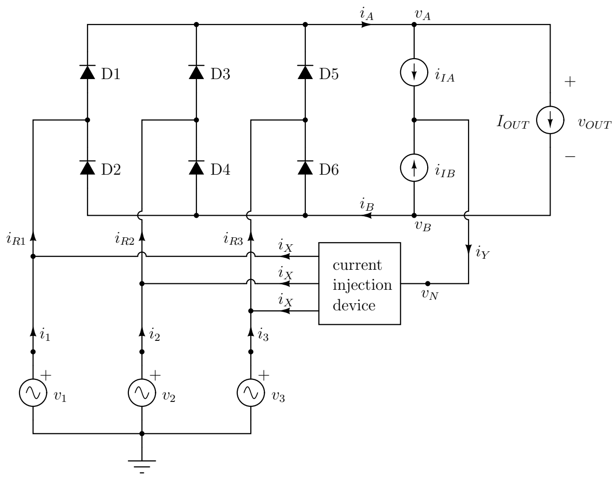
The Third Harmonic Current Injection

how to improve the THD?

1. patch the gaps in the input currents
2. do some shaping

what is this all about?



current injection device

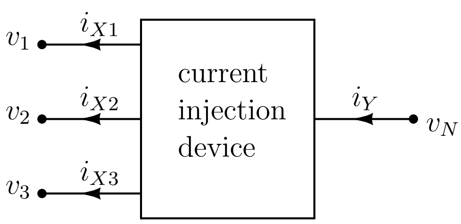
a magnetic device that provides:

$$i_X = \frac{1}{3}\, i_Y$$

$$v_N = \frac{1}{3}\,(v_1 + v_2 + v_3) = 0$$

- patching even where not needed . . .
- more to be said about the device . . .
- hard to put parallel events into a sequence . . .
- but, let's take a closer look . . .

let's complicate a little bit



some circuit theory . . .

8 variables introduced
4 element equations needed
the remaining 4 will be covered by KCL and KVL

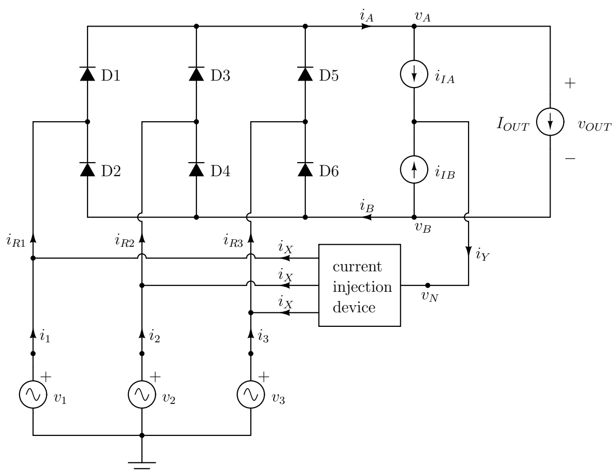
$$i_{X1} = \tfrac{1}{3}\, i_Y$$
$$i_{X2} = \tfrac{1}{3}\, i_Y$$
$$i_{X3} = \tfrac{1}{3}\, i_Y$$
$$v_N = \tfrac{1}{3}\,(v_1 + v_2 + v_3)$$

the device is **resistive**, **please remember**

$$v_N\, i_Y - v_1\, i_1 - v_2\, i_2 - v_3\, i_3 = 0$$

the device is **non-dissipative**, **please remember this**

back to the circuit . . .



some equations, KCL, . . .

$$i_k = i_{Rk} - i_X, \quad k \in \{1, 2, 3\}$$

$$i_X = \frac{1}{3}\, i_Y$$

$$i_{IA} = i_{IB} = \frac{1}{2}\, i_Y$$

$$i_A = I_{OUT} + i_{IA} = I_{OUT} + \frac{1}{2}\, i_Y$$

$$i_B = I_{OUT} - i_{IB} = I_{OUT} - \frac{1}{2}\, i_Y$$

some equations normalized

$$j_X \triangleq \frac{i_X}{I_{OUT}}$$

$$j_k = j_{Rk} - j_X, \quad k \in \{1,\, 2,\, 3\}$$

$$j_X = \frac{1}{3}\, j_Y$$

$$j_{IA} = j_{IB} = \frac{1}{2}\, j_Y$$

$$j_A = 1 + j_{IA} = 1 + \frac{1}{2}\, j_Y$$

$$j_B = 1 - j_{IB} = 1 - \frac{1}{2}\, j_Y$$

j_Y limitations

for the continuous conduction mode (CCM):

$$j_A > 0 \quad \Rightarrow \quad 1 + \frac{1}{2}\, j_Y > 0$$

$$j_B > 0 \quad \Rightarrow \quad 1 - \frac{1}{2}\, j_Y > 0$$

$$-2 < j_Y < 2$$

or

$$-2\, I_{OUT} < i_Y < 2\, I_{OUT}$$

how to get j_{Rk}, $k \in \{1,\, 2,\, 3\}$?

