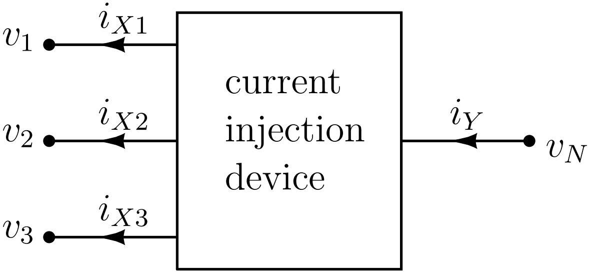
Current Injection Devices

current injection device (CID)?



element equations

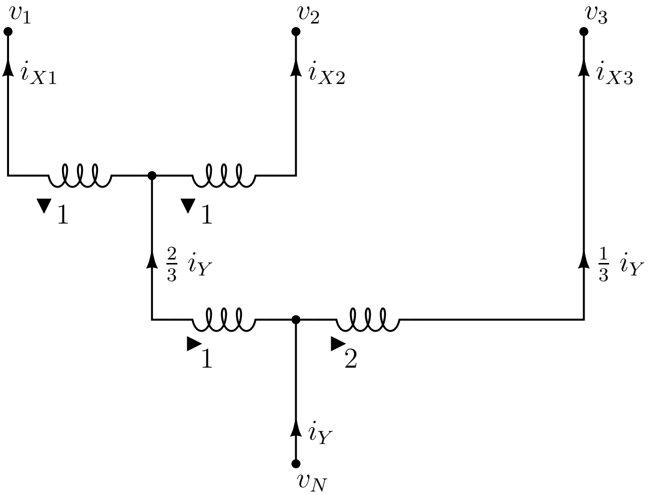
$$i_{X1} = i_{X2} = i_{X3} = \frac{1}{3}\, i_Y$$

$$v_N = \frac{1}{3}\,(v_1 + v_2 + v_3)$$

which is:

1. **resistive**, i.e. no $\frac{d}{dt}$ in element equations
2. **non-dissipative**, i.e. $v_N\, i_Y - v_1\, i_{X1} - v_2\, i_{X2} - v_3\, i_{X3} = 0$

possible? out-of-trash solution, CID #0



what is this for? a specific application . . .

intended for:

$$v_1 = V_m \cos(\omega_0 t)$$

$$v_2 = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

and

$$i_Y = \frac{3}{2} I_{OUT} \cos(3\omega_0 t)$$

$$i_X = \frac{1}{2} I_{OUT} \cos(3\omega_0 t)$$

VA-rating?

- a rough measure of the component size
- magnetic components should not:
  1. cause explosions
  2. cause fire
- which means:
  1. $|\vec{B}| < B_{sat}$
  2. $J_{RMS} < J_{max}$
- to simplify, let's neglect dependence on frequency
- we deal mainly with ω_0 and $3\omega_0$
- as stated, let's neglect derate with frequency: skin effect, proximity effect, eddy currents, hysteresis losses, ...
- in every transformer, there are windings and the core ...

current handling capability; preventing fire

the windings should fit into the core window

$$\sum_{k=1}^{n_w} N_k \, I_{k\,RMS} \leq k_{ff} \, J_{max} \, A_{window}$$

where

$$k_{ff} \, J_{max} \, A_{window}$$

is the "current handling capability" of the core

more turns, more stress on current handling

a word about flux . . .

flux linkage

$$\lambda_k = \int v_k\,(t)\ dt$$

don't mention integrating constants, please

assuming perfect coupling

$$\frac{\lambda_1}{N_1} = \ldots = \frac{\lambda_{n_w}}{N_{n_w}} = \Phi$$

where Φ is the core flux

flux handling capability; preventing explosions

the core should not saturate

$$\frac{\lambda_{k\,max}}{N_k} \leq \Phi_{max}$$

for $k \in \{1, \ldots n_w\}$ where

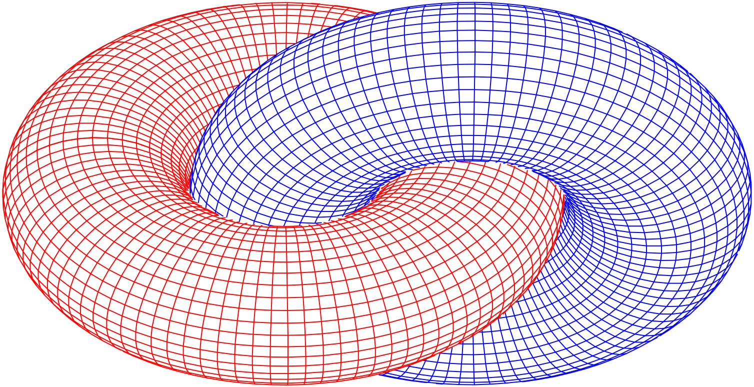
$$\Phi_{max} = A_{core}\, B_{sat}$$

is the "flux handling capability" of the core

more turns, less stress on flux handling

there is a trade-off over N_k variable

current and flux



let's put them together . . .

