The Discontinuous Conduction Mode
what really happens?

- in the case $V_{A, 1}$ and $V_{B, 1}$ remain the same as in the CCM, the amplitude of $i_{Y}$ won't be limited
- something, though, limits the amplitude; try and see ...
- let's look at the circuit and search for the answers:

1. what limits the amplitude of $i_{Y}$ ?
2. is it safe to operate in this mode?

3 . is there any use of this operating mode?
$v_{1}, i_{1}, I_{O U T} \approx 3 \mathrm{~A}$, spectra

$v_{2}, i_{2}, I_{\text {OUT }} \approx 3 \mathrm{~A}$


$v_{1}, i_{1}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{1}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{2}, i_{2}, I_{\text {OUT }} \approx 3 \mathrm{~A}$, spectra

$v_{2}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{3}, i_{3}, I_{\text {OUT }} \approx 3 \mathrm{~A}$, spectra

$v_{1}, i_{1}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{1}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{3}, i_{3}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{3}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{1}, i_{1}, I_{\text {OUT }} \approx 6 \mathrm{~A}$, spectra

$v_{2}, i_{2}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{2}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{3}, i_{3}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{3}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{1}, i_{1}, I_{O U T} \approx 9 \mathrm{~A}$, spectra


$v_{3}, i_{3}, I_{\text {OUT }} \approx 6 \mathrm{~A}$, spectra

$v_{1}, i_{1}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

$v_{1}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

$v_{2}, i_{2}, I_{\text {OUT }} \approx 9 \mathrm{~A}$
$v_{2}, i_{2}, I_{\text {OUT }} \approx 9 \mathrm{~A}$, spectra

$v_{2}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

$v_{3}, i_{3}, I_{O U T} \approx 9 \mathrm{~A}$, spectra

$v_{O U T}, i_{O U T}, I_{O U T} \approx 3 \mathrm{~A}$


$v_{3}, i_{3}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

$v_{3}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

$v_{\text {OUT }}, i_{\text {OUT }}, I_{\text {OUT }} \approx 3 \mathrm{~A}$, spectra

$v_{\text {OUT }}, i_{\text {OUT }}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{\text {OUT }}, i_{\text {OUT }}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

does it worth?

| $I_{\text {OUT }}[\mathrm{A}]$ | $k$ | $I_{k R M S}[\mathrm{~A}]$ | $V_{k R M S}[\mathrm{~V}]$ | $S[\mathrm{VA}]$ | $P[\mathrm{~W}]$ |
| :---: | :--- | ---: | ---: | ---: | ---: |
| $\approx 3 \mathrm{~A}$ | 1 | 2.90 | 99.78 | 289.66 | 282.36 |
|  | 2 | 2.91 | 99.47 | 289.58 | 282.42 |
|  | 3 | 2.88 | 100.12 | 288.67 | 281.42 |
| $\approx 6 \mathrm{~A}$ | 1 | 5.66 | 95.89 | 542.29 | 533.10 |
|  | 2 | 5.66 | 95.75 | 541.53 | 532.38 |
|  | 3 | 5.65 | 97.01 | 548.57 | 539.83 |
| $\approx 9 \mathrm{~A}$ | 1 | 8.38 | 91.67 | 767.90 | 752.36 |
|  | 2 | 8.48 | 92.14 | 781.43 | 767.26 |
|  | 3 | 8.42 | 94.72 | 797.43 | 785.45 |

does it worth?

| $I_{\text {OUT }}[\mathrm{A}]$ | $V_{\text {OUT }}[\mathrm{V}]$ | $P_{\text {OUT }}[\mathrm{W}]$ | $P_{I N}[\mathrm{~W}]$ | $\eta[\%]$ |
| ---: | ---: | ---: | ---: | ---: |
| 3.08 | 253.32 | 781.38 | 846.21 | 92.34 |
| 6.06 | 232.91 | 1410.35 | 1605.31 | 87.86 |
| 9.41 | 216.86 | 2041.09 | 2305.07 | 88.55 |

really low $\eta$; quite unexpected! is this a mistake?

