creating harmonics

“future work”

- ▶ only currents focused; how to get them?
- ▶ how to improve the efficiency?
- ▶ is there a way to lower the $THD = 5.12\%$ further?

published in ...

Predrag Pejović, Žarko Janda

**“An Analysis of Three Phase Low Harmonic Rectifiers
Applying the Third Harmonic Current Injection”**

IEEE Transactions on Power Electronics,
vol. 14, no. 3, pp. 397–407, May 1999

but this research was initiated by a comparison of two circuits ...
which is our next topic ...