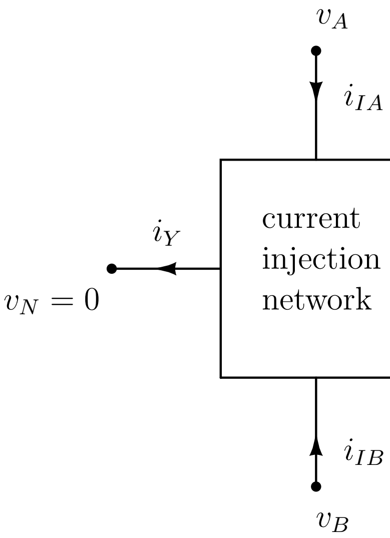
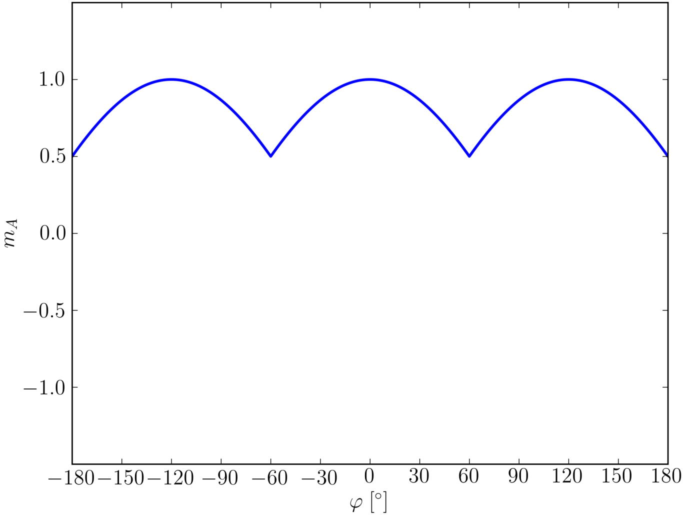
Current Injection Networks

how to get i_Y?



m_A, waveform



m_A, analytical

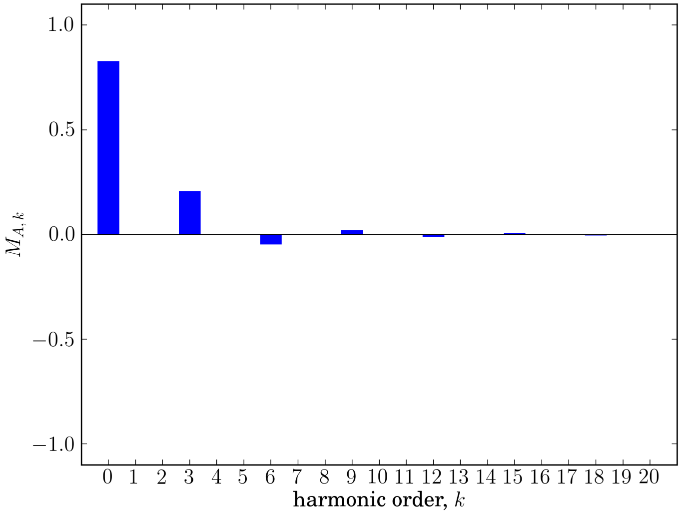
$$m_A = \max\left(m_1, m_2, m_3\right)$$

$$m_A = \frac{3\sqrt{3}}{2\pi}\left(1 + 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{9k^2-1}\cos\left(3k\omega_0 t\right)\right)$$

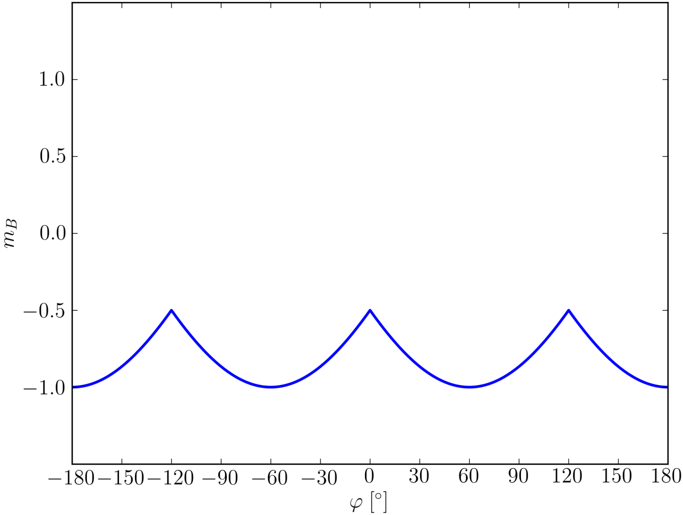
$$m_A = \sum_{k=0}^{\infty} M_{A,\,k}\cos\left(3k\omega_0 t\right)$$

$$M_{A,\,k} = \begin{cases} \dfrac{3\sqrt{3}}{2\pi} & \text{for} \quad k = 0 \\[2ex] \dfrac{3\sqrt{3}}{\pi}\,\dfrac{(-1)^{k+1}}{9k^2-1} & \text{for} \quad k \in \mathbb{N} \end{cases}$$

m_A, spectrum, real (cosine) part



m_B, waveform



m_B, analytical

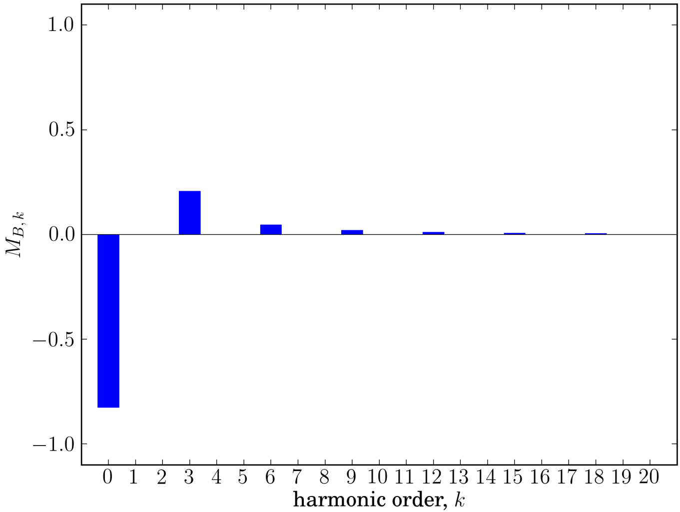
$$m_B = \min\left(m_1, m_2, m_3\right)$$

$$m_B = \frac{3\sqrt{3}}{2\pi}\left(-1 + 2\sum_{k=1}^{\infty}\frac{1}{9k^2-1}\cos\left(3k\omega_0 t\right)\right)$$

$$m_B = \sum_{k=0}^{\infty} M_{B,\,k}\cos\left(3k\omega_0 t\right)$$

$$M_{B,\,k} = \begin{cases} -\dfrac{3\sqrt{3}}{2\pi} & \text{for} \quad k = 0 \\ \dfrac{3\sqrt{3}}{\pi}\dfrac{1}{9k^2-1} & \text{for} \quad k \in \mathbb{N} \end{cases}$$

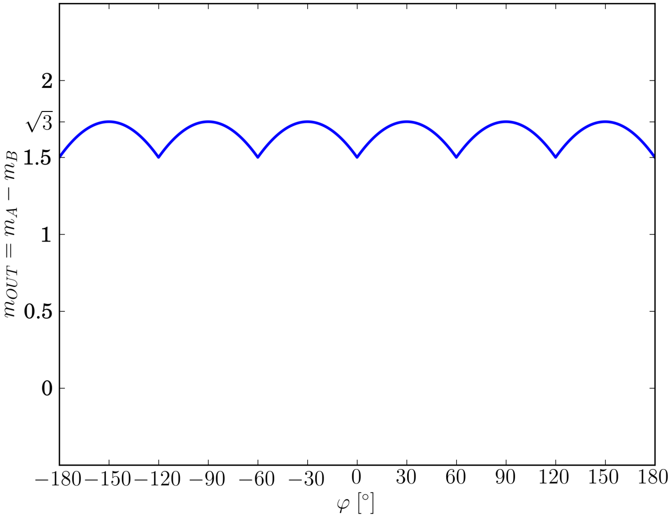
m_B, spectrum, real (cosine) part



important to note!

$$M_{B,k} = \begin{cases} M_{A,k} & \text{for} \quad k = 2\,n - 1 \\ -M_{A,k} & \text{for} \quad k = 2\,n \end{cases} \qquad \text{for } n \in \mathbb{N}$$

m_{OUT}, waveform



m_{OUT}, analytical

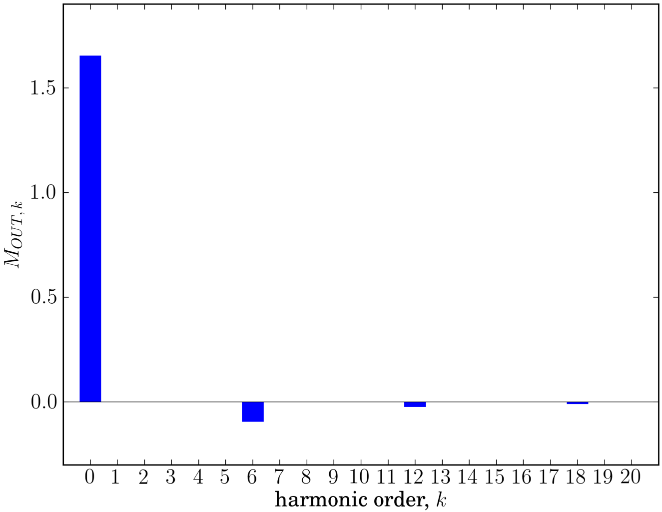
$$m_{OUT} = m_A - m_B = \max(m_1, m_2, m_3) - \min(m_1, m_2, m_3)$$

$$m_{OUT} = \frac{3\sqrt{3}}{\pi}\left(1 - 2\sum_{k=1}^{\infty} \frac{1}{36k^2 - 1} \cos(6k\omega_0 t)\right)$$

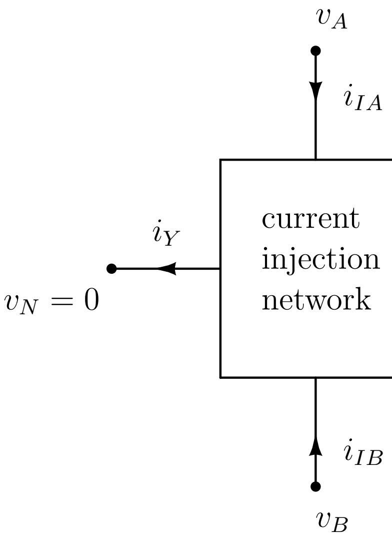
$$m_{OUT} = \sum_{k=0}^{\infty} M_{OUT,k} \cos(6k\omega_0 t)$$

$$M_{OUT,k} = \begin{cases} \dfrac{3\sqrt{3}}{\pi} & \text{for} \quad k = 0 \\ -\dfrac{6\sqrt{3}}{\pi}\dfrac{1}{36k^2 - 1} & \text{for} \quad k \in \mathbb{N} \end{cases}$$

m_{OUT}, spectrum, real (cosine) part



and what is our goal?



aiming ...

$$i_Y = \frac{3}{2}\, I_{OUT} \cos\left(3\omega_0 t\right)$$

$$i_{IA} = i_{IB} = \frac{1}{2}\, i_Y$$

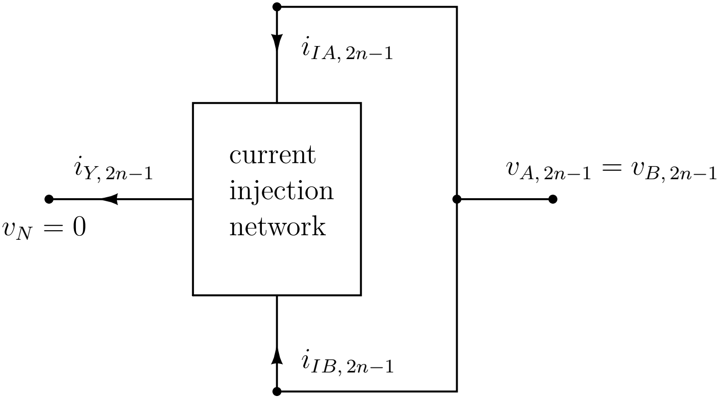
out of v_A and v_B with given waveforms and spectra, having $v_N = 0$

a few words about power

- $P_{INJ} = \frac{3}{35}\,P_{IN} \approx 8.571\%\,P_{IN}$
- $P_{INJ} = \frac{3}{32}\,P_{OUT} = 9.375\%\,P_{OUT}$
- P_{INJ} taken by the current injection network form the rectifier output
- $v_N = 0$, no way to inject the power back to the mains
- besides, $i_X = \frac{1}{2}\,I_{OUT}\cos(3\omega_0 t)$, again no way to restore P_{INJ}

- **there has to be something dissipative in the current injection network!**

equivalent circuit at odd triples of the line frequency



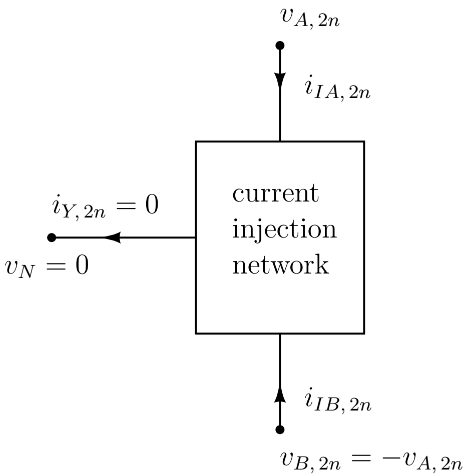
since $M_{B,\,2n-1} = M_{A,\,2n-1}$

odd symmetry

if the circuit is symmetric:

$$i_{IA,\,2n-1} = i_{IB,\,2n-1} = \frac{1}{2}\, i_{Y,\,2n-1}$$

equivalent circuit at even triples of the line frequency



since $M_{B,\,2n} = -M_{A,\,2n}$

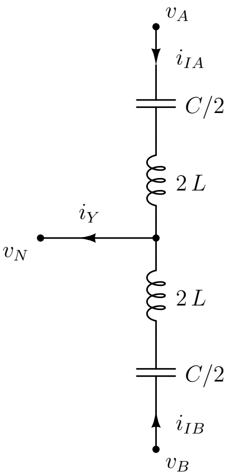
even symmetry

if the circuit is symmetric:

$$i_{IB,\,2n} = -i_{IA,\,2n}$$

$$i_{Y,\,2n} = 0$$

circuit #1



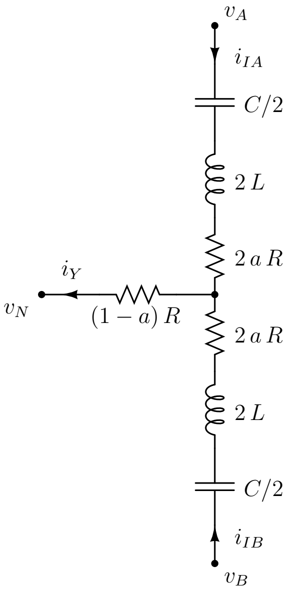
published in ...