$v_{\text {OUT }}, i_{\text {OUT }}, I_{\text {OUT }} \approx 9 \mathrm{~A}$, spectra

does it worth?

| $I_{\text {OUT }}[\mathrm{A}]$ | $k$ | $P F$ | $T H D\left(i_{k}\right)[\%]$ | $T H D\left(v_{k}\right)[\%]$ |
| :---: | :---: | ---: | ---: | ---: |
| $\approx 3 \mathrm{~A}$ | 1 | 0.9748 | 17.76 | 4.22 |
|  | 2 | 0.9753 | 17.86 | 3.91 |
|  | 3 | 0.9749 | 17.74 | 4.14 |
| $\approx 6 \mathrm{~A}$ | 1 | 0.9830 | 14.69 | 3.76 |
|  | 2 | 0.9831 | 14.19 | 4.21 |
|  | 3 | 0.9841 | 14.19 | 3.80 |
| $\approx 9 \mathrm{~A}$ | 1 | 0.9798 | 11.98 | 5.47 |
|  | 2 | 0.9819 | 11.53 | 5.47 |
|  | 3 | 0.9850 | 11.69 | 4.32 |

user's point of view ... and a conclusion

- the input currents are not so good, but better than without injection
- THD is in the range from $10 \%$ to $20 \%$
- absolutely no notches in the input voltages
- spikes in the output voltage, rich with harmonics
- the spikes decrease with increases of the output current
- the spikes increase the output voltage average, VOUT
- there are some losses in the system, unexpected?
- the efficiency is much lower than expected!
- at 9 A the rectifier operates close to the CCM
- spikes are the answer! what causes them?
- let's take a closer look ...
$v_{A}, i_{A}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{A}, i_{A}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{A}, i_{A}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

let's start from the end...

Q: what the end is?
A: CCM-DCM boundary, close to $I_{O U T}=9 \mathrm{~A}$ in our case at that point:

$$
i_{Y}=2 I_{O U T} \cos \left(3 \omega_{0} t\right)
$$

normalize, ...

$$
j_{Y}=J_{Y m} \cos \left(3 \omega_{0} t\right)=2 \cos \left(3 \omega_{0} t\right)
$$

$J_{Y m}=2$ instead of $\frac{3}{2}$, which would be optimal
$v_{B}, i_{B}, I_{\text {OUT }} \approx 3 \mathrm{~A}$

$v_{B}, i_{B}, I_{O U T} \approx 6 \mathrm{~A}$

$v_{B}, i_{B}, I_{O U T} \approx 9 \mathrm{~A}$

efficiency at the boundary ...

$$
\begin{gathered}
P_{I N J}=\frac{1}{2} J_{Y m} M_{A, 1} \\
P_{I N J}=\frac{1}{2} \times 2 \times \frac{3 \sqrt{3}}{8 \pi}=\frac{3 \sqrt{3}}{8 \pi} \\
P_{O U T}=\frac{3 \sqrt{3}}{\pi} \\
\eta=\frac{P_{O U T}}{P_{O U T}+P_{I N J}}=\frac{8}{9} \approx 88.89 \%
\end{gathered}
$$

$j_{1}$ at the boundary (and not just there) ...

$T H D$ and the $P F$ at the boundary

$$
\begin{gathered}
T H D=\sqrt{\frac{224 \pi^{2}}{2187}-1} \approx 10.43 \% \\
P F=\frac{27 \sqrt{3}}{4 \pi \sqrt{14}} \approx 0.9946
\end{gathered}
$$

not that bad...
but, the efficiency is bad!
parameters for $Q=0$ ?
this boundary requires a different value of $\rho \ldots$

$$
\rho=\frac{\max \left(m_{A V}\right)}{\max \left(j_{Y}\right)}=\frac{1}{4} \times \frac{1}{2}=\frac{1}{8}
$$

at the boundary ...

$$
\begin{gathered}
J_{R M S}=\sqrt{\frac{10}{9}-\frac{2}{\sqrt{3} \pi}} \\
J_{1 m}=\frac{\sqrt{3}}{\pi}+\frac{2}{3} \\
J_{1 R M S}=\frac{\sqrt{3}}{\pi \sqrt{2}}+\frac{\sqrt{2}}{3}
\end{gathered}
$$

partial conclusions ...
there is some interest in the DCM, due to the relatively acceptable THD