$$j_{R1} = d_1\, j_A - d_2\, j_B$$

$$j_{R2} = d_3\, j_A - d_4\, j_B$$

$$j_{R3} = d_5\, j_A - d_6\, j_B$$

or, in general terms

$$j_{Rk} = d_{2k-1}\, j_A - d_{2k}\, j_B$$

for

$$k \in \{1,\, 2,\, 3\}$$

the input currents

plugging in expressions for i_{Rk} and i_X

$$j_k = d_{2k-1}\left(1 + \frac{1}{2}\, j_Y\right) - d_{2k}\left(1 - \frac{1}{2}\, j_Y\right) - \frac{1}{3}\, j_Y$$

there are three cases to be considered:

$$j_k = \begin{cases} 1 + \frac{1}{6}\, j_Y & \text{for} \quad d_{2k-1} = 1 \quad \text{and} \quad d_{2k} = 0 \\ -1 + \frac{1}{6}\, j_Y & \text{for} \quad d_{2k-1} = 0 \quad \text{and} \quad d_{2k} = 1 \\ -\frac{1}{3}\, j_Y & \text{for} \quad d_{2k-1} = 0 \quad \text{and} \quad d_{2k} = 0 \end{cases}$$

some symmetry

since

$$d_1\left(\omega_0 t\right) = d_3\left(\omega_0 t + \frac{2\pi}{3}\right) = d_5\left(\omega_0 t + \frac{4\pi}{3}\right)$$

$$d_2\left(\omega_0 t\right) = d_4\left(\omega_0 t + \frac{2\pi}{3}\right) = d_6\left(\omega_0 t + \frac{4\pi}{3}\right)$$

to have

$$j_1\left(\omega_0 t\right) = j_2\left(\omega_0 t + \frac{2\pi}{3}\right) = j_3\left(\omega_0 t + \frac{4\pi}{3}\right)$$

we need

$$j_Y\left(\omega_0 t\right) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right) = j_Y\left(\omega_0 t + \frac{4\pi}{3}\right)$$

more about j_Y

to get

$$j_Y\left(\omega_0 t\right) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right) = j_Y\left(\omega_0 t + \frac{4\pi}{3}\right)$$

we need

$$j_Y\left(\omega_0 t\right) = j_Y\left(\omega_0 t + \frac{2\pi}{3}\right)$$

meaning that j_Y is periodic with $\frac{1}{3}\, T_0$, i.e. the fundamental frequency of i_Y is $3\, f_0$!

thus, **the third-harmonic current injection** is the simplest case

let's assume i_Y

$$i_Y = k\, I_{OUT}\, \cos\left(3\omega_0 t - \phi\right)$$

$$\boxed{j_Y = k\, \cos\left(3\omega_0 t - \phi\right)}$$

range:

1. for the continuous conduction mode $-2 < k < 2$
2. let us assume $-\pi < \phi < \pi$

which couple $(k,\, \phi)$ minimizes the THD?

optimization over k and ϕ

$$j_1 = \begin{cases} 1 + \frac{1}{6}\, j_Y & \text{for} \quad d_1 = 1 \quad \text{and} \quad d_2 = 0 \\ -1 + \frac{1}{6}\, j_Y & \text{for} \quad d_1 = 0 \quad \text{and} \quad d_2 = 1 \\ -\frac{1}{3}\, j_Y & \text{for} \quad d_1 = 0 \quad \text{and} \quad d_2 = 0 \end{cases}$$

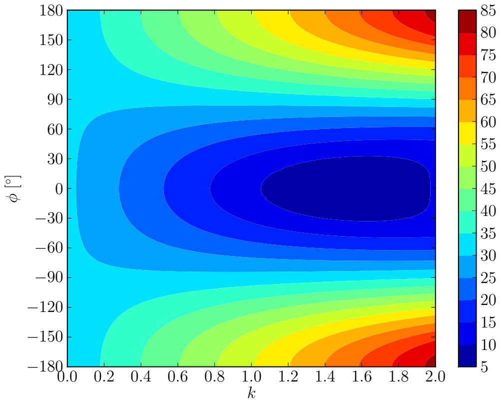
and

$$j_Y = k\, \cos\left(3\omega_0 t - \phi\right)$$

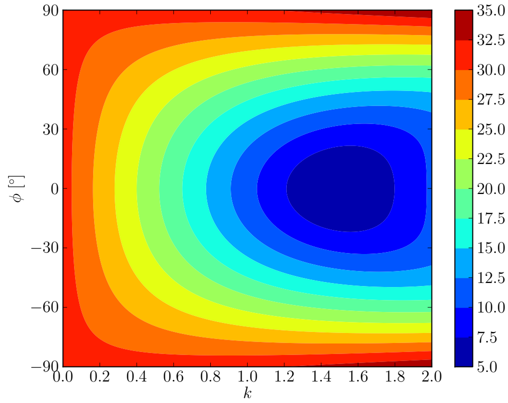
let's go to work!

goal: minimize the THD

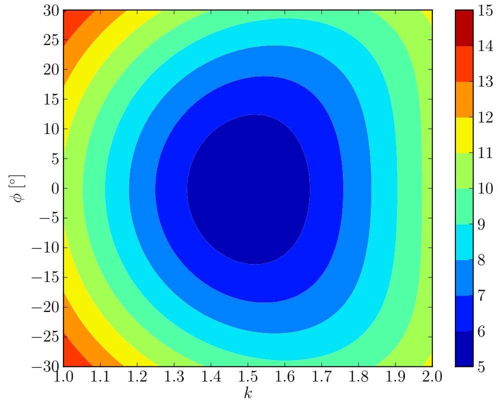
numerical optimization, 1, THD [%]