W. B. Lawrance, W. Mielczarski

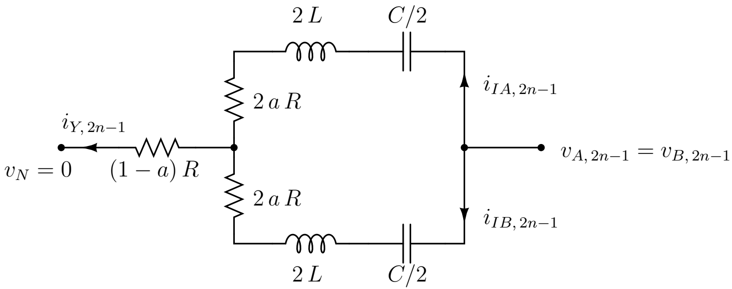
"Harmonic current reduction in a three-phase diode bridge rectifier"

IEEE Transactions on Industrial Electronics,
pp. 571–576, vol. 39, no. 6, Dec. 1992

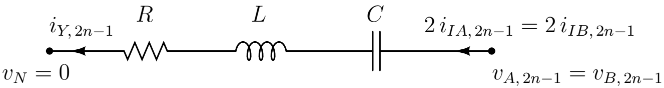
circuit #1, realistic



circuit #1, at odd $3\omega_0$

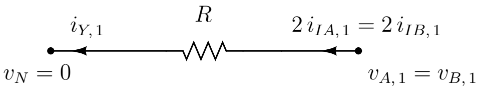


circuit #1, at odd $3\omega_0$, reduced



resonance at $3\omega_0$

$$\boxed{3\omega_0 = \frac{1}{\sqrt{L\,C}}}$$



let's get R

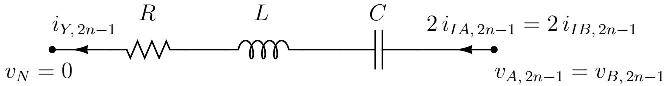
$$V_{A,1} = V_{B,1} = \frac{3\sqrt{3}}{8\pi} V_m \quad v_{A,1} = v_{B,1} = \frac{3\sqrt{3}}{8\pi} V_m \cos(3\omega_0 t)$$

$$I_{Y,1} = \frac{3}{2} I_{OUT} \quad i_{Y,1} = \frac{3}{2} I_{OUT} \cos(3\omega_0 t)$$

$$\boxed{R = \frac{V_{A,1}}{I_{Y,1}} = \frac{\sqrt{3}}{4\pi} \frac{V_m}{I_{OUT}}}$$

$$\boxed{\rho = R \frac{I_{OUT}}{V_m} = \frac{\sqrt{3}}{4\pi} \approx 0.13783}$$

circuit #1, let's get back at odd $3\omega_0$



$$\underline{Z}_{odd,\,2n-1} = \frac{\underline{V}_{A,\,2n-1}}{\underline{I}_{Y,\,2n-1}} = R + (2n-1)\,j\,3\omega_0 L + \frac{1}{(2n-1)\,j\,3\omega_0 C}$$

some more math . . .

$$R_0 \triangleq \sqrt{\frac{L}{C}}$$

$$3\omega_0 = \frac{1}{\sqrt{L\,C}}$$

$$L = \frac{R_0}{3\omega_0}, \quad 3\omega_0 L = R_0$$

$$C = \frac{1}{3\omega_0 R_0}, \quad 3\omega_0 C = \frac{1}{R_0}$$

and just some more ...

for k odd, $k = 2n - 1$:

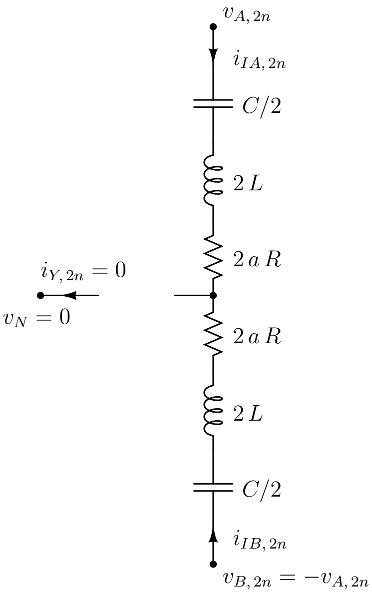
$$\underline{Z}_{odd,\,k} = R + R_0 \left(jk + \frac{1}{jk} \right)$$

$$\boxed{\underline{Z}_{odd,\,k} = R \left(1 + j\,Q \left(k - \frac{1}{k} \right) \right)}$$

$$\boxed{Q \triangleq \frac{R_0}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}}$$

and a has no effect at all

circuit #1, at even $3\omega_0$



some math, again …

$$\underline{Z}_{even,\,2n} = \frac{\underline{V}_{A,\,2n}}{\underline{I}_{A,\,2n}} = \frac{\underline{V}_{B,\,2n}}{\underline{I}_{B,\,2n}}$$

$$\underline{Z}_{even,\,2n} = 2\,a\,R + (2n)\,j\,3\omega_0\,(2L) + \frac{1}{(2n)\,j\,3\omega_0\,(C/2)}$$

$$\underline{Z}_{even,\,2n} = 2\,a\,R + (2n)\,j\,2R_0 + \frac{1}{(2n)\,j\,(1/(2R_0))}$$

and just some more ...

for k even, $k = 2n$:

$$\underline{Z}_{even,\,k} = 2\,a\,R + 2\,R_0\left(jk + \frac{1}{jk}\right)$$

$$\boxed{\underline{Z}_{even,\,k} = 2\,R\left(a + j\,Q\left(k - \frac{1}{k}\right)\right)}$$

and a has some effect now

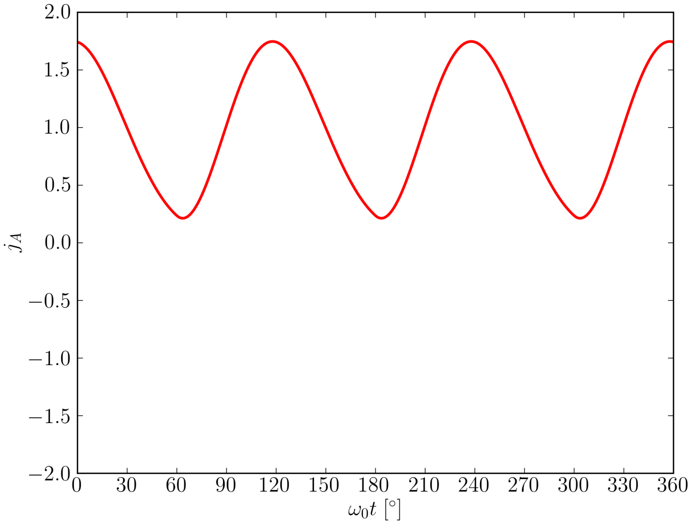
how far to go with Q?

- $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
- increase in Q increases selectivity, reduces higher-order harmonics
- increase in Q increases voltage stress on the capacitors
- aim is to use electrolytic capacitors, unipolar

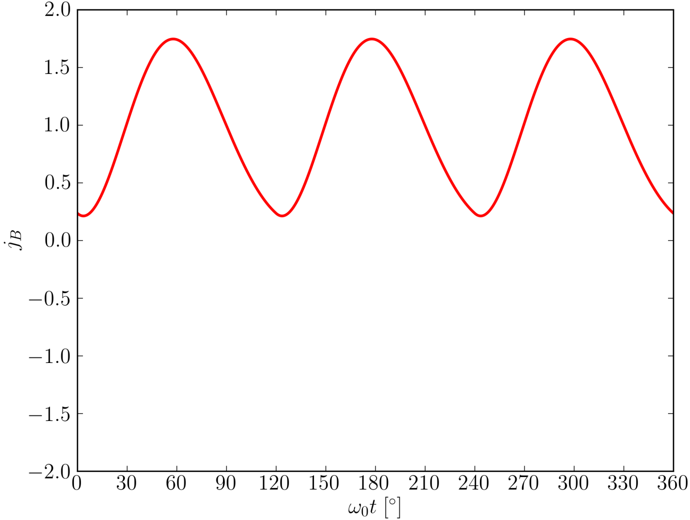
$$\left(3\,\omega_0\,\frac{C}{2}\right)^{-1} \times \frac{3}{4}\,I_{OUT} < \frac{3\sqrt{3}}{2\pi}\,V_m$$

$$\boxed{Q < 4}$$

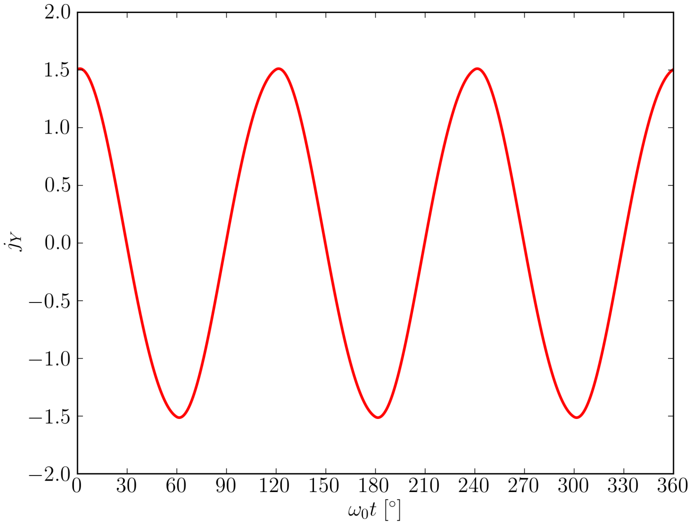
simulation, j_A, $Q = 2$, $a = 0.5$, circuit #1



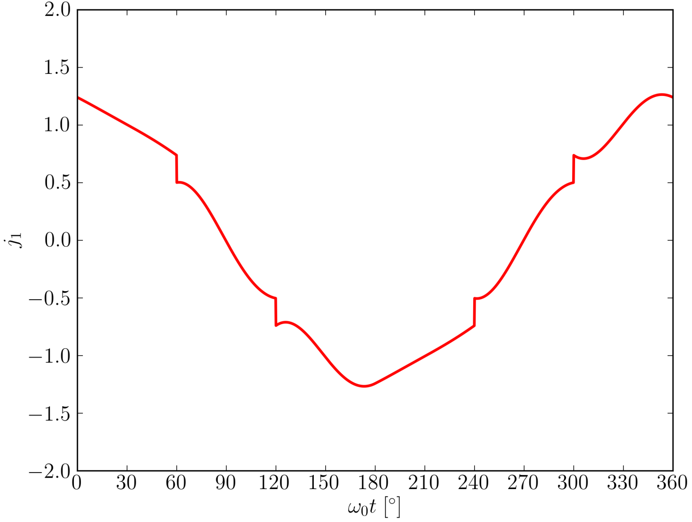
simulation, j_B, $Q = 2$, $a = 0.5$, circuit #1



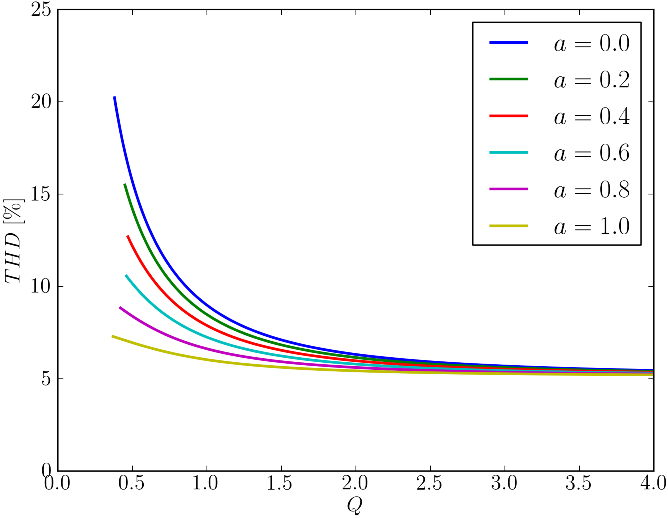
simulation, j_Y, $Q = 2$, $a = 0.5$, circuit #1



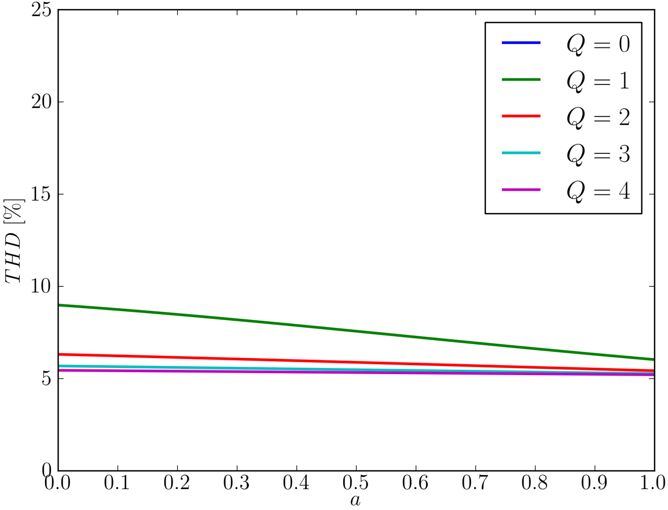
simulation, j_1, $Q = 2$, $a = 0.5$, circuit #1