- the boundary between the CCM and the DCM suffers from poor efficiency, not of practical interest
- we should analyze the DCM somewhere away from the boundary...
at the boundary ..
to get to the boundary

$$
\rho=\frac{M_{A, 1}}{J_{Y m}}=\frac{3 \sqrt{3}}{8 \pi} \times \frac{1}{2}=\frac{3 \sqrt{3}}{16 \pi}
$$

and at the boundary ...
wxMaxima will be heavily needed from here...

$$
\begin{gathered}
J_{R M S}=\frac{\sqrt{7}}{3} \\
J_{1 m}=\frac{9 \sqrt{3}}{4 \pi} \\
J_{1 R M S}=\frac{9 \sqrt{3}}{4 \pi \sqrt{2}}
\end{gathered}
$$

and what if $Q=0$ ?

$\eta, T H D$, and $P F$ at the boundary, $Q=0$

$$
\begin{gathered}
\eta=\frac{P_{\text {OUT }}}{P_{I N}}=\frac{M_{\text {OUT }}}{\frac{3}{2} J_{1 m}}=\frac{6 \sqrt{3}}{2 \pi+3 \sqrt{3}} \approx 90.53 \% \\
T H D=\sqrt{\frac{16 \pi^{2}-24 \pi \sqrt{3}-27}{4 \pi^{2}+12 \pi \sqrt{3}+27}} \approx 4.93 \% \\
P F=\sqrt{\frac{4 \pi^{2}+12 \pi \sqrt{3}+27}{20 \pi^{2}-12 \pi \sqrt{3}}} \approx 0.9988
\end{gathered}
$$

$T H D$ and $P F$ okay, but $\eta$ is not $\ldots$
better THD and PF are achieved in CCM with better $\eta$, this mode does not make any sense in practice ...
case $Q \rightarrow \infty, R \rightarrow 0$

- $Q=R_{0} / R, R_{0}=\sqrt{L / C}$
- let's analyze large $R_{0}$ and low $R$ case ... not just huge $Q$
- in that case $j_{Y} \approx 2 \cos \left(3 \omega_{0} t\right)$
- and the spikes limit the amplitude of $j_{Y} \ldots$
- ... since there is no other cause
- let's model the spikes somehow ..
- maybe with Dirac impulses?
- that's why I stressed the "flat" spectrum
- let's take a look ...
$v_{A}$ and $v_{B}, I_{O U T} \approx 3 \mathrm{~A}$

$v_{A}$ and $v_{B}, I_{\text {OUT }} \approx 6 \mathrm{~A}$

$v_{A}$ and $v_{B}, I_{\text {OUT }} \approx 9 \mathrm{~A}$

definitely, the spikes ...
definitely, the spikes are the answer for the DCM:
- the spikes reduce the $3^{\text {rd }}$ harmonic (at $3 \omega_{0}$ ) in $m_{A}$ and $m_{B}$
- the spikes depend on the output current
- the spikes are disappearing as we are getting close to the CCM
- the spikes introduce new harmonics, needed to distort $i_{Y}$
- the spikes have flat-looking spectrum ...
- ... which makes them suitable to model with Dirac $\delta$ impulses!
- and I personally like $\delta$ impulses and Dirac's approach .
- P. A. M. Dirac: "The aim of science is to make difficult things understandable in a simpler way; the aim of poetry is to state simple things in an incomprehensible way."
$v_{A}$ and $v_{B}, I_{O U T} \approx 3 \mathrm{~A}$

$v_{A}$ and $v_{B}, I_{O U T} \approx 6 \mathrm{~A}$

$v_{A}$ and $v_{B}, I_{O U T} \approx 9 \mathrm{~A}$

definitions of $m_{A 0}$ and $m_{B 0}$, no spikes

$$
\begin{aligned}
& m_{A 0}=\max \left(m_{1}, m_{2}, m_{3}\right) \\
& m_{B 0}=\min \left(m_{1}, m_{2}, m_{3}\right)
\end{aligned}
$$

the same as $m_{A}$ and $m_{B}$ before $\ldots$
thus, they have the same spectra, ...
let's model the spikes ..

$$
\begin{gathered}
m_{A}=m_{A 0}+M_{X} \sum_{n=-\infty}^{+\infty} \delta\left(\omega_{0} t-\frac{\pi}{3}-n \frac{2 \pi}{3}\right) \\
m_{B}=m_{B 0}-M_{X} \sum_{n=-\infty}^{+\infty} \delta\left(\omega_{0} t-n \frac{2 \pi}{3}\right)
\end{gathered}
$$

where $V_{X}$ is yet to be determined...
a word about physical dimension of $\delta$ impulses ...