let the winding indexed 1 be "privileged"

$$\frac{\lambda_{1\,max}}{N_1} \sum_{k=1}^{n_w} N_k \, I_{k\,RMS} \leq A_{core} \, B_{sat} \, k_{ff} \, J_{max} \, A_{window}$$

$$\lambda_{1\,max} \sum_{k=1}^{n_w} \frac{N_k}{N_1} \, I_{k\,RMS} \leq k_{ff} \, J_{max} \, B_{sat} \, A_{window} \, A_{core}$$

$$\lambda_{1\,max} \sum_{k=1}^{n_w} n_k \, I_{k\,RMS} \leq k_{ff} \, J_{max} \, B_{sat} \, A_{window} \, A_{core}$$

where

$$n_k \triangleq \frac{N_k}{N_1}$$

are the turns ratios, normalized to N_1, $n_1 = 1$

a single-phase two-winding transformer, sinusoidal case

$$v_1 = V_1 \sin(\omega_0 t)$$

$$\lambda_{1\,max} = \frac{V_1}{\omega_0} = \frac{\sqrt{2}\, V_{1\,RMS}}{\omega_0}$$

$$\lambda_{1\,max} \sum_{k=1}^{2} n_k\, I_{k\,RMS} = \frac{V_1}{\omega_0}\, \left(I_{1\,RMS} + n_2\, I_{2\,RMS}\right)$$

$$n_2\, I_{2\,RMS} = I_{1\,RMS}$$

$$\lambda_{1\,max} \sum_{k=1}^{2} n_k\, I_{k\,RMS} = \frac{2\sqrt{2}}{\omega_0}\, V_{1\,RMS}\, I_{1\,RMS} = \frac{2\sqrt{2}}{\omega_0}\, S$$

finally, VA-rating

for an equivalent size single-phase transformer

$$S = \frac{\omega_0}{2\sqrt{2}}\, k_{ff}\, J_{max}\, B_{sat}\, A_{window}\, A_{core}$$

reduced for non-sinusoidal cases

$$\boxed{S = \frac{\omega_0}{2\sqrt{2}}\, \lambda_{1\,max} \sum_{k=1}^{n_w} n_k\, I_{k\,RMS}}$$

please note: ω_0 applies for the original single-phase transformer, where the core is rated

assumed: k_{ff}, J_{max}, B_{sat} constant, not dependent on frequency, waveform, ...

since we are already here, VA-rating of inductors

$$v_L = \frac{d\lambda_L}{dt} = L\,\frac{di_L}{dt}$$

$$\lambda_L = L\,i_L$$

$$\lambda_{max} = L\,i_{L\,max}$$

$$S_L = \frac{\omega_0}{2\sqrt{2}}\,\lambda_{max}\,I_{L\,RMS}$$

$$\boxed{S_L = \frac{\omega_0}{2\sqrt{2}}\,L\,i_{L\,max}\,I_{L\,RMS}}$$

no matter what the actual inductance is . . .

VA-rating, three-phase

not a big deal, just assume symmetry and generalize ...

$$\boxed{S = \frac{3\,\omega_0}{2\sqrt{2}}\,\lambda_{1\,max}\sum_{k=1}^{n_w} n_k\,I_{k\,RMS}}$$

some results to be used . . .

basis to normalize VA-ratings

$$P_{OUT} = \frac{3\sqrt{3}}{\pi} V_m I_{OUT} \quad \text{or} \quad P_{IN} = \frac{105\sqrt{3}}{32\pi} V_m I_{OUT}$$

some RMSs

$$I_{Y\,RMS} = \frac{3}{2\sqrt{2}} I_{OUT}$$

$$I_{X\,RMS} = \frac{1}{2\sqrt{2}} I_{OUT}$$

let's get back to CID #1

1:1 transformer

$$S_{1:1} = \frac{\omega_0}{2\sqrt{2}} \times \frac{V_m\sqrt{3}}{2\,\omega_0} \times 2 \times \frac{I_{OUT}}{2\sqrt{2}}$$

$$S_{1:1} = \frac{\sqrt{3}}{8}\,V_m\,I_{OUT} = \frac{\pi}{24}\,P_{OUT} \approx 13.09\,\%\;P_{OUT}$$

$$S_{1:1} = \frac{4\pi}{105}\,P_{IN} \approx 11.97\,\%\;P_{IN}$$

back to CID #1 ...