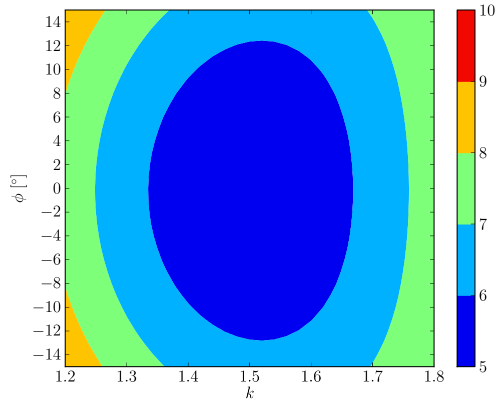
numerical optimization, 2, THD [%]



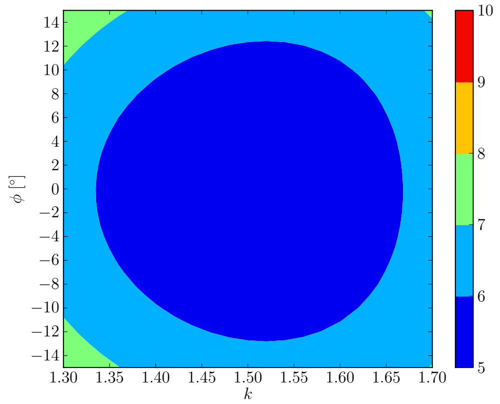
numerical optimization, 3, THD [%]



numerical optimization, 4, THD [%]



numerical optimization, 5, THD [%]



numerical optimization, conclusions

- improvement possible!
- $5\% < THD < 6\%$ achievable
- $k \approx 1.5$, $\phi \approx 0$
- it is promissing to continue
- analytical optimization?
- circuit?
- **let's continue . . .**

analytical optimization, start

$$j_1 = \begin{cases} 1 + \frac{1}{6}\, j_Y & \text{for} \quad d_1 = 1 \quad \text{and} \quad d_2 = 0 \\ -1 + \frac{1}{6}\, j_Y & \text{for} \quad d_1 = 0 \quad \text{and} \quad d_2 = 1 \\ -\frac{1}{3}\, j_Y & \text{for} \quad d_1 = 0 \quad \text{and} \quad d_2 = 0 \end{cases}$$

and

$$j_Y = k\, \cos\left(3\omega_0 t - \phi\right)$$

analytical optimization, rms and the fundamental

after lots of work (fortunately, wxMaxima's):

$$J_{RMS} = \frac{\sqrt{k^2 + 24}}{6}$$

$$J_{1Cm} = \frac{3\,k\,\cos\phi + 48}{8\,\sqrt{3}\,\pi}$$

$$J_{1Sm} = \frac{3\,\sqrt{3}\,k\,\sin\phi}{8\,\pi}$$

$$J_{1RMS} = \sqrt{\frac{J_{1Cm}^2 + J_{1Sm}^2}{2}}$$

$$J_{1RMS} = \frac{\sqrt{768 + 27\,k^2 + 96\,k\,\cos\phi - 24\,k^2\,(\cos\phi)^2}}{8\,\sqrt{2}\,\pi}$$

analytical optimization, make it simple

$$THD = \sqrt{\left(\frac{J_{RMS}}{J_{1RMS}}\right)^2 - 1}$$

make it simple: to minimize THD is to minimize

$$y\left(k,\, \phi\right) \triangleq \left(\frac{J_{RMS}}{J_{1RMS}}\right)^2$$

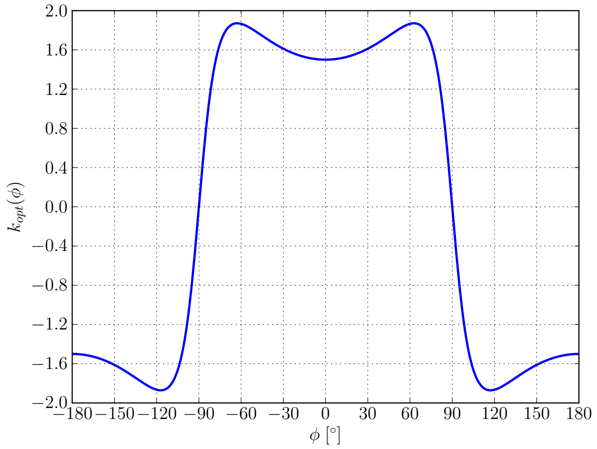
$$y\left(k,\, \phi\right) = \frac{2\,\pi^2\left(16\,k^2 + 384\,\pi^2\right)}{6912 + 243\,k^2 + 864\,k\,\cos\phi - 216\,k^2\,\left(\cos\phi\right)^2}$$