## a DSP note ...

- when it comes to $\delta$ impulses, the DSP approach becomes extremely error prone
- be VERY careful in forming the spectra, a tiny error could destroy the results
- this is not a story, this is an experience ...
- much worse than Gibbs phenomenon...
- the higher the level of discontinuity - the worse
- with $\delta$ impulses spectral leakage is a problem, since the is a lot to leak
the spectra, $m_{B} \ldots$

$$
\begin{gathered}
m_{B}=\min \left(m_{1}, m_{2}, m_{3}\right)-M_{X} \sum_{n=-\infty}^{+\infty} \delta\left(\omega_{0} t-n \frac{2 \pi}{3}\right) \\
m_{B}=M_{B, 0}+\sum_{k=1}^{\infty} M_{B, k} \cos \left(3 k \omega_{0} t\right) \\
M_{B, 0}=-\frac{3 \sqrt{3}}{2 \pi}-\frac{3}{2 \pi} M_{X} \\
M_{B, k}=\frac{3 \sqrt{3}}{\pi} \frac{1}{9 k^{2}-1}-\frac{3}{\pi} M_{X}
\end{gathered}
$$



$$
\begin{gathered}
m_{A}=\max \left(m_{1}, m_{2}, m_{3}\right)+M_{X} \sum_{n=-\infty}^{+\infty} \delta\left(\omega_{0} t-\frac{\pi}{3}-n \frac{2 \pi}{3}\right) \\
m_{A}=M_{A, 0}+\sum_{k=1}^{\infty} M_{A, k} \cos \left(3 k \omega_{0} t\right) \\
M_{A, 0}=\frac{3 \sqrt{3}}{2 \pi}+\frac{3}{2 \pi} M_{X} \\
M_{A, k}=\frac{3 \sqrt{3}}{\pi} \frac{(-1)^{k+1}}{9 k^{2}-1}+\frac{3}{\pi}(-1)^{k} M_{X}
\end{gathered}
$$

since we are already here, $m_{O U T}$, the spectrum $\ldots$

$$
\begin{gathered}
m_{\text {OUT }}=m_{A}-m_{B} \\
m_{\text {OUT }}=M_{\text {OUT }, 0}+\sum_{k=1}^{\infty} M_{\text {OUT }, k} \cos \left(3 k \omega_{0} t\right) \\
M_{\text {OUT }, 0}=\frac{3 \sqrt{3}}{\pi}+\frac{3}{\pi} M_{X} \\
M_{\text {OUT }, k}= \begin{cases}0 & \text { for } k \text { odd } \\
-\frac{6 \sqrt{3}}{\pi} \frac{1}{9 k^{2}-1}+\frac{6}{\pi} M_{X} & \text { for } k \text { even }\end{cases}
\end{gathered}
$$

a double check ...
at $\rho=0$ :

$$
M_{\text {OUT }}=\frac{27 \sqrt{3}}{8 \pi} \approx 1.8607
$$

at $\rho=\frac{3 \sqrt{3}}{16 \pi}$ :

$$
M_{\text {OUT }}=\frac{27 \sqrt{3}}{8 \pi}-\frac{3 \sqrt{3}}{16 \pi} \times 2=\frac{3 \sqrt{3}}{\pi} \approx 1.6540
$$

$M_{\text {OUT }}$ variation within $+12.5 \% \ldots$

$$
M_{O U T}=\frac{27 \sqrt{3}}{8 \pi}-2 \rho
$$

denormalize...

$$
\begin{gathered}
\frac{V_{O U T}}{V_{m}}=\frac{27 \sqrt{3}}{8 \pi}-2 \frac{R I_{O U T}}{V_{m}} \\
V_{O U T}=\frac{27 \sqrt{3}}{8 \pi} V_{m}-2 R I_{O U T}
\end{gathered}
$$

the output impedance is $2 R, \ldots$
$\delta$ conclusions ...