1:2 transformer

$$S_{1:2} = \frac{\omega_0}{2\sqrt{2}} \times \frac{V_m}{2\,\omega_0} \times 2 \times \frac{I_{OUT}}{2\sqrt{2}}$$

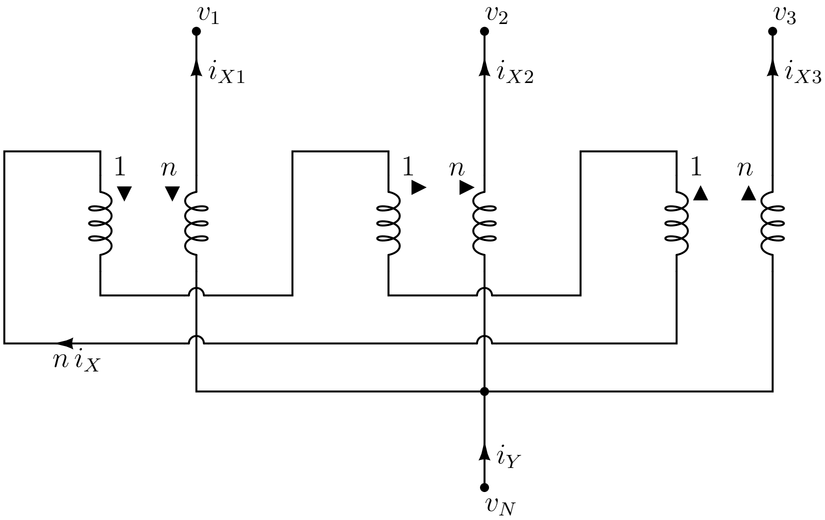
$$S_{1:2} = \frac{1}{8}\,V_m\,I_{OUT} = \frac{\pi}{24\sqrt{3}}\,P_{OUT} \approx 7.56\,\%\,P_{OUT}$$

$$S_{1:2} = \frac{4\pi}{105\sqrt{3}}\,P_{IN} \approx 6.91\,\%\,P_{IN}$$

a wrong measure: $S_{1:1} + S_{1:2} \approx 20.65\,\%\,P_{OUT}$, but not so bad

... in this way we compare the transformers.

a solution: theoretical (CID #2)



VA-rating, #2

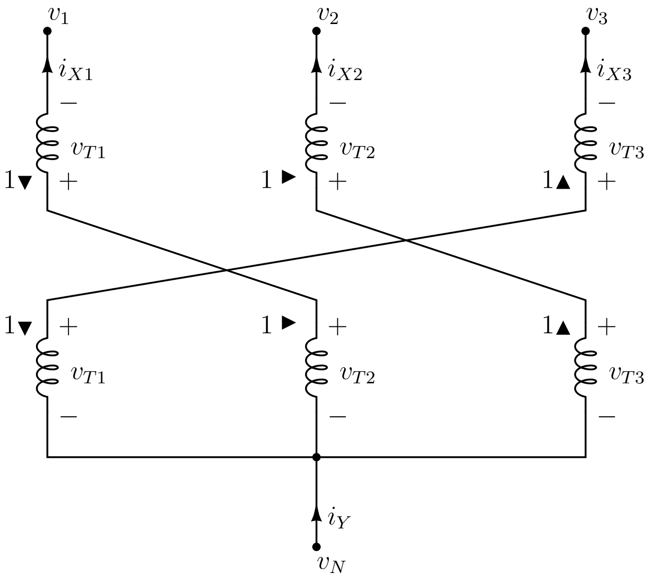
$$S_{\#2} = \frac{3\omega_0}{2\sqrt{2}} \times \frac{V_m}{\omega_0} \times 2 \times \frac{I_{OUT}}{2\sqrt{2}}$$

$$S_{\#2} = \frac{3}{4} V_m I_{OUT} = \frac{\pi}{4\sqrt{3}} P_{OUT} \approx 45.34\,\%\ P_{OUT}$$

$$S_{\#2} = \frac{8\pi}{35\sqrt{3}} P_{IN} \approx 41.46\,\%\ P_{IN}$$

pretty big rating, too big ...

a solution: the real one (CID #3)



it is simple with i_{X1}, i_{X2}, and i_{X3} ...

$$i_{X3} = i_{X1}$$

$$i_{X1} = i_{X2}$$

$$i_{X2} = i_{X3}$$

$$i_Y = i_{X1} + i_{X2} + i_{X3}$$

so, finally

$$i_{X1} = i_{X2} = i_{X3} = \frac{1}{3}\, i_Y$$

v_{T1}, v_{T2}, v_{T3}, v_N ...

$$v_1 = v_N + v_{T2} - v_{T1}$$

$$v_2 = v_N + v_{T3} - v_{T2}$$

$$v_3 = v_N + v_{T1} - v_{T3}$$

and

$$0 = v_{T1} + v_{T2} + v_{T3}$$

v_{T1}, v_{T2}, v_{T3}, v_N ...