analytical optimization, let's fix ϕ

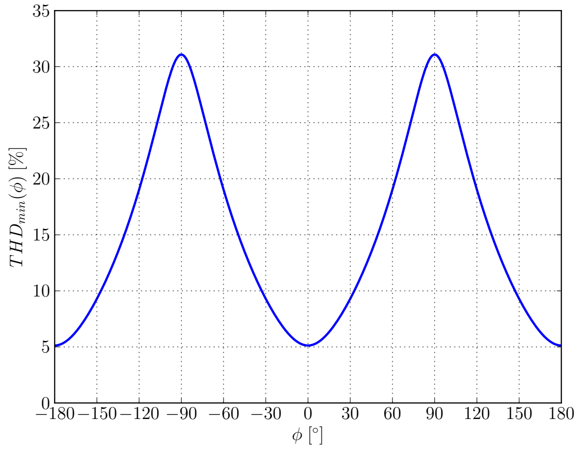
$$\frac{\partial\, y\left(k,\, \phi\right)}{\partial k} = 0$$

$$k_{opt}\left(\phi\right) = \frac{\sqrt{576\,\left(\cos\phi\right)^4 + 624\,\left(\cos\phi\right)^2 + 25} - 24\,\left(\cos\phi\right)^2 - 5}{4\,\cos\phi}$$

$k_{opt}\left(\phi\right)$



$THD\left(k_{opt}\left(\phi\right),\, \phi\right)$



THD_{MIN}, k_{OPT}, ϕ_{OPT}

$$THD_{MIN} = \min\left(THD\left(k_{opt}\left(\phi\right),\, \phi\right)\right) = THD\left(k_{OPT},\, \phi_{OPT}\right)$$

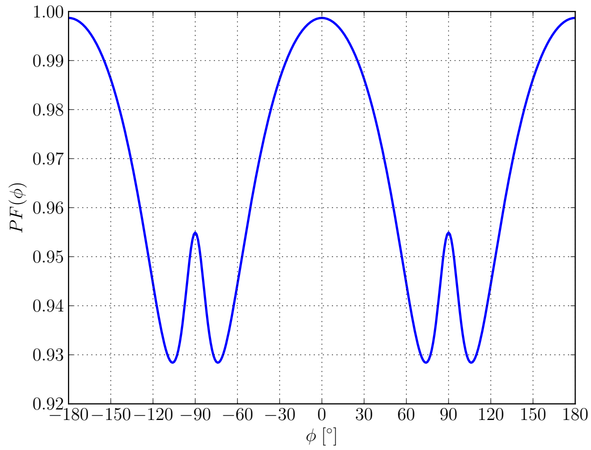
$$\phi_{OPT} = 0$$

$$k_{OPT} = k_{opt}(0) = \frac{3}{2}$$

$$\boxed{THD_{MIN} = \sqrt{\frac{32\,\pi^2}{315} - 1} \approx 5.12\,\%}$$

and that's all the third harmonic current injection can do

$PF\left(k_{opt}\left(\phi\right),\, \phi\right)$



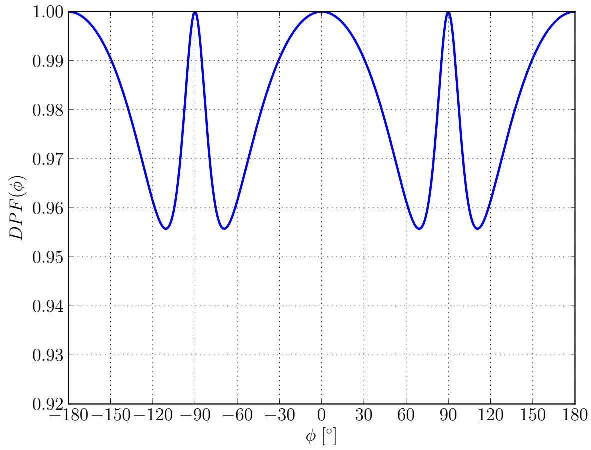
$PF\left(k_{OPT},\, \phi_{OPT}\right)$

$$PF\left(k,\, \phi\right) = \frac{\sqrt{6}\,\left(3\,k\,\cos\phi + 48\right)}{8\,\pi\,\sqrt{k^2 + 24}}$$

$$\boxed{PF\left(\tfrac{3}{2},\, 0\right) = \frac{3\sqrt{70}}{8\,\pi} \approx 0.9987}$$

we could have optimized $PF\left(k,\, \phi\right)$ ...

$DPF\left(k_{opt}\left(\phi\right),\, \phi\right)$

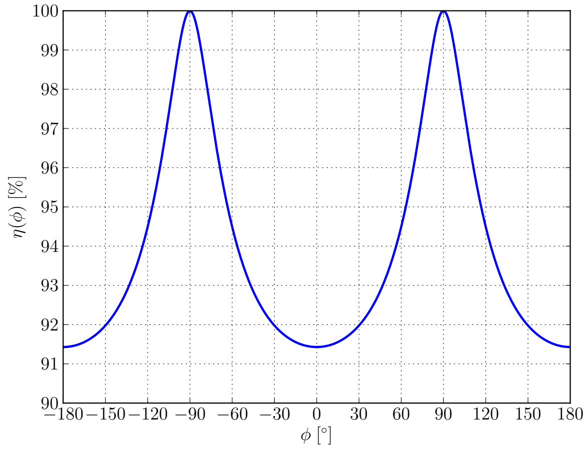


$DPF\left(k_{OPT},\, \phi_{OPT}\right)$

$$DPF\left(k,\, \phi\right) = \frac{\sqrt{3}\left(k\, \cos\phi + 16\right)}{\sqrt{768 + 27\, k^2 + 96\, k\, \cos\phi - 24\, k^2\, \left(\cos\phi\right)^2}}$$