- the spikes are modelled by $\delta$ impulses
- prediction of the output voltage obtained, $V_{\text {OUT }} \ldots$
- prediction of the output impedance, $2 R \ldots$
- prediction of the efficiency obtained...
- however, this is just an approximation ...
- ... although experimentally verifiable
- how good the approximation is?
- is there a better model?
- avoid exact solution, it is not available in a closed form ...
- ... like it was in the CCM
- how about "simulation"?
- but not just in a form of a cheap experiment ...
an equivalent circuit to study the DCM

about DA and DB
- DA and $v_{A 0}$ represent $v_{1}, v_{2}, v_{3}$, and D1, D3, D5
- DB and $v_{B 0}$ represent $v_{1}, v_{2}, v_{3}$, and D2, D4, D6
- DA and DB model the DCM
- from 6 diodes to 2
- from $2^{6}=64$ states to $2^{2}=4$
- and out of these four, one is irrelevant...
- an improvement ... helps us understand...
- only 3 states to take care of!
- but that's not that only ...

$$
\begin{gathered}
P_{\text {OUT }}=M_{\text {OUT }}=\frac{27 \sqrt{3}}{8 \pi}-2 \rho \\
P_{\text {IN }}=P_{\text {OUT }}+\frac{1}{2} \times \rho \times 2^{2}=\frac{27 \sqrt{3}}{8 \pi} \\
\eta=1-\frac{16 \pi}{27 \sqrt{3}} \rho
\end{gathered}
$$

$\eta$ passes the double check at the CCM-DCM boundary ...

- originally, there are 6 diodes
- deep theory says there are $2^{6}=64$ states
- fortunately, we do not care about all of them ...
- in the CCM only 6 states occur
- in the DCM there are more than $6 \ldots$
- first, let's reduce the problem as much as we can ...
- but not more than that ...
some voltages defined...
as defined earlier ... just rename ...

$$
\begin{aligned}
& v_{A 0}=\max \left(v_{1}, v_{2}, v_{3}\right) \\
& v_{B 0}=\min \left(v_{1}, v_{2}, v_{3}\right)
\end{aligned}
$$

and another voltage waveform which would be needed...

$$
v_{A V 0}=\frac{v_{A 0}+v_{B 0}}{2}
$$

how to get the currents?
in the same way as before:

$$
\begin{aligned}
& i_{1}=d_{1} i_{A}-d_{2} i_{B}-\frac{1}{3} i_{Y} \\
& i_{2}=d_{3} i_{A}-d_{4} i_{B}-\frac{1}{3} i_{Y} \\
& i_{3}=d_{5} i_{A}-d_{6} i_{B}-\frac{1}{3} i_{Y}
\end{aligned}
$$

where $d_{n}$ functions are as defined earlier, $n \in\{1, \ldots 6\}$

$$
\begin{aligned}
i_{A} & =I_{O U T}+\frac{1}{2} i_{Y} \\
i_{B} & =I_{O U T}-\frac{1}{2} i_{Y}
\end{aligned}
$$

$$
\begin{aligned}
& i_{1}=\left(d_{1}-d_{2}\right) I_{\text {OUT }}+\frac{3\left(d_{1}+d_{2}\right)-2}{6} i_{Y} \\
& i_{2}=\left(d_{3}-d_{4}\right) I_{\text {OUT }}+\frac{3\left(d_{3}+d_{4}\right)-2}{6} i_{Y} \\
& i_{3}=\left(d_{5}-d_{6}\right) I_{\text {OUT }}+\frac{3\left(d_{5}+d_{6}\right)-2}{6} i_{Y}
\end{aligned}
$$

back to the circuit ...

and some circuit theory, capacitors ..