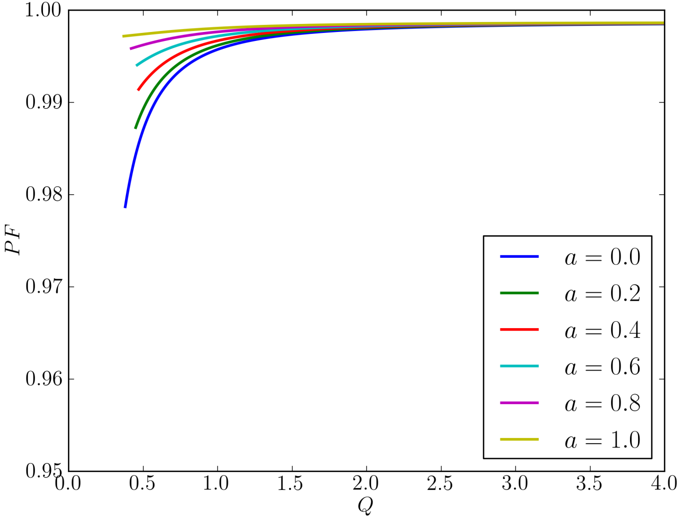
$THD(Q)$, a parameter, circuit #1



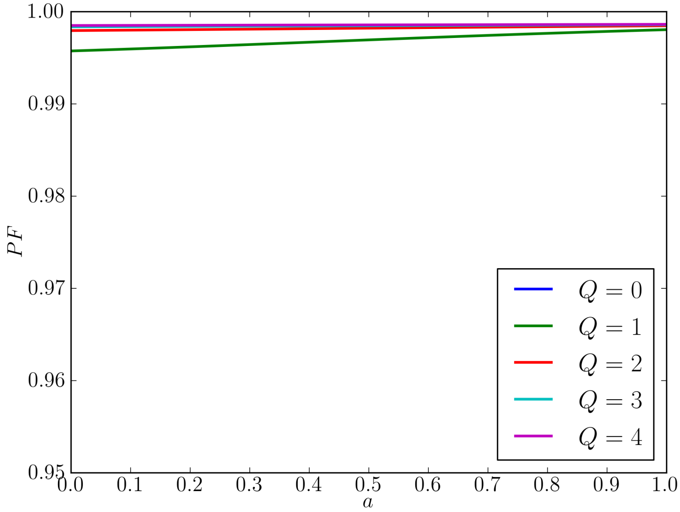
$THD(a)$, Q parameter, circuit #1



$PF(Q)$, a parameter, circuit #1



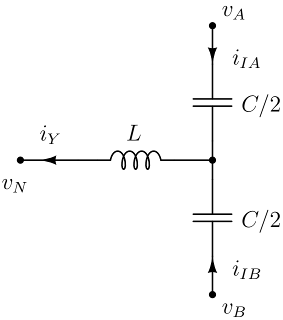
$PF(a)$, Q parameter, circuit #1



some comments . . .

- the diagrams end when the DCM is reached
- DCM? in CCM $i_A > 0$ and $i_B > 0$ all the time
- increased Q improves response
- increased a improves response

circuit #2



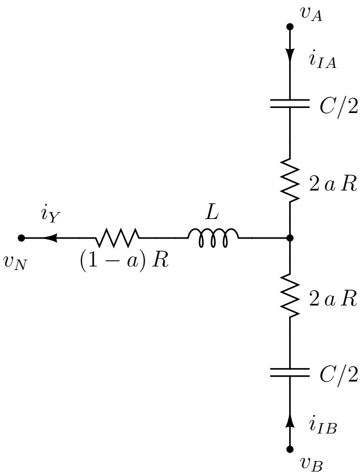
published in . . .

S. Kim, P. Enjeti, P. Packebush, I. Pitel

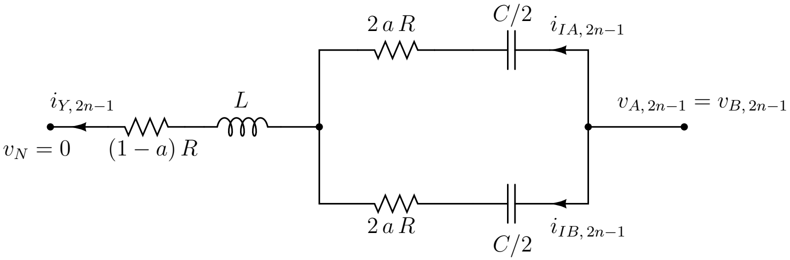
“A new approach to improve power factor and reduce harmonics in a three-phase diode rectifier type utility interface”

IEEE Transactions on Industry Applications, pp. 1557–1564, vol. 30, no. 6, Nov./Dec. 1994

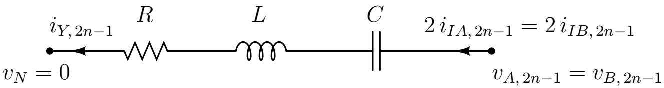
circuit #2, realistic



circuit #2, at odd $3\omega_0$



circuit #2, at odd $3\omega_0$, reduced



resonance, R, impedance, ...

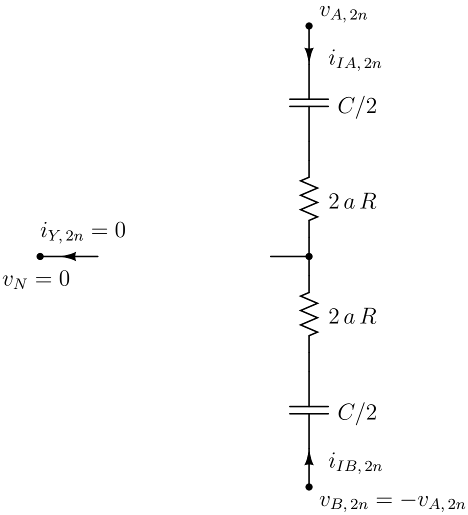
$$3\omega_0 = \frac{1}{\sqrt{L\,C}}$$

$$R = \frac{V_{A,\,1}}{I_{Y,\,1}} = \frac{\sqrt{3}}{4\pi}\,\frac{V_m}{I_{OUT}} \qquad \rho = R\,\frac{I_{OUT}}{V_m} = \frac{\sqrt{3}}{4\pi} \approx 0.13783$$

$$\underline{Z}_{odd,\,k} = R\left(1 + j\,Q\left(k - \frac{1}{k}\right)\right)$$

the same as for the circuit #1; for off triples of ω_0, I mean

circuit #2, at even $3\omega_0$



and now, something completely different ...

$$\underline{Z}_{even,\,2n} = \frac{\underline{V}_{A,\,2n}}{\underline{I}_{A,\,2n}} = \frac{\underline{V}_{B,\,2n}}{\underline{I}_{B,\,2n}} = 2\,a\,R + \frac{1}{(2n)\,j\,3\omega_0\,(C/2)}$$

$$\underline{Z}_{even,\,2n} = 2\,a\,R + \frac{1}{(2n)\,j\,(1/(2R_0))}$$

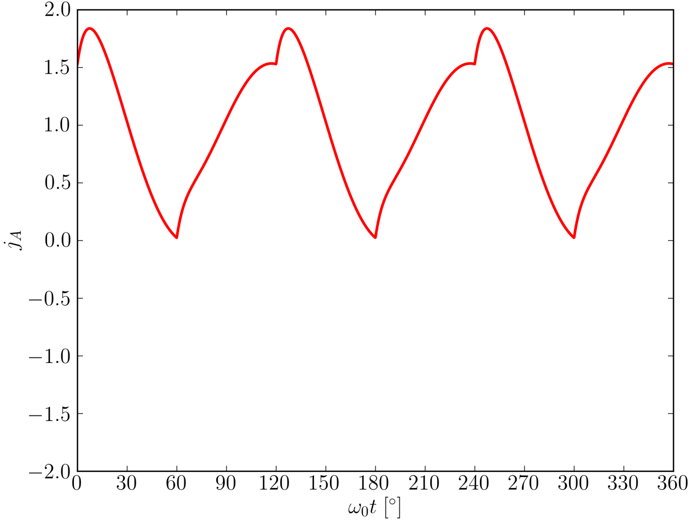
and some polish ...

for k even, $k = 2n$:

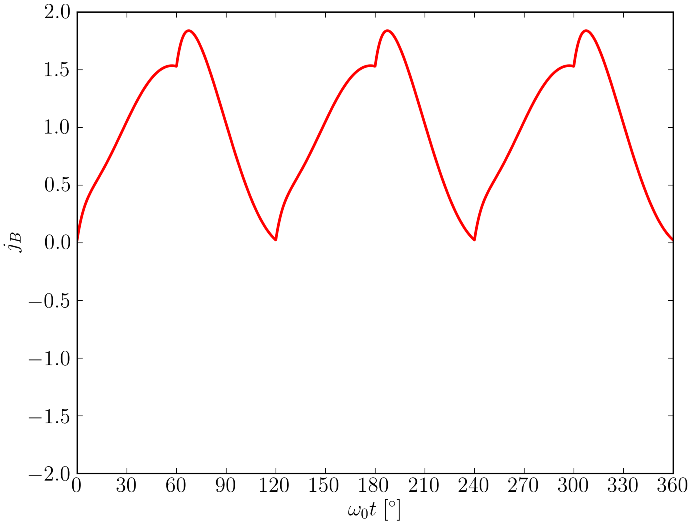
$$\underline{Z}_{even,\,k} = 2\,a\,R + 2\,R_0\,\frac{1}{jk}$$

$$\boxed{\underline{Z}_{even,\,k} = 2\,R\left(a - j\,Q\,\frac{1}{k}\right)}$$

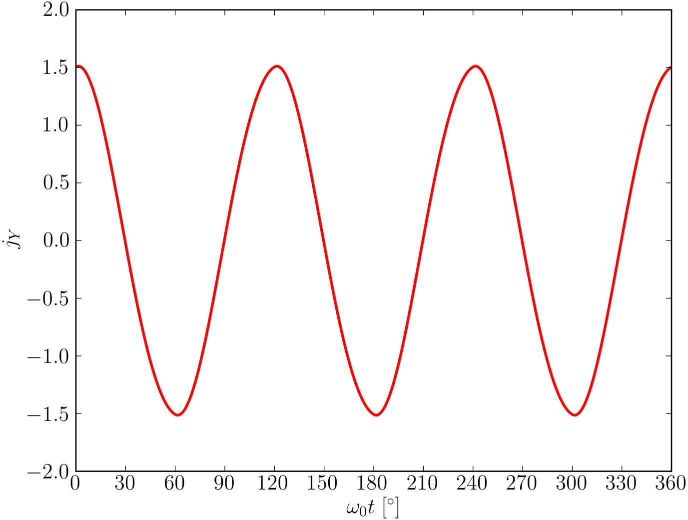
simulation, j_A, $Q = 2$, $a = 0.5$, circuit #2



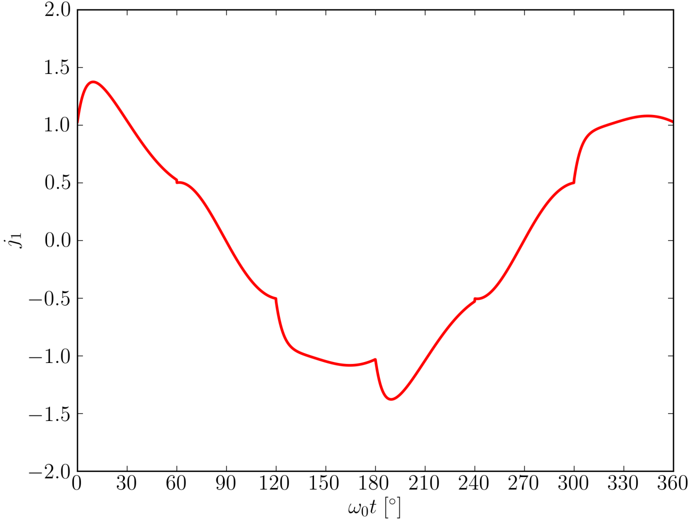
simulation, j_B, $Q = 2$, $a = 0.5$, circuit #2



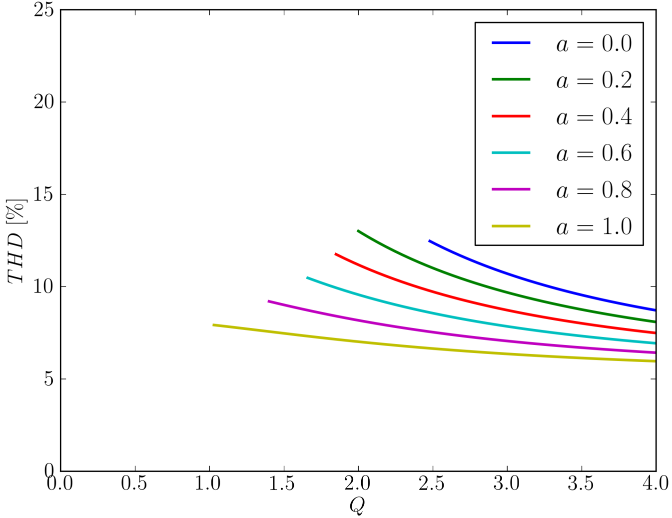
simulation, j_Y, $Q = 2$, $a = 0.5$, circuit #2