$$\boxed{DPF\left(\tfrac{3}{2},\, 0\right) = 1}$$

$\eta\left(k_{opt}\left(\phi\right),\, \phi\right)$



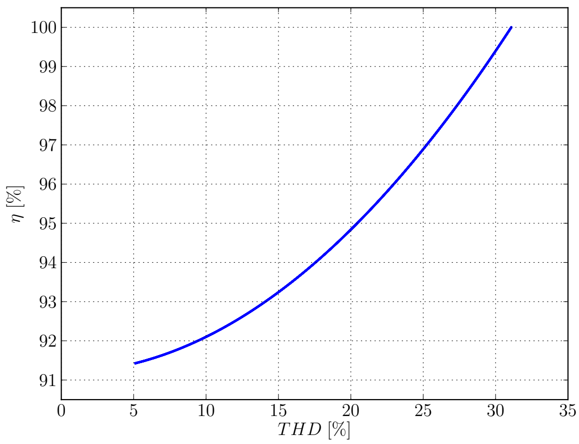
$\eta\left(k_{OPT},\, \phi_{OPT}\right)$

$$\eta\left(k,\, \phi\right) = \frac{48}{3\, k\, \cos\phi + 48}$$

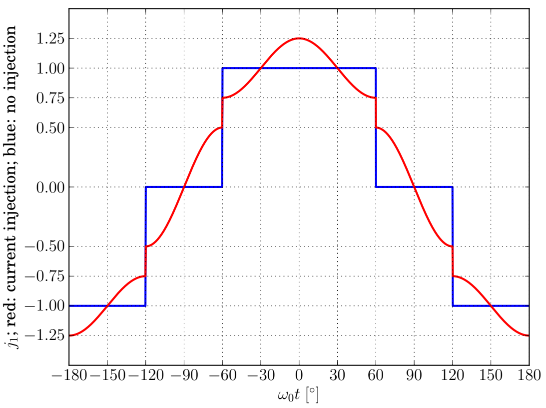
$$\boxed{\eta\left(\tfrac{3}{2},\, 0\right) = \frac{32}{35} \approx 91.43\,\%}$$

optimizing $\eta\left(k,\, \phi\right)$?

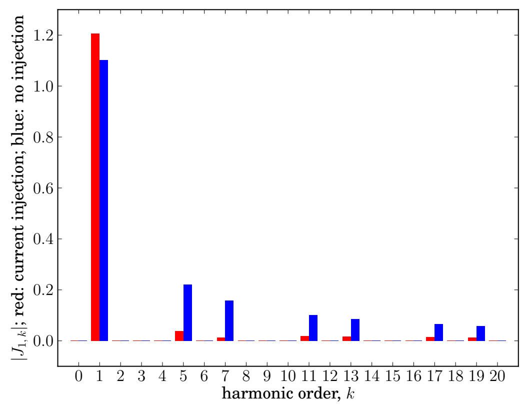
$\eta\left(THD\right)$, the tradeoff



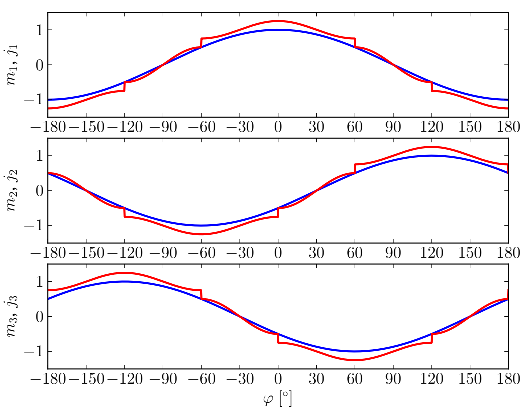
j_1, with and without injection



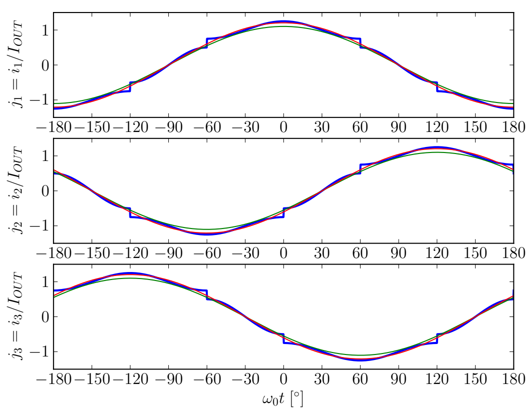
$\left|J_{1,k}\right|$, with and without injection