- the capacitors share the same current, $i_{Y} / 2$
- thus, their voltages differ for a constant ...
- the circuit (with DA and DB on) has three poles, one of them in $s=0$
- the capacitors have the same AC components in their voltages
- but the DC components are different!
- it is not a big deal to find the DC components ... well ...
- especially since $V_{C A}=-V_{C B}$ due to the symmetry $\ldots$
- but to find the AC component is a problem ...
- which we are going to solve!
equations:

$$
\begin{aligned}
& L \frac{d i_{Y}}{d t}=-R i_{Y}-v_{C}+v_{A V 0} \\
& C \frac{d v_{C}}{d t}=i_{Y}
\end{aligned}
$$

conditions:

$$
\begin{aligned}
-2 I_{O U T} \leq i_{Y} & \text { if violated switch to state }-1 \\
i_{Y} \leq 2 I_{O U T} & \text { if violated switch to state }+1
\end{aligned}
$$

instead of $2^{6}=64$ we deal with 4 states now

- states, I mean diode state combinations
- and it is not 4 , but 3 for $I_{O U T}>0$
- which is why the equivalent circuit has been introduced...
- we need $i_{Y} \ldots$
- $v_{A}$ and $v_{B}$ would also be useful $\ldots$
- let's find $i_{Y}, v_{A}$, and $v_{B}$ in each of the states $\ldots$
and some circuit theory ...
- at most, for DA and DB on, the circuit is of the third order
- when a diode goes off, algebraic degeneration over $i_{L}=i_{Y}$ occurs, since $i_{Y}= \pm 2 I_{O U T}$ in such a case ...
- ... but that's not a big problem; the capacitors are ...
after some dirty job...
let's introduce

$$
v_{C}=\frac{v_{C A}+v_{C B}}{2}
$$

which turns out to be the AC component of the voltages across $C_{A}$ and $C_{B}$, since the DC components are the opposite $\ldots$
it can be shown that the resistance distribution parameter does not have any influence...
the circuit exposes algebraic degeneration if a diode is off, as already stated ...
a lot of effort to solve in a sort of elegant way ...
state $-1, \mathrm{DA}$ is off, DB is on

$$
\begin{aligned}
& i_{Y}=-2 I_{\text {OUT }} \\
& C \frac{d v_{C}}{d t}=-2 I_{\text {OUT }}
\end{aligned}
$$

condition:
$v_{C}>v_{A V 0}+2 R I_{O U T} \quad$ if violated switch to state 0

$$
\begin{aligned}
& i_{Y}=2 I_{O U T} \\
& C \frac{d v_{C}}{d t}=2 I_{O U T}
\end{aligned}
$$

condition:

$$
v_{C}<v_{A V 0}-2 R I_{O U T} \quad \text { if violated switch to state } 0
$$

new normalization, motivation

- existing normalization of currents, with $I_{\text {base }}=I_{O U T}$ is inadequate ...
- the problem is in the dependence of $\rho$ on $I_{O U T}$, $\rho=R I_{\text {OUT }} / V_{m}$
- $R$ remains constant, while $I_{O U T}$ varies
- it is inconvenient to consider variations of $I_{O U T}$ as variations of $\rho$, but not that big of a deal...
- we need a solid foundation for $I_{\text {base }}$
- besides, $R_{0} \triangleq \sqrt{L / C}$ plays a significant role now ...
- it's time to renew normalization ...
the resonance parameter, $r$

$$
\begin{aligned}
\omega_{R} & \triangleq \frac{1}{\sqrt{L C}} \\
r \triangleq & \triangleq \frac{\omega_{R}}{3 \omega_{0}}
\end{aligned}
$$

in resonance $r=1$, the CIN should be designed to meet this, this is the resonance constraint

$$
\begin{aligned}
& j_{Y}=-2 J_{O U T} \\
& \frac{d m_{C}}{d \varphi}=-6 r J_{O U T}
\end{aligned}
$$

condition:
$m_{C}>m_{A V 0}+2 \rho J_{O U T} \quad$ if violated switch to state 0

| state | DA | DB | $v_{A}$ | $v_{B}$ |
| ---: | :---: | :---: | :--- | :--- |
| 0 | on | on | $v_{A 0}$ | $v_{B 0}$ |
| -1 | off | on | $v_{A}=v_{A D C M}$ | $v_{B 0}$ |
| +1 | on | off | $v_{A 0}$ | $v_{B}=v_{B D C M}$ |

$$
\begin{aligned}
& v_{A D C M}=-v_{B 0}-4 R I_{O U T}+2 v_{C} \\
& v_{B D C M}=-v_{A 0}+4 R I_{O U T}+2 v_{C}
\end{aligned}
$$