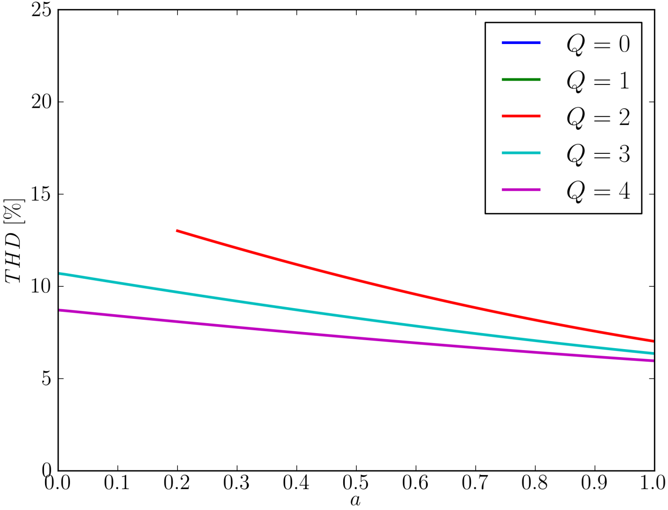
simulation, j_1, $Q = 2$, $a = 0.5$, circuit #2



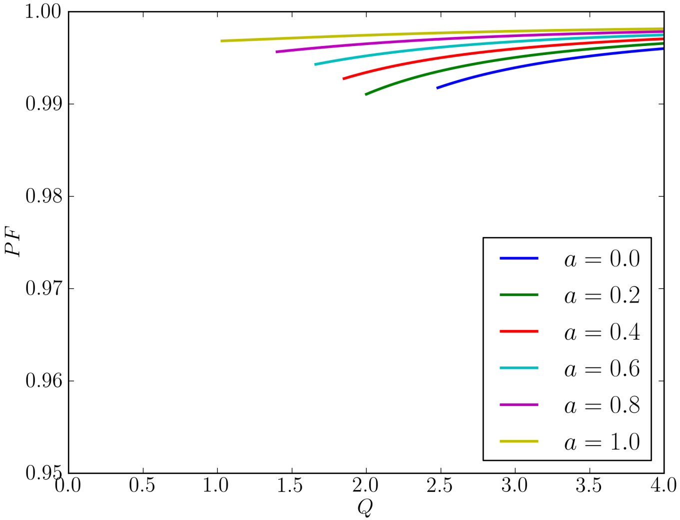
$THD(Q)$, a parameter, circuit #2



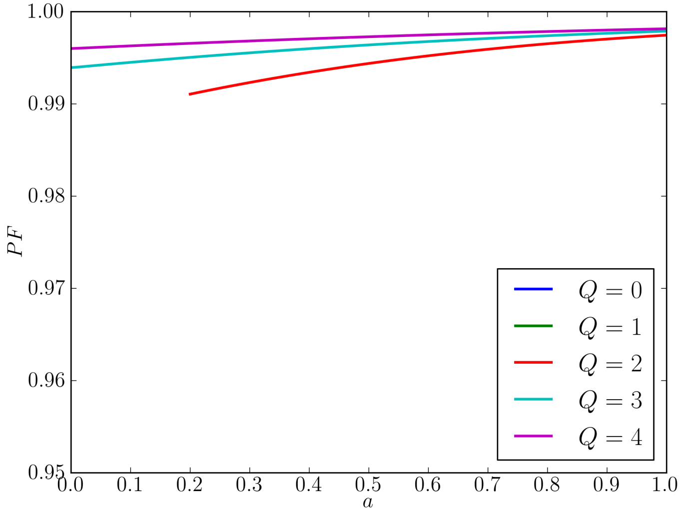
$THD(a)$, Q parameter, circuit #2



$PF(Q)$, a parameter, circuit #2



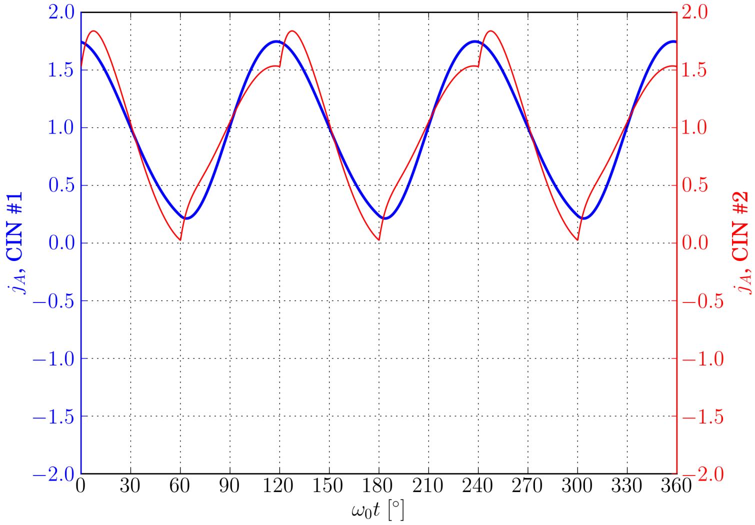
$PF(a)$, Q parameter, circuit #2



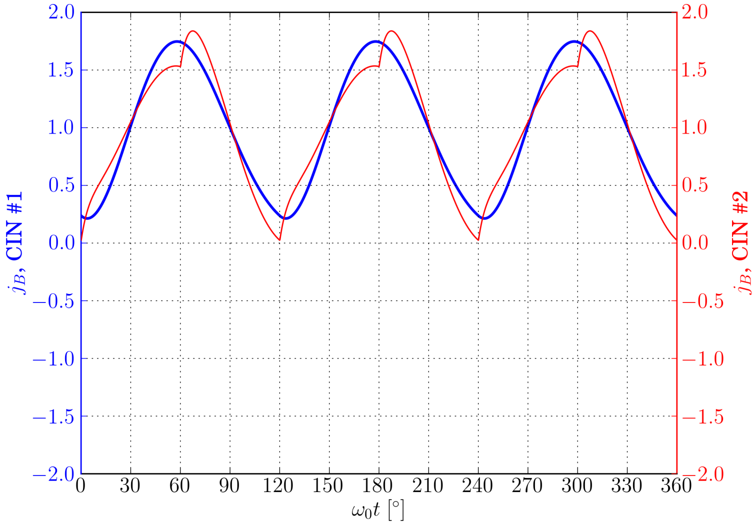
some comments . . .

- the diagrams end when the DCM is reached
- DCM? in CCM $i_A > 0$ and $i_B > 0$ all the time
- increased Q improves response
- increased a improves response
- much worse than the circuit #1
- reduced CCM range

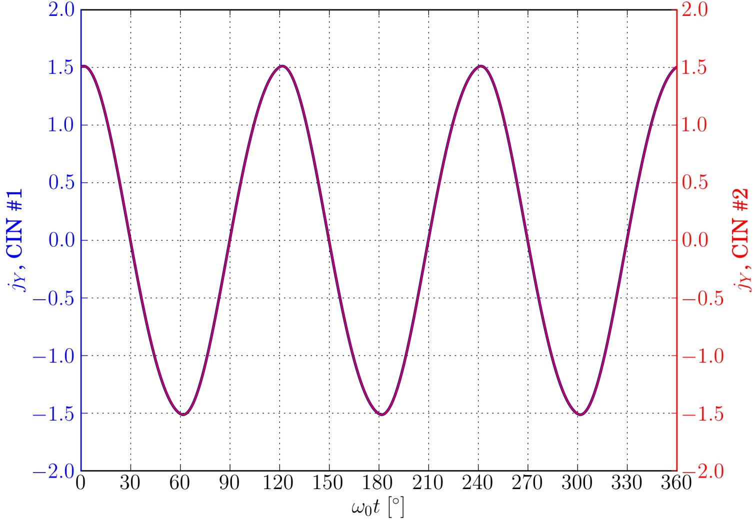
comparison, j_A, $Q = 2$, $a = 0.5$



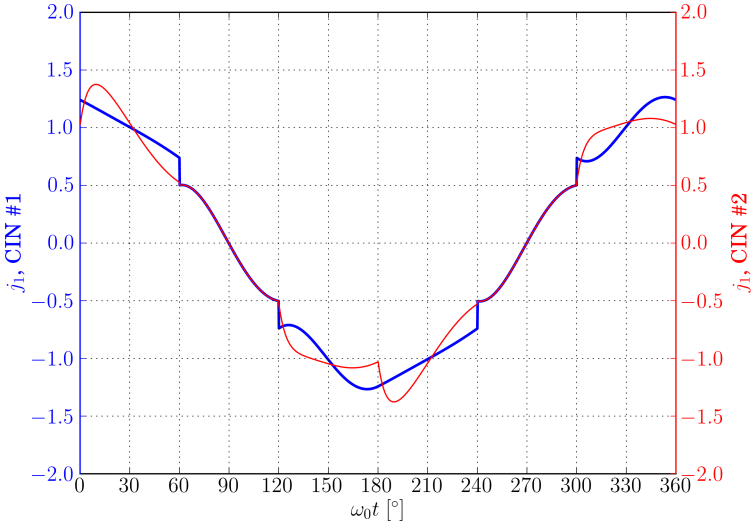
comparison, j_B, $Q = 2$, $a = 0.5$



comparison, j_Y, $Q = 2$, $a = 0.5$



comparison, j_1, $Q = 2$, $a = 0.5$

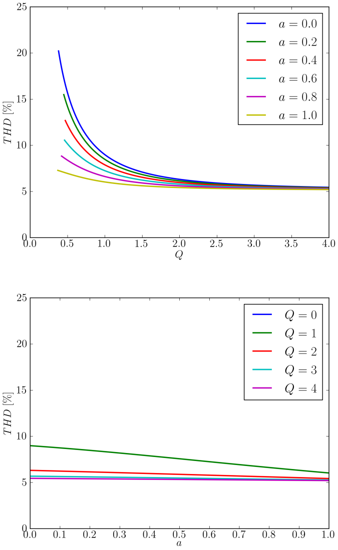


comparison at $Q = 2$ and $a = 0.5$

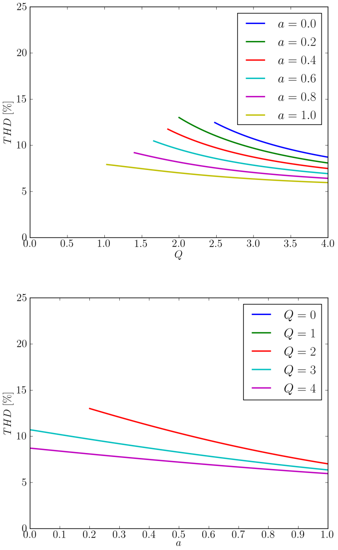
CID #	$THD\,(i_k)$	PF
1	5.88 %	0.9982
2	10.35 %	0.9944

comparison, THD

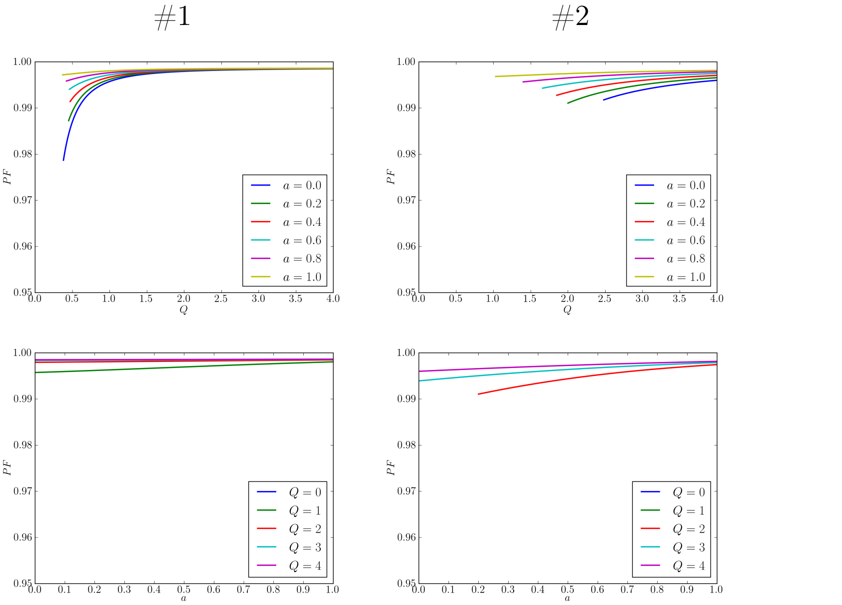
#1



#2



comparison, PF



some comments . . . and a comparison

- comparison between the two circuits . . .
- fair comparison, Q and a are the same
  1. capacitors are the same
  2. VA-ratings of the inductors "the same" $2\,S_{L,\,\#1} = S_{L,\,\#2}$
- although #2 is likely to have lower a, inductors . . .
- circuit #2 performs worse:
  1. higher THD
  2. lower PF
  3. pronounced DCM problems
  4. higher Q required
- **but published later!**

this story was published in . . .

Predrag Pejović, Žarko Janda

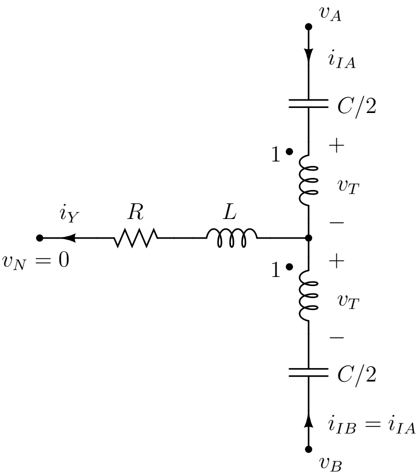
“An Analysis of Three Phase Low Harmonic Rectifiers Applying the Third Harmonic Current Injection”

IEEE Transactions on Power Electronics,
vol. 14, no. 3, pp. 397–407, May 1999

conclusions after the analyses

- even triples of ω_0 cause big trouble:
  1. high THD
  2. lower PF
  3. DCM
- **is there a way to get rid of the even triples completely?**

circuit #3, asymmetric



published in . . .