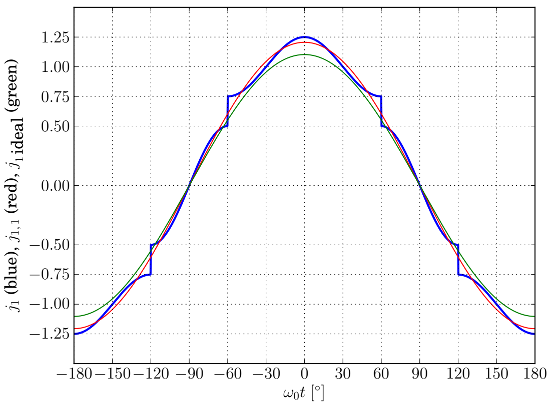
m_k and j_k, $k \in 1,\, 2,\, 3$



j_k, with ideal waveforms



j_k, with ideal waveforms, a closer look



increase in the amplitude

$$J_{1m} = \frac{35\sqrt{3}}{16\,\pi} \approx 1.2060$$

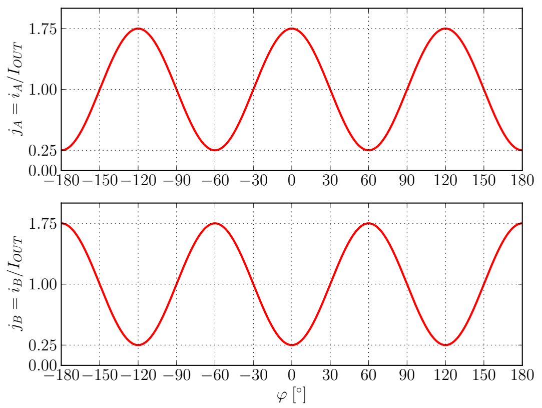
$$J_{1m\,ideal} = \frac{2\sqrt{3}}{\pi} \approx 1.1027$$

$$J_{RMS} = \frac{\sqrt{105}}{12} \approx 0.8539$$

$$J_{1RMS} = \frac{35\sqrt{6}}{32\,\pi} \approx 0.8528$$

$$J_{1RMS\,ideal} = \frac{\sqrt{6}}{\pi} \approx 0.77970$$

j_A and j_B



j_A and j_B, mean and rms

$$J_A = \overline{j_A} = \langle j_A \rangle = 1$$

$$J_B = \overline{j_B} = \langle j_B \rangle = 1$$

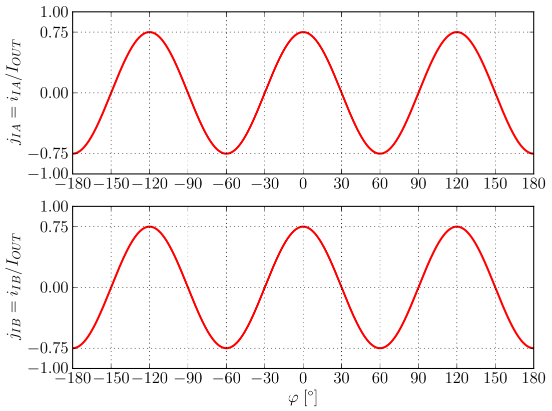
as it was before, but

$$J_{A\,RMS} = \sqrt{\frac{41}{32}} \approx 1.1319 > 1$$

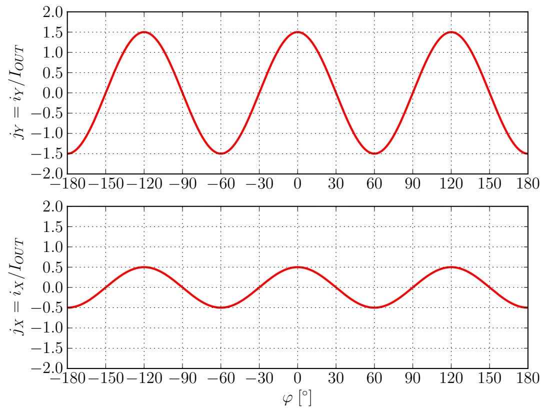
$$J_{B\,RMS} = \sqrt{\frac{41}{32}} \approx 1.1319 > 1$$

$v_D = V_D + R_D\, i_D$, somewhat increased losses in the diodes

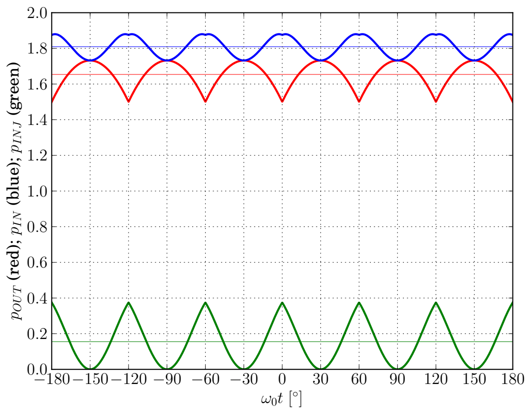
j_{IA} and j_{IB}



j_Y and j_X



the cost ...



achieved

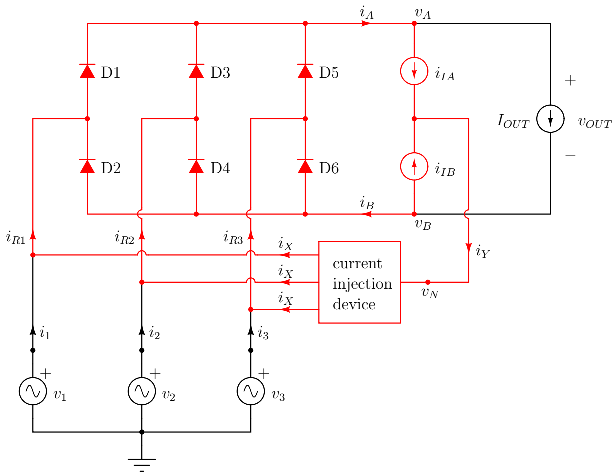
$$THD = \sqrt{\frac{32\,\pi^2}{315} - 1} \approx 5.12\,\%$$