new normalization

$$
\begin{gathered}
V_{\text {base }}=V_{m} \\
R_{\text {base }} \triangleq R_{0}=\sqrt{\frac{L}{C}} \\
I_{\text {base }}=\frac{V_{m}}{R_{0}} \\
J_{\text {OUT }}=\frac{I_{\text {OUT }}}{I_{\text {base }}}=\frac{R_{0} I_{\text {OUT }}}{V_{m}} \\
\rho=\frac{R}{R_{0}}=\frac{1}{Q}
\end{gathered}
$$

state $0, \mathrm{DA}$ is on, DB is on, normalized

## equations:

$$
\begin{aligned}
& \frac{d j_{Y}}{d \varphi}=3 r\left(-\rho j_{Y}-m_{C}+m_{A V 0}\right) \\
& \frac{d m_{C}}{d \varphi}=3 r j_{Y}
\end{aligned}
$$

conditions:

$$
\begin{aligned}
-2 J_{O U T} \leq j_{Y} & \text { if violated switch to state }-1 \\
j_{Y} \leq 2 J_{O U T} & \text { if violated switch to state }+1
\end{aligned}
$$

state $+1, \mathrm{DA}$ is off, DB is on, normalized

$$
\begin{aligned}
& j_{Y}=2 J_{O U T} \\
& \frac{d m_{C}}{d \varphi}=6 r J_{O U T}
\end{aligned}
$$

condition:
$m_{C}<m_{A V 0}-2 \rho J_{O U T} \quad$ if violated switch to state 0
$m_{A}$ and $m_{B}$

| state | DA | DB | $m_{A}$ | $m_{B}$ |
| ---: | :---: | :---: | :--- | :--- |
| 0 | on | on | $m_{A 0}$ | $m_{B 0}$ |
| -1 | off | on | $m_{A}=m_{A D C M}$ | $m_{B 0}$ |
| +1 | on | off | $m_{A 0}$ | $m_{B}=m_{B D C M}$ |

$$
\begin{aligned}
& m_{A D C M}=-m_{B 0}-4 \rho J_{O U T}+2 m_{C} \\
& m_{B D C M}=-m_{A 0}+4 \rho J_{O U T}+2 m_{C}
\end{aligned}
$$

the steady state acceleration method published in ...

Marija Stojsavljević, Predrag Pejović
"An Extrapolation Method for Accelerated Convergence to Steady State Solution of Power Electronics Circuits"

Power Conversion and Intelligent Motion, PCIM Europe 2005, pp. 574-578, Nuremberg, Germany, June 2005
$M_{\text {OUT }}$ versus $J_{\text {OUT }}$, the simulation result

$M_{\text {OUT max }}$ versus $J_{\text {OUT }}$, the simulation result


- all the equations derived ...
- just to solve them ...
- equations piecewise-linear, nonhomogeneous ...
- trapezoidal rule to integrate ...
- simple discretization ...
- but the steady state is required!
- which is a problem of itself!
- and remains to be a problem ...
- a new steady state acceleration method had to be derived to solve the model in a reasonable time ...
- the original intention was to include the method in this presentation...

THD versus $J_{O U T}$, the simulation result

$\eta$ versus $J_{O U T}$, the simulation result

$T H D$ versus $r$, the simulation result


- simulation may be used even to draw fairly general conclusions
- but this requires analytical preparation and normalization
- obtained diagrams should be denormalized to apply for a specific circuit
- agreement with $\delta$ impulse approach?
- disagreement only at low IOUT
- which was expected ... after we got the results

Predrag Božović, Predrag Pejović
"Current Injection Based Low Harmonic Three Phase Diode Bridge Rectifier Operating in Discontinuous Conduction Mode"

IEE Proceedings Electric Power Applications, vol. 152, no. 2, pp. 199-208, March 2005
without any problem!
after the DCM ...

- finally, there is some understanding of the DCM ..
- but is there any use of it?
- at first, it seems pretty useless...
- but do we study only the things to be applied at the very moment?
- actually, we do!
- but there is resistance emulation ...
- where these concepts turned out to be useful
although this was not an original idea
- and this is our next topic ...