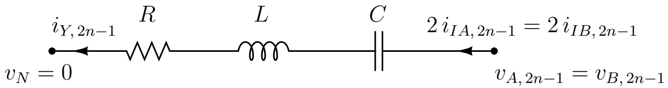
Predrag Pejović, Žarko Janda

“An Improved Current Injection Network for Three Phase High Power Factor Rectifiers that Apply the Third Harmonic Current Injection”

IEEE Transactions on Industrial Electronics,
vol. 47, no. 2, pp. 497–499, April 2000

and rejected for EPE’99, in “as is” form

circuit #3, at odd $3\omega_0$, reduced



resonance, R, impedance, ...

$$3\omega_0 = \frac{1}{\sqrt{L\,C}}$$

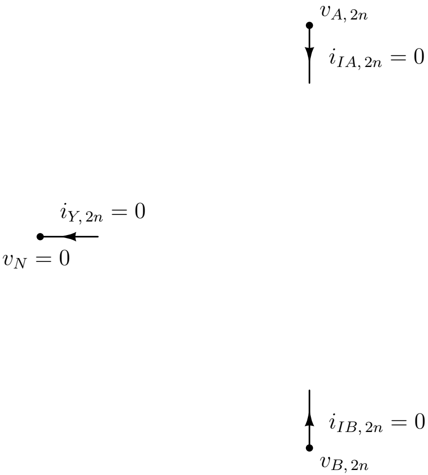
$$R = \frac{V_{A,\,1}}{I_{Y,\,1}} = \frac{\sqrt{3}}{4\pi}\,\frac{V_m}{I_{OUT}} \qquad \rho = R\,\frac{I_{OUT}}{V_m} = \frac{\sqrt{3}}{4\pi} \approx 0.13783$$

$$\underline{Z}_{odd,\,k} = R\left(1 + j\,Q\left(k - \frac{1}{k}\right)\right)$$

$$\underline{Z}_{even,\,k} = \infty$$

1. for “odd triples” the same as for the both of already analyzed circuits
2. for “even triples” quite different, open circuit

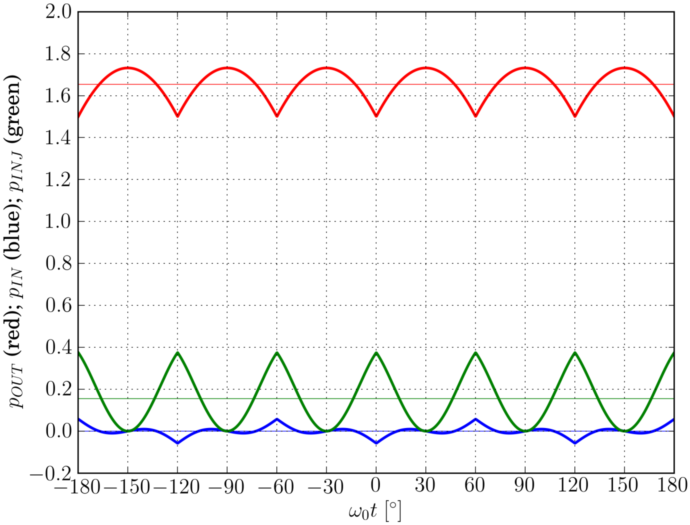
circuit #2, at even $3\omega_0$



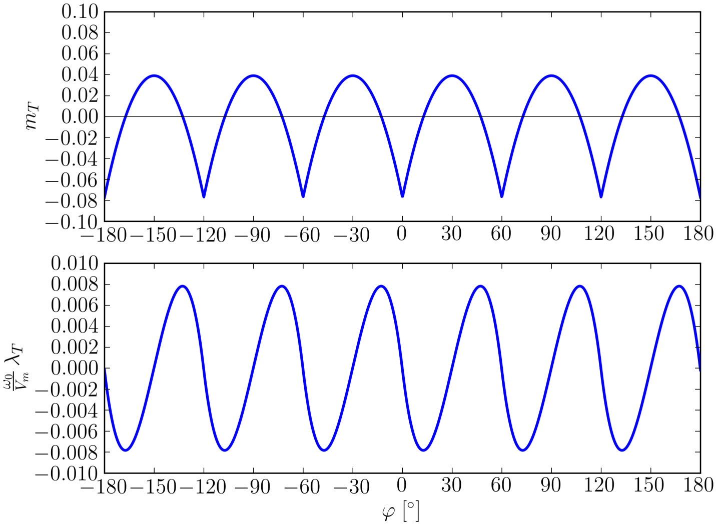
some notes

- a is omitted; actually, makes no difference; there is nothing at even $3\omega_0$, where a has an effect
- having one inductor is an advantage
- what is the VA-rating of the 1:1 transformer?
  1. $I_{T\,RMS} = \frac{3}{4\sqrt{2}} I_{OUT}$
  2. $v_T = \frac{1}{2}\left(v_{OUT} - V_{OUT}\right)$ (prove!)
  3. $\lambda_{T\,max}$ to be found;
     however: small amplitude, sixth harmonic dominant

power at the 1:1 transformer



$\lambda_{T\,max}$, numerical estimate



$\lambda_{T\,max}$, VA-rating ...

$$\lambda_{T\,max} = \frac{\sqrt{3}}{2\pi}\left(\sqrt{\pi^2 - 9} - 3\arccos\left(\frac{3}{\pi}\right)\right)\frac{V_m}{\omega_0} \approx 0.00783\,\frac{V_m}{\omega_0}$$

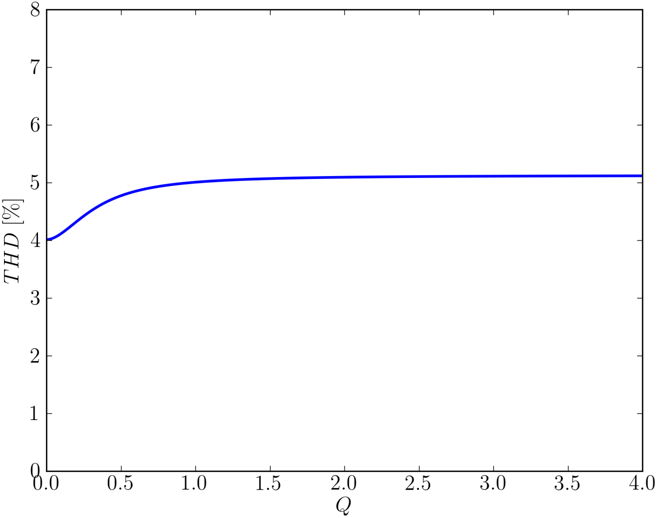
consider this as having fun: exact calculations with approximate figures

$$S_T = \frac{3\omega_0}{8}\,\lambda_{T\,max}\,I_{OUT}$$

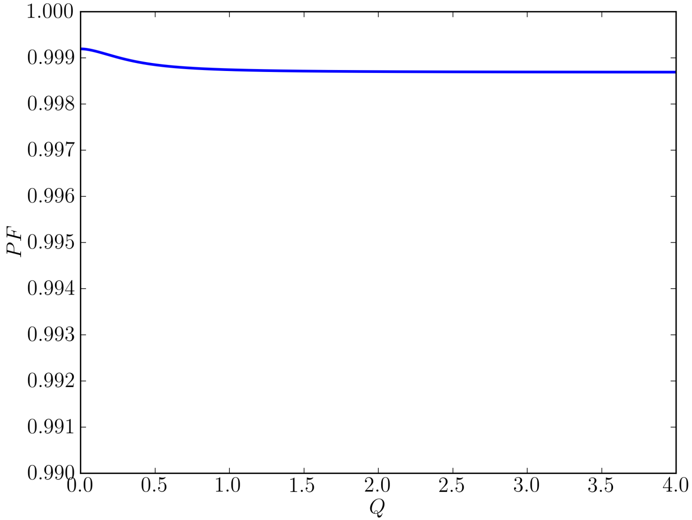
and after normalization to P_{OUT} and P_{IN}

$$\boxed{S_T \approx 0.18\,\%\;P_{OUT} \approx 0.16\,\%\;P_{IN}}$$

$THD(Q)$, derate with Q ... derate?

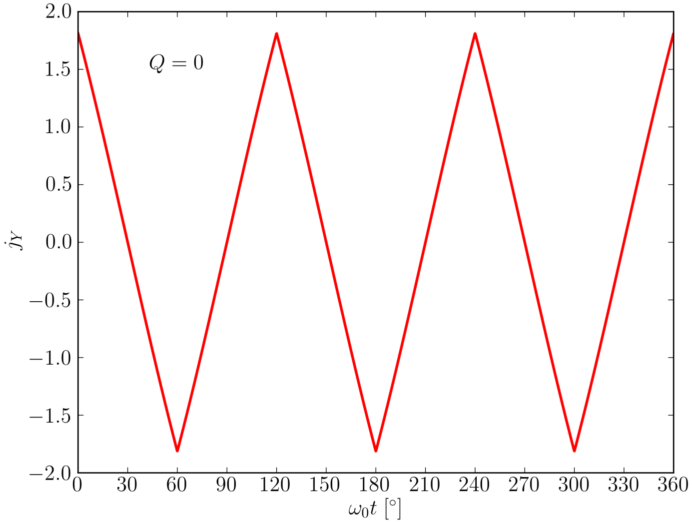


$PF(Q)$



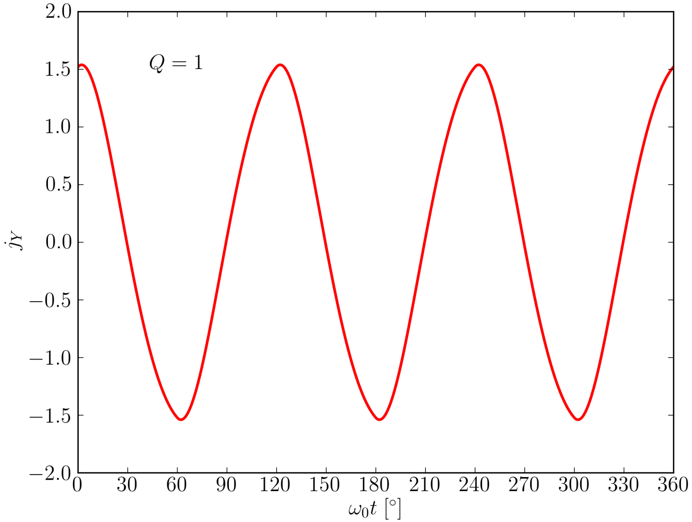
1.000
0.999
0.998
0.997
0.996
0.995
0.994
0.993
0.992
0.991
0.990
PF
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Q

j_Y, $Q = 0$



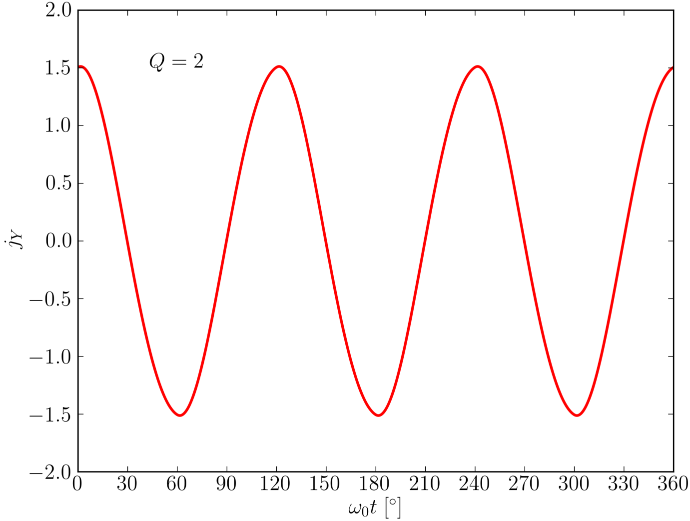
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
j_Y
$Q = 0$
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_Y, $Q = 1$



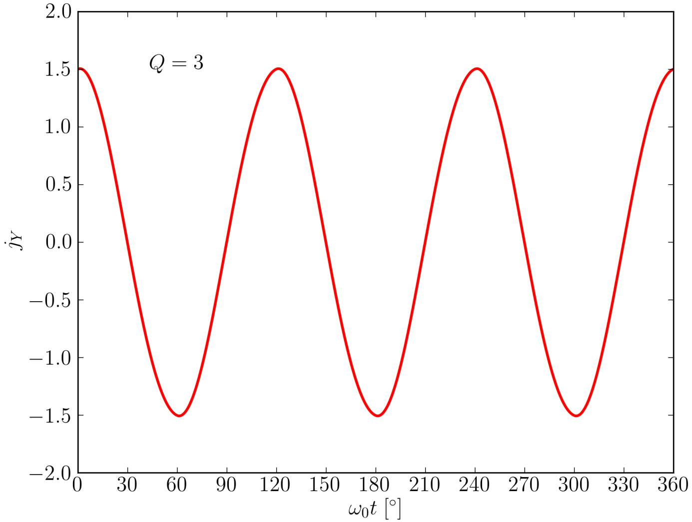
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
j_Y
$Q = 1$
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_Y, $Q = 2$



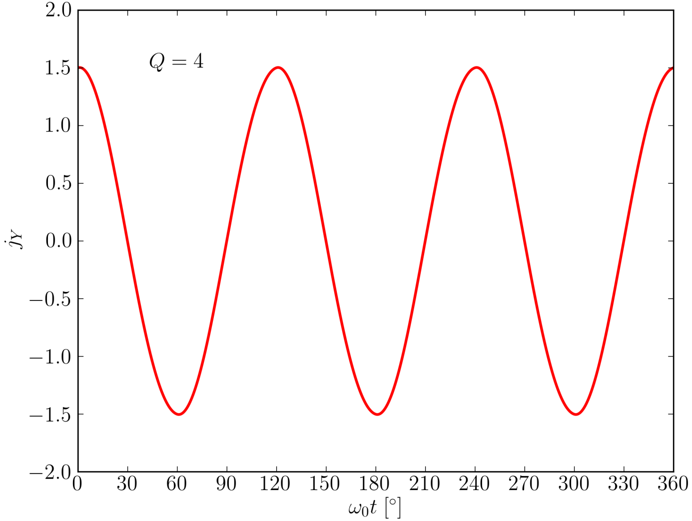
$Q = 2$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
j_Y
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_Y, $Q = 3$