$$PF = \frac{3\sqrt{70}}{8\,\pi} \approx 0.9987$$

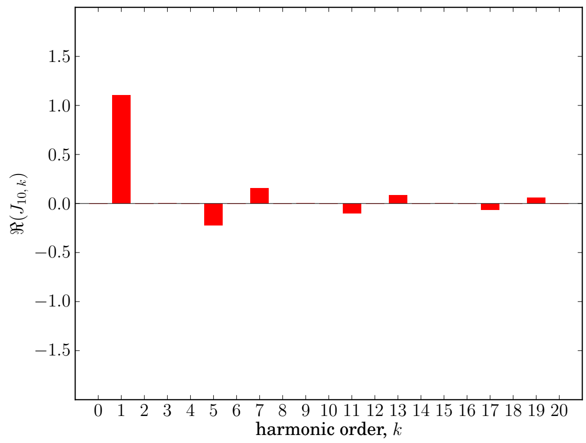
$$DPF = 1$$

$$\eta = \frac{32}{35} \approx 91.43\,\%$$

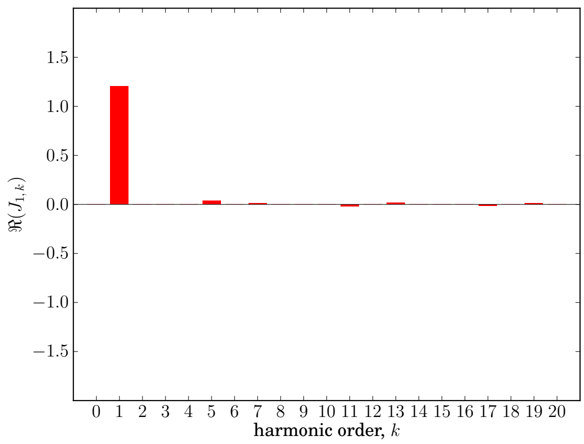
a common misconception ...



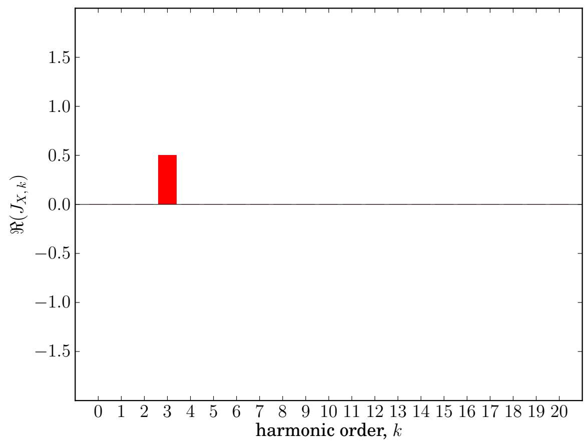
let's walk around, j_{10}, real (cosine) part



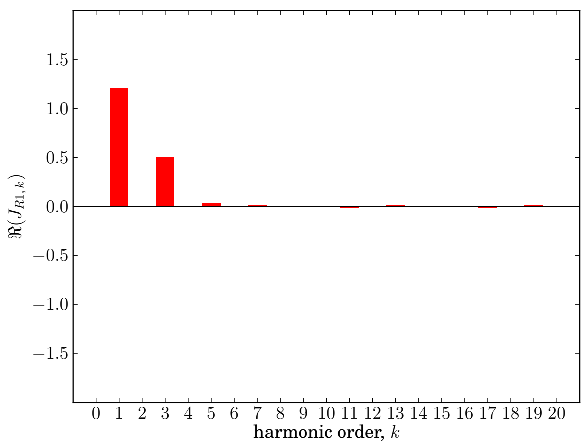
let's walk around, j_1, real (cosine) part



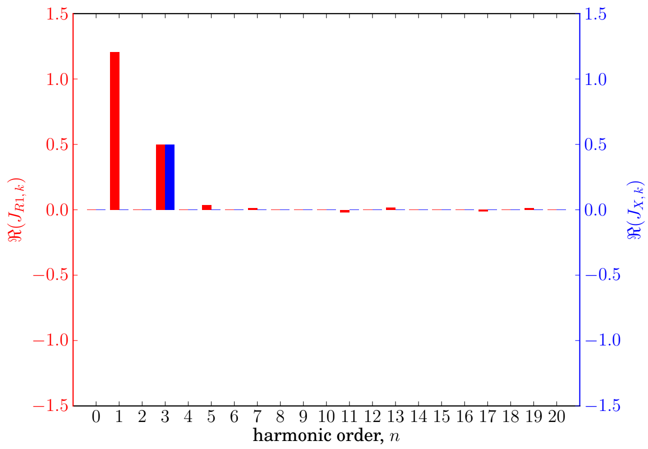
let's walk around, j_X, real (cosine) part



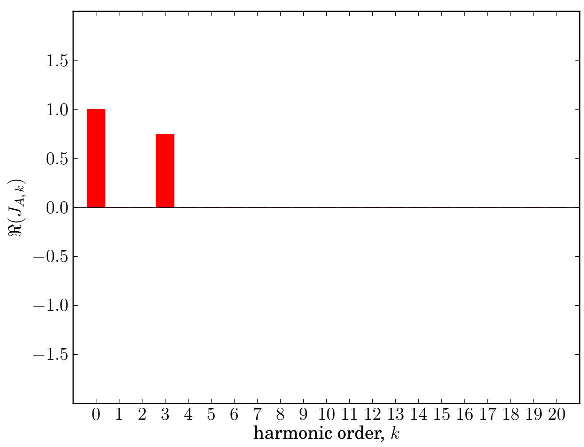
let's walk around, j_{R1}, real (cosine) part



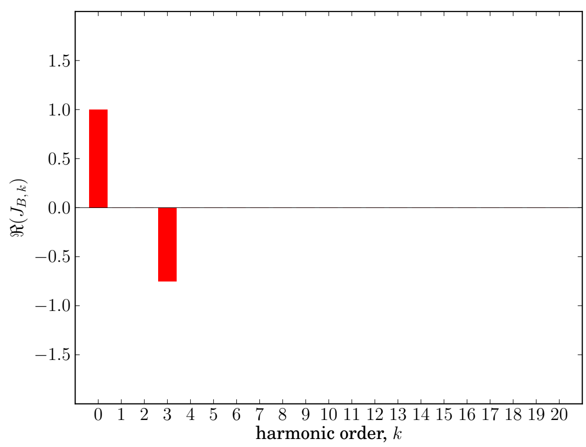
how j_1 is fixed?



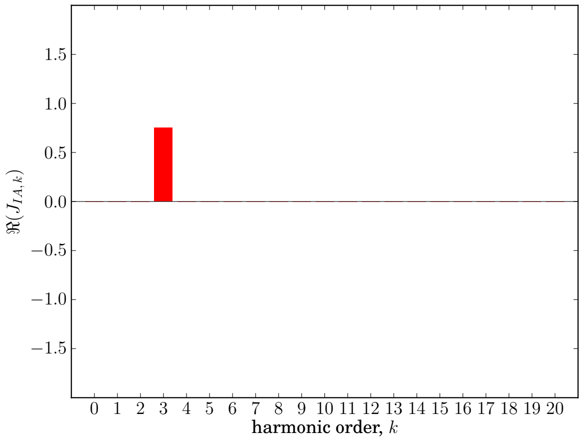
let's walk around, j_A, real (cosine) part



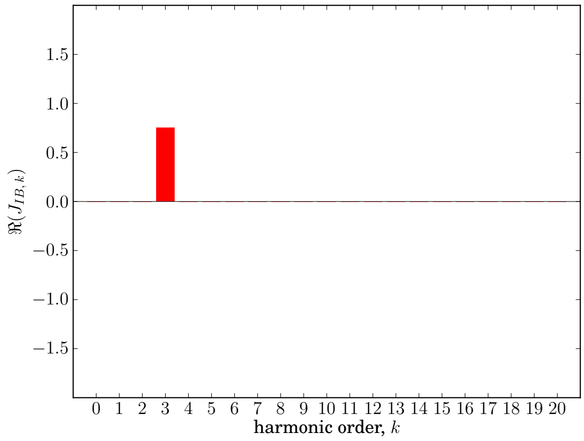
let's walk around, j_B, real (cosine) part



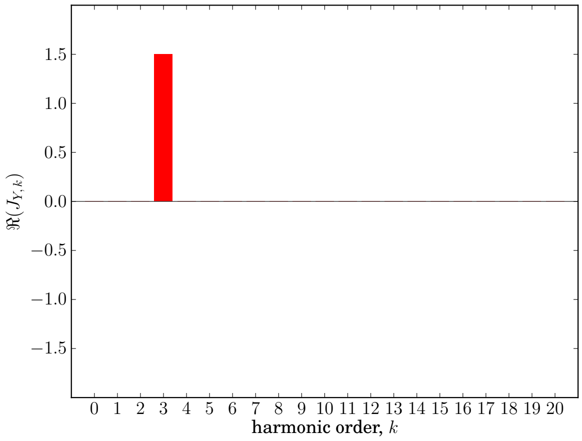
let's walk around, j_{IA}, real (cosine) part



let's walk around, j_{IB}, real (cosine) part



let's walk around, j_Y, real (cosine) part



what the 3rd harmonic current injection is not ...

- the third harmonic current injection is not a compensation for the third harmonics in the rectifier input currents
- there were **no** third harmonics in the input currents **before** the injection was applied
- **neither** there are **any** third harmonics in the input currents **after** the injection was applied
- the third harmonics circle through and around the diode bridge
- the diode bridge creates the third harmonics and "consumes" them
- the diode bridge is a nonlinear system, capable of creating harmonics

"future work"

- only currents focused; how to get them?
- how to improve the efficiency?
- is there a way to lower the $THD = 5.12\,\%$ further?

published in ...

Predrag Pejović, Žarko Janda

"An Analysis of Three Phase Low Harmonic Rectifiers Applying the Third Harmonic Current Injection"

IEEE Transactions on Power Electronics,
vol. 14, no. 3, pp. 397–407, May 1999

but this research was initiated by a comparison of two circuits ...

which is our next topic ...