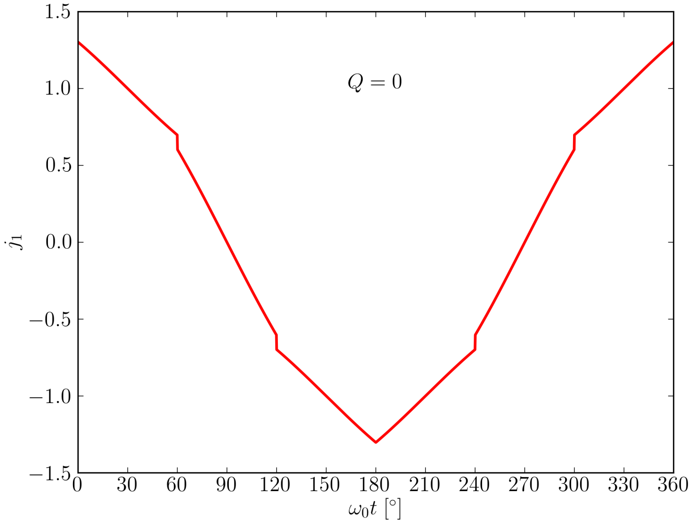
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
$Q = 3$
j_Y
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_Y, $Q = 4$



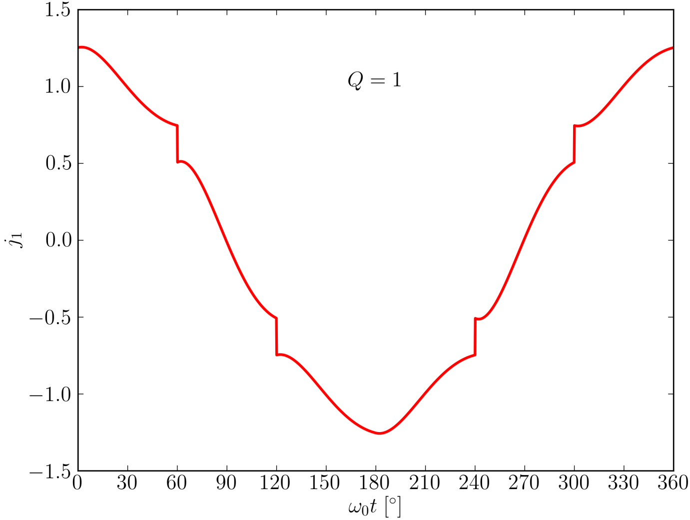
$Q = 4$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
j_Y
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_1, $Q = 0$



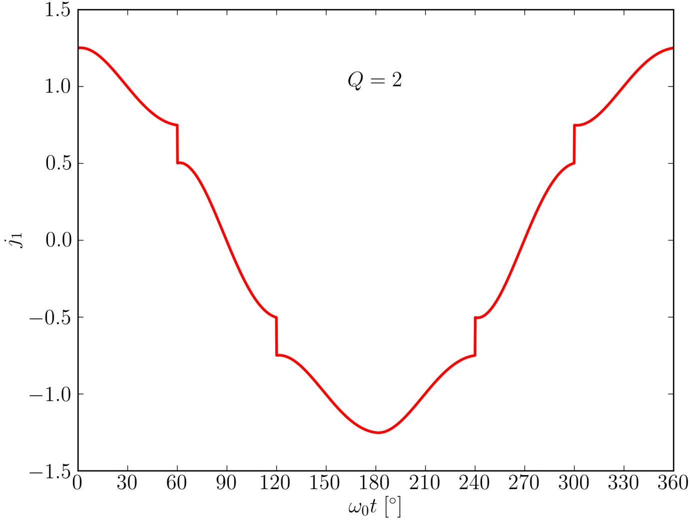
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j_1
$Q = 0$
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_1, $Q = 1$



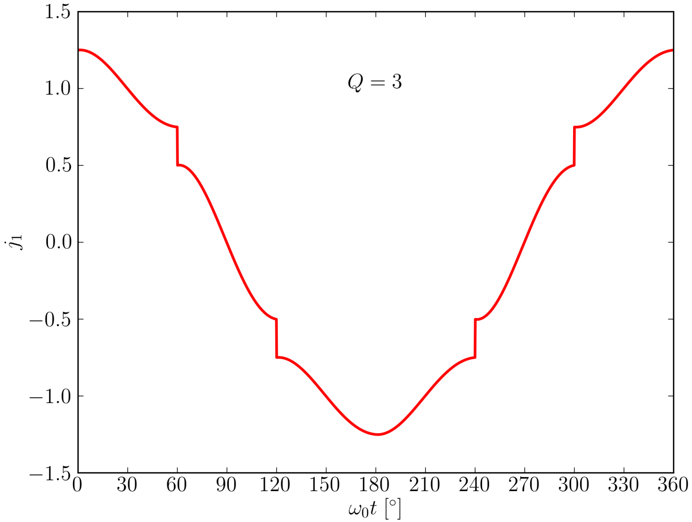
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j_1
$Q = 1$
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_1, $Q = 2$



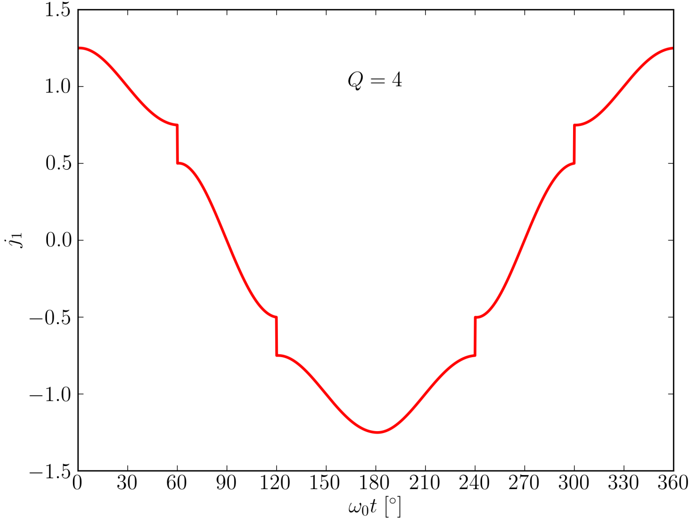
$Q = 2$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j_1
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

j_1, $Q = 3$



Q = 3
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j₁
0
30
60
90
120
150
180
210
240
270
300
330
360
ω₀t [°]

j_1, $Q = 4$

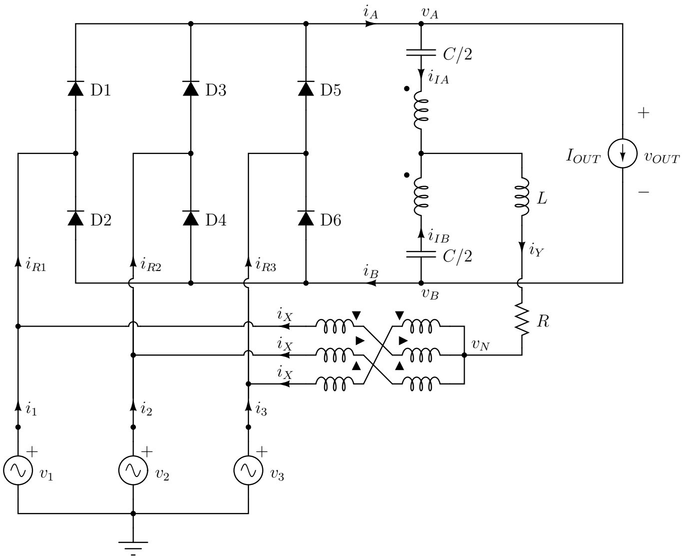


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j_1
$Q = 4$
0
30
60
90
120
150
180
210
240
270
300
330
360
$\omega_0 t$ [°]

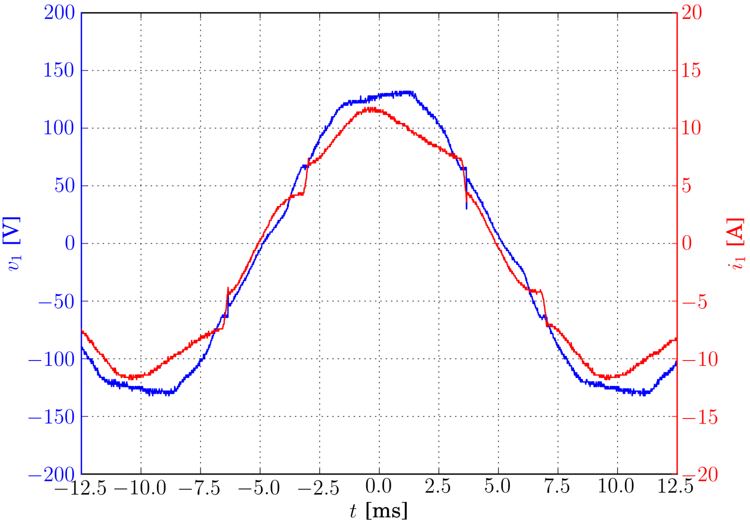
some figures . . .

Q	THD	PF
0.0	4.02	0.9992
1.0	5.01	0.9987
2.0	5.10	0.9987
3.0	5.11	0.9987
4.0	5.12	0.9987

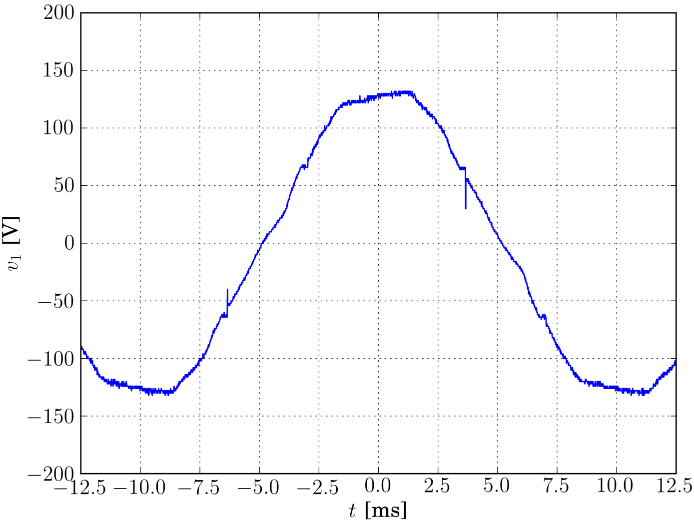
rectifier as a whole . . .



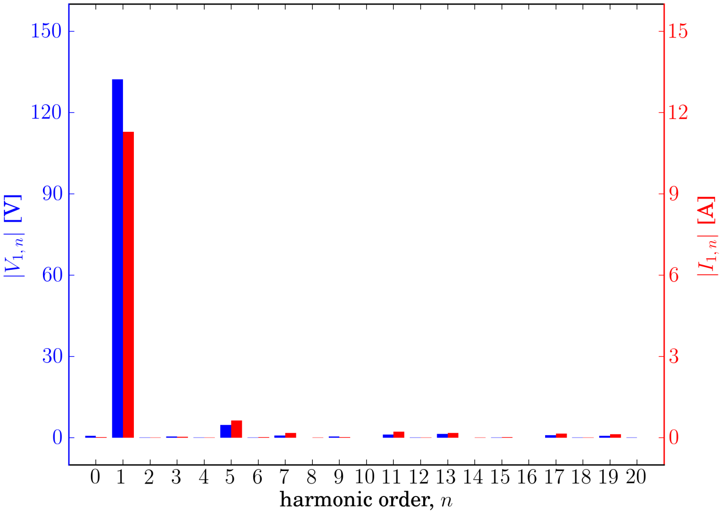
v_1 and i_1, experimental



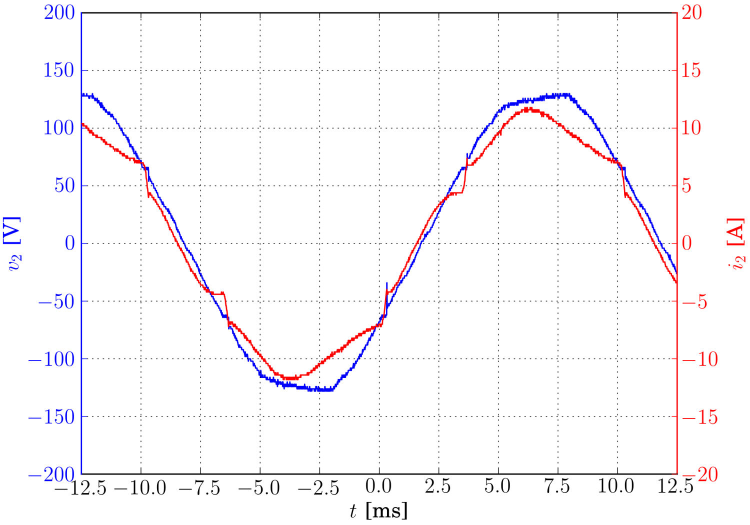
v_1, experimental



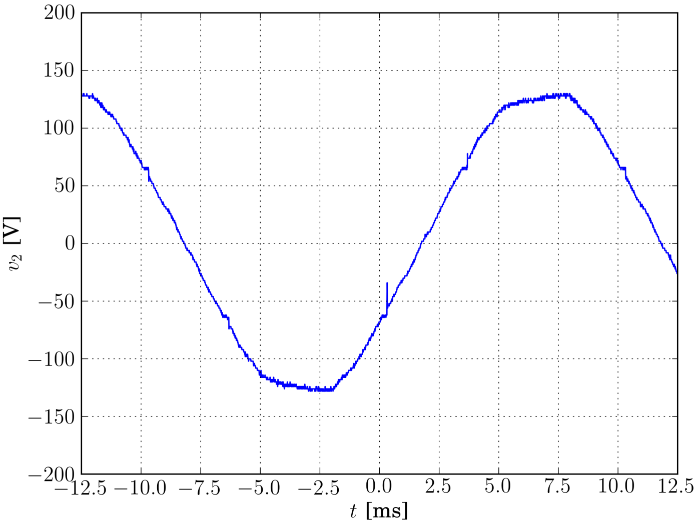
v_1 and i_1, experimental, spectra



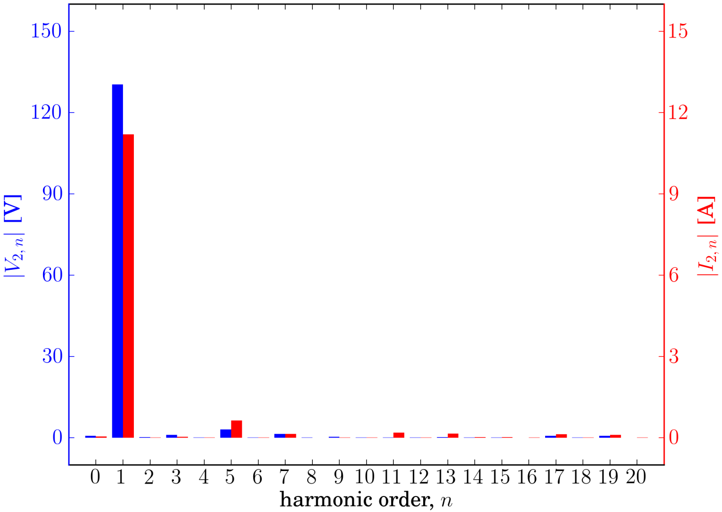
v_2 and i_2, experimental



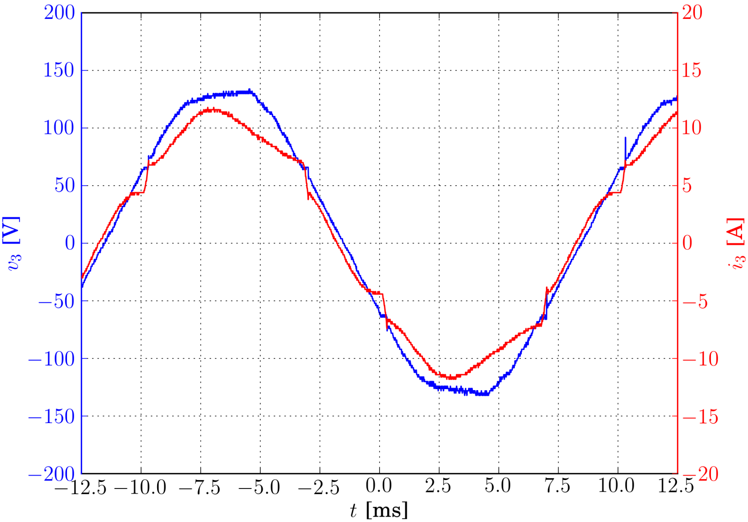
v_2, experimental



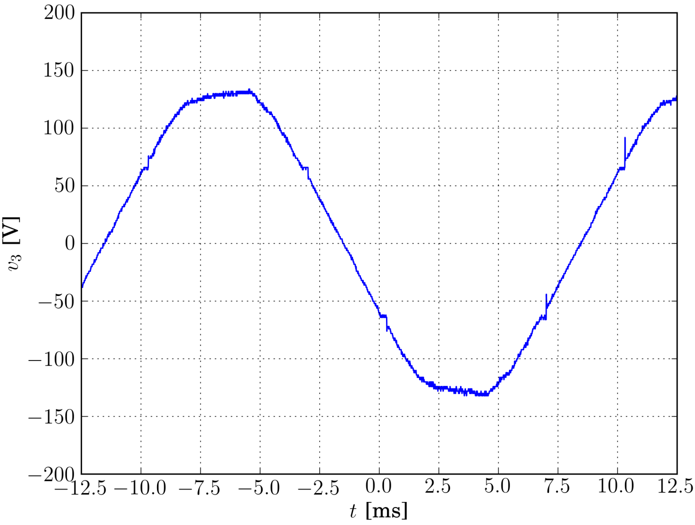
v_2 and i_2, experimental, spectra



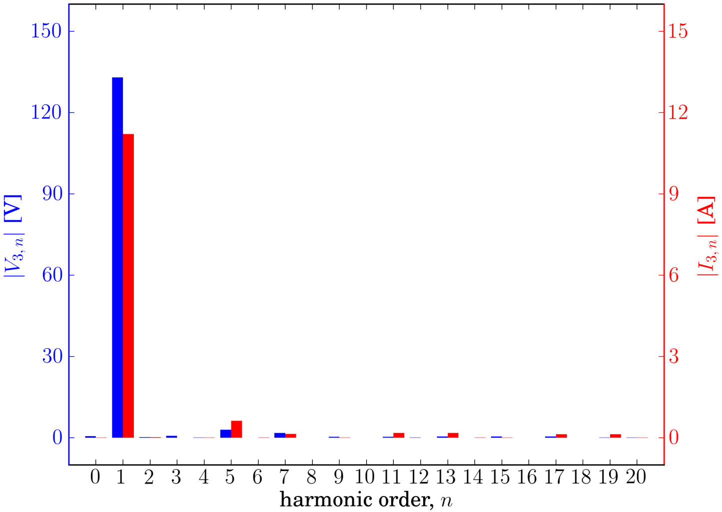
v_3 and i_3, experimental



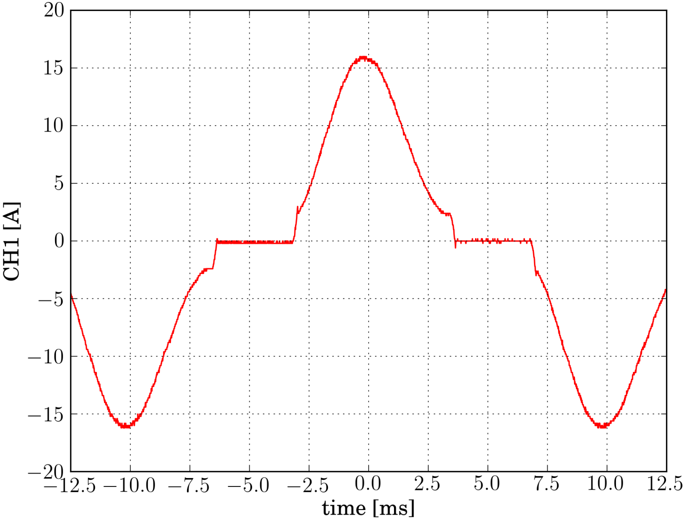
v_3, experimental



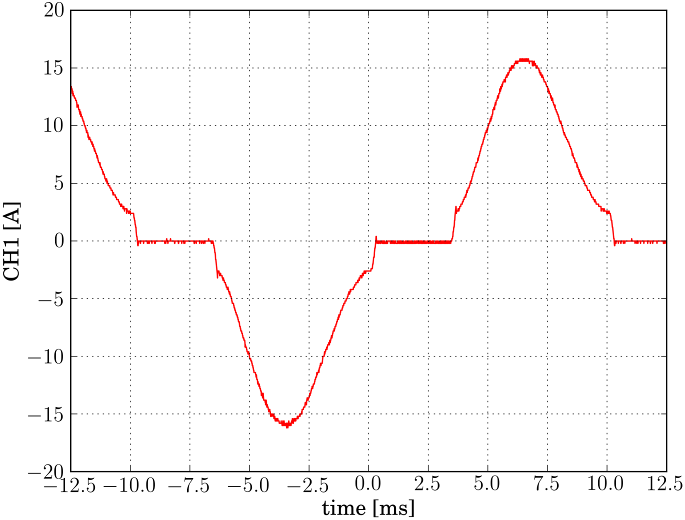
v_3 and i_3, experimental, spectra



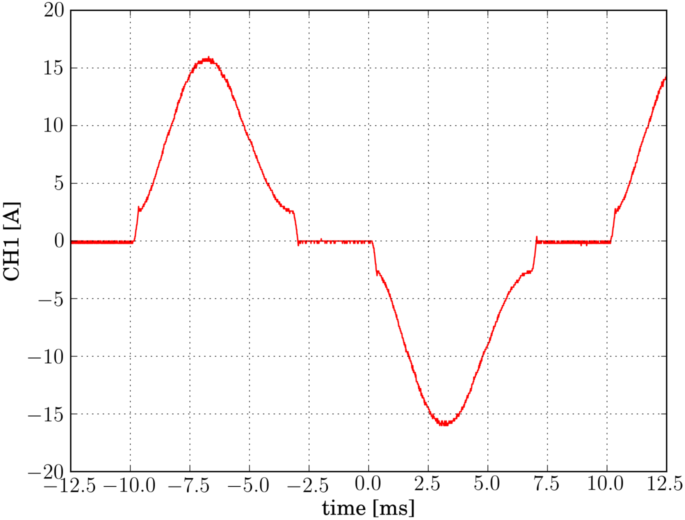
i_{R1}, experimental



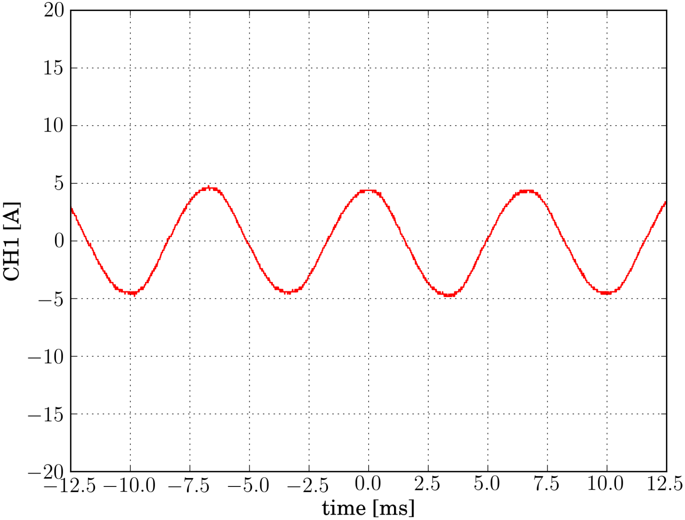
i_{R2}, experimental



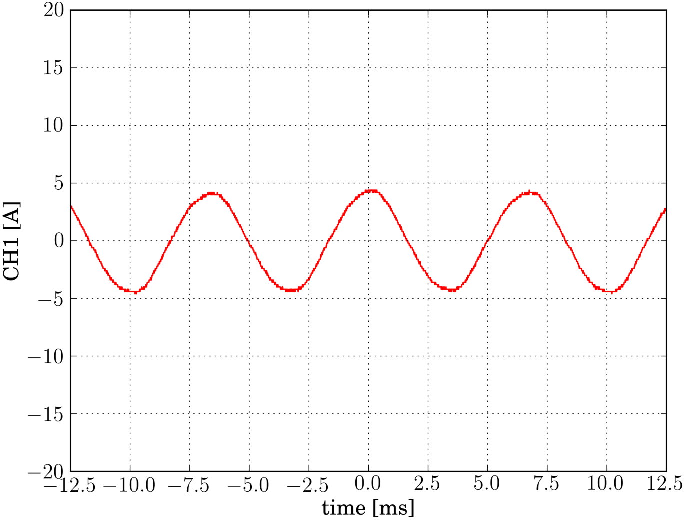
i_{R3}, experimental



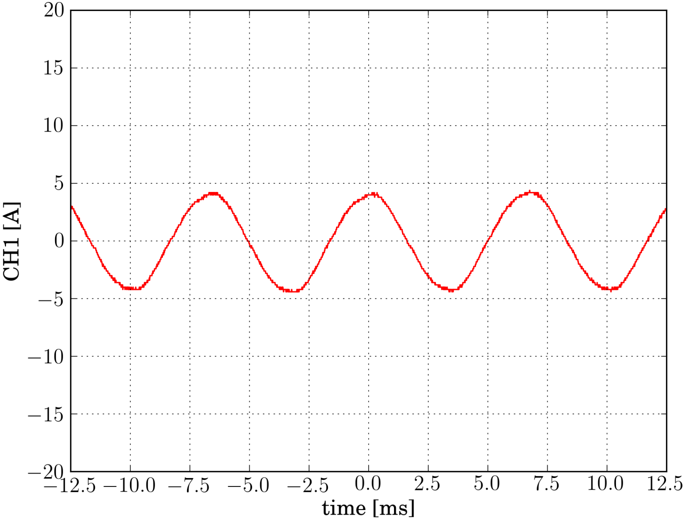
i_{X1}, experimental



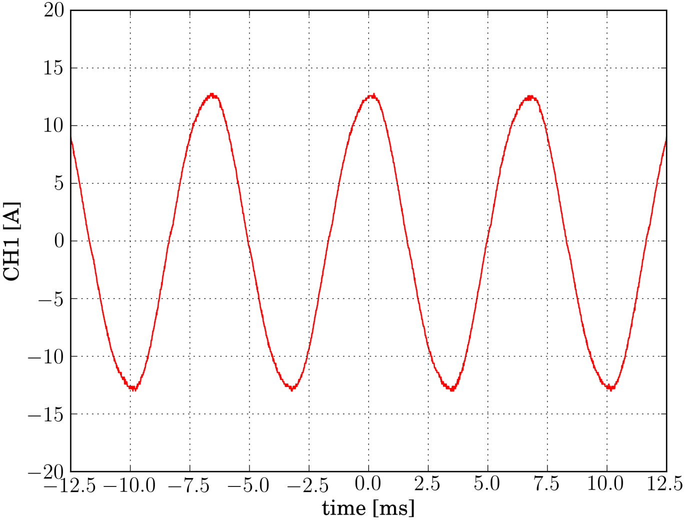
i_{X2}, experimental



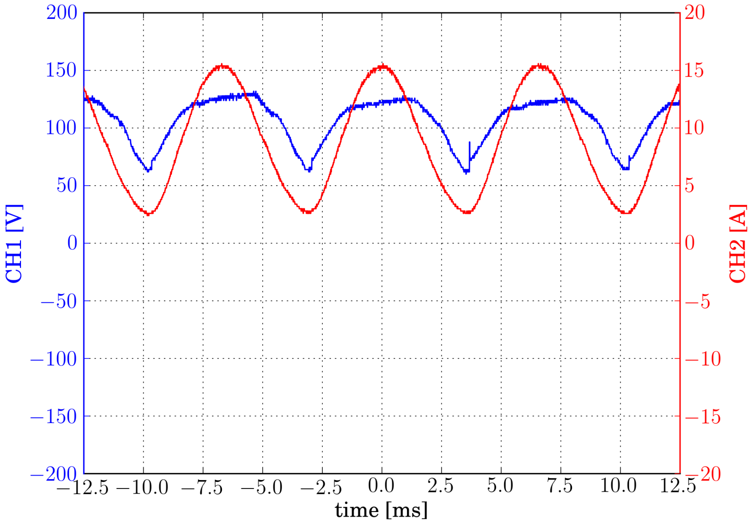
i_{X3}, experimental



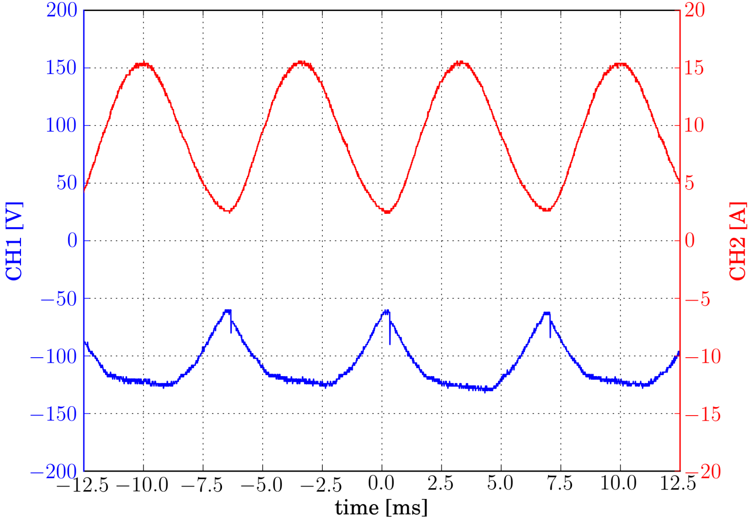
i_Y, experimental



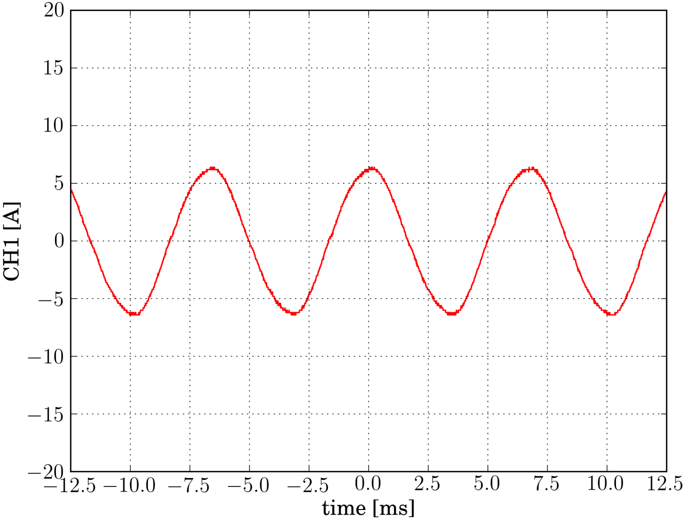
v_A and i_A, experimental



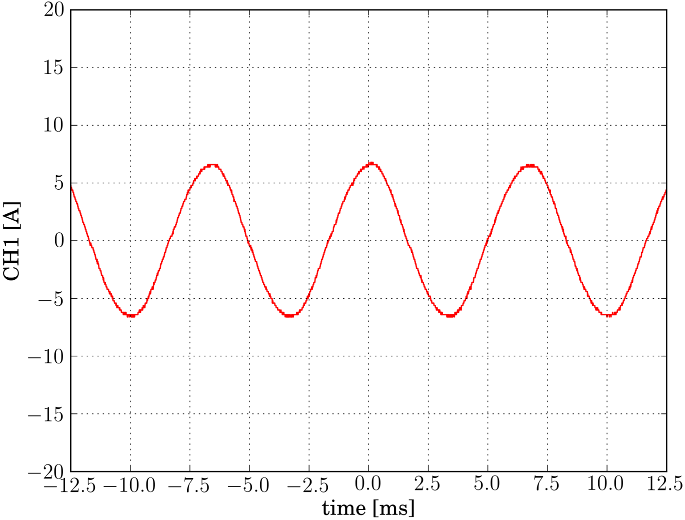
v_B and i_B, experimental



i_{IA}, experimental



i_{IB}, experimental



experimental results . . .

k	$I_{k\,RMS}$ [A]	$V_{k\,RMS}$ [V]	S [VA]	P [W]
1	8.00	93.63	748.95	744.85
2	7.94	92.25	732.07	728.56
3	7.94	94.08	747.11	743.89

experimental results . . .

k	PF	$THD(i_k)$ [%]	$THD(v_k)$ [%]
1	0.9945	7.15	4.39
2	0.9952	6.95	3.20
3	0.9957	6.84	3.17

experimental results . . .

$$I_{OUT} = 9.53\,\text{A}$$

$$V_{OUT} = 213.48\,\text{V}$$

$$P_{OUT} = 2035.32\,\text{W}$$

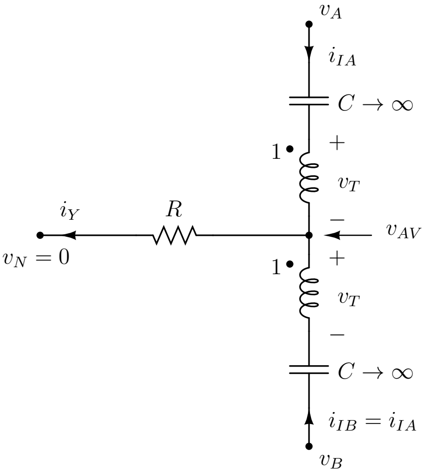
$$P_{IN} = 2217.30\,\text{W}$$

$$\eta = 91.793\,\%$$

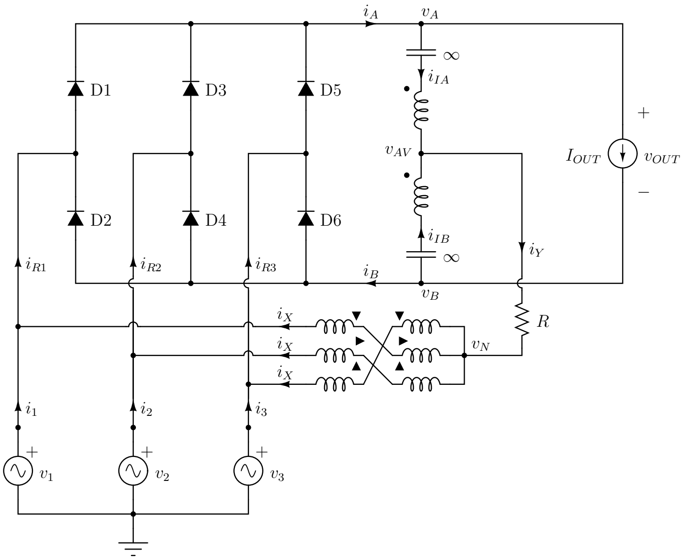
a special case, $Q = 0$

- inductorless design
- $THD \approx 4.0155\%$
- $PF \approx 0.9992$
- no resonance constraints
- suitable for switching resistance emulation

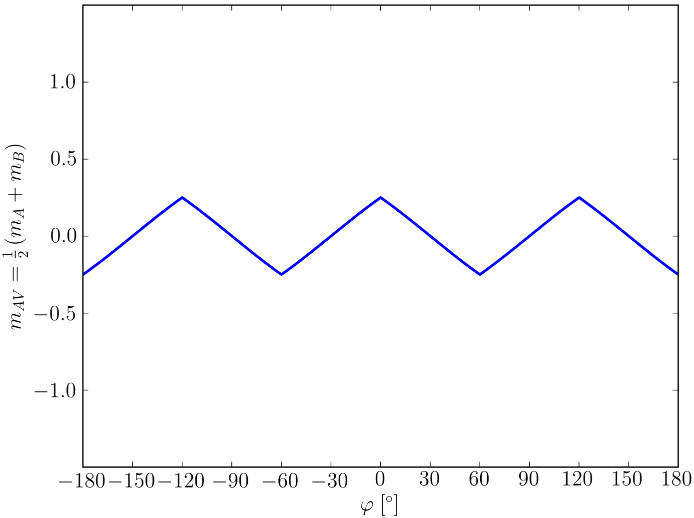
circuit #3, no resonance



the whole circuit . . .

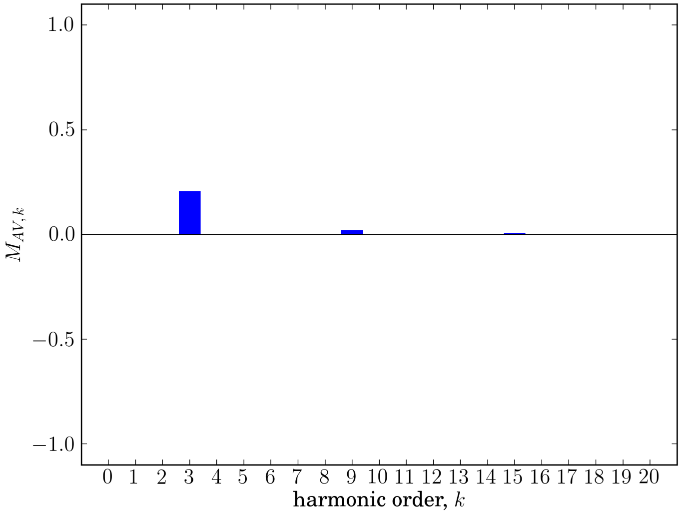


m_{AV}

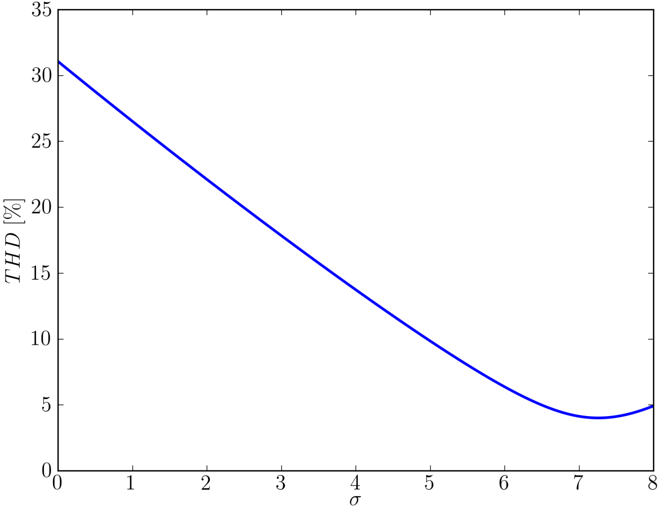


1.0
0.5
0.0
−0.5
−1.0
$m_{AV} = \frac{1}{2}(m_A + m_B)$
−180 −150 −120 −90 −60 −30 0 30 60 90 120 150 180
φ [°]

m_{AV}, spectrum



$THD(\sigma)$



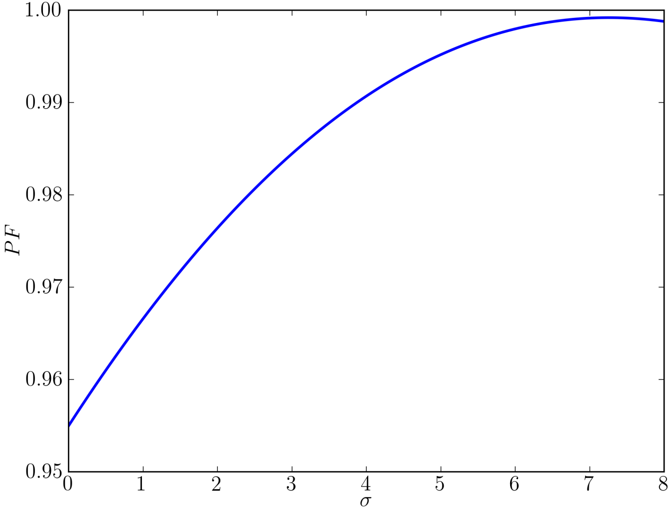
analytical optimization . . .

$$THD_{min} = \frac{\sqrt{4\pi^4 - 27\pi^2 + 216\sqrt{3}\pi - 1296}}{2\pi^2 - 3\sqrt{3}\pi + 36} \approx 4.01\%$$

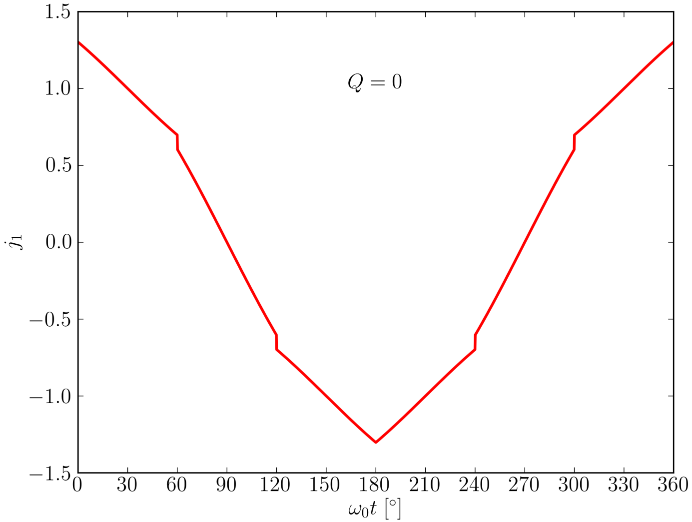
$$\sigma_{opt} = \frac{4\pi}{\sqrt{3}}$$

$$\sigma_{opt} = \frac{1}{\rho}$$

$PF(\sigma)$



j_1, σ_{opt}



Q = 0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
j1
0
30
60
90
120
150
180
210
240
270
300
330
360
ω0t [°]

conclusions

- circuits #1, #2, and #3 compared
- circuit #3 provides the best performance:
  1. the smallest THD
  2. single inductor
  3. good dependence on Q
  4. no dependence on a
  5. not having problems with the DCM
  6. special version, $Q = 0$, without resonance
- all future designs will assume circuit #3
- circuits #1 and #2 abandoned

“future work”

1. is there a way to improve the THD further?
2. is there a simple way to restore the power taken by the current injection network?
3. what happens in the DCM? any interest in that?