$$v_{T1} = \frac{v_3 - v_1}{3}$$

$$v_{T2} = \frac{v_1 - v_2}{3}$$

$$v_{T3} = \frac{v_2 - v_3}{3}$$

$$v_N = \frac{v_1 + v_2 + v_3}{3} = 0$$

VA-rating, #3

$$S_{\#3} = \frac{3\omega_0}{2\sqrt{2}} \times \frac{V_m\sqrt{3}}{3\omega_0} \times 2 \times \frac{I_{OUT}}{2\sqrt{2}}$$

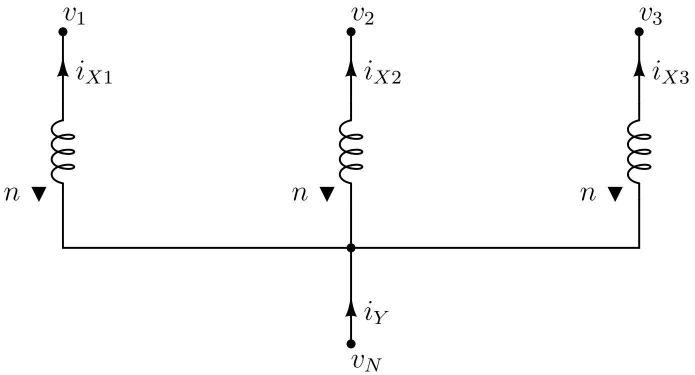
$$S_{\#3} = \frac{\sqrt{3}}{4} V_m \, I_{OUT} = \frac{\pi}{12} P_{OUT} \approx 26.18\,\% \; P_{OUT}$$

$$S_{\#3} = \frac{8\pi}{105} P_{IN} \approx 23.94\,\% \; P_{IN}$$

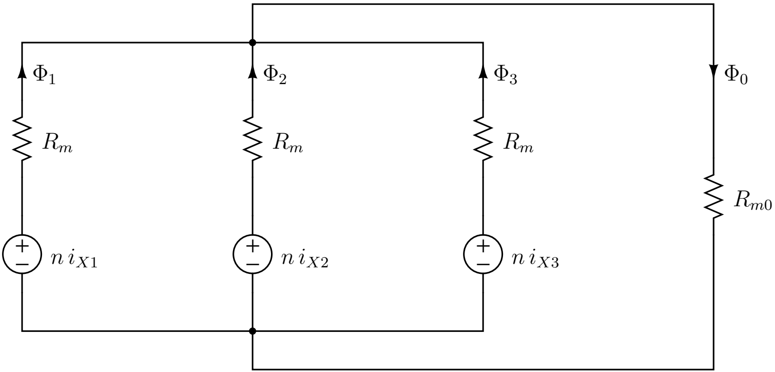
not so big rating, much better than #2

applied in all of the experiments (to be) presented here

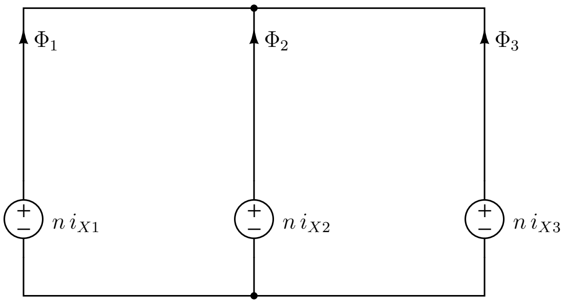
a solution: wishful thinking (CID #4)



why the thinking is wishful?



“somewhat” idealized . . .



element equations from "somewhat idealized" . . .

$$n\, i_{X1} = n\, i_{X2} = n\, i_{X3}$$

$$i_{X1} = i_{X2} = i_{X3} = i_X = i_Y/3$$

$$\Phi_1 + \Phi_2 + \Phi_3 = 0$$

$$n\,\frac{d\,\Phi_1}{dt} + n\,\frac{d\,\Phi_2}{dt} + n\,\frac{d\,\Phi_3}{dt} = 0$$

$$(v_N - v_1) + (v_N - v_2) + (v_N - v_3) = 0$$

$$v_N = \frac{v_1 + v_2 + v_3}{3} = 0$$

yup, that's it! . . . or is it?

VA-rating, #4

$$S_{\#4} = \frac{3\omega_0}{2\sqrt{2}} \times \frac{V_m}{\omega_0} \times \frac{I_{OUT}}{2\sqrt{2}}$$

$$S_{\#4} = \frac{3}{8} V_m I_{OUT} = \frac{\pi}{8\sqrt{3}} P_{OUT} \approx 22.67\,\%\, P_{OUT}$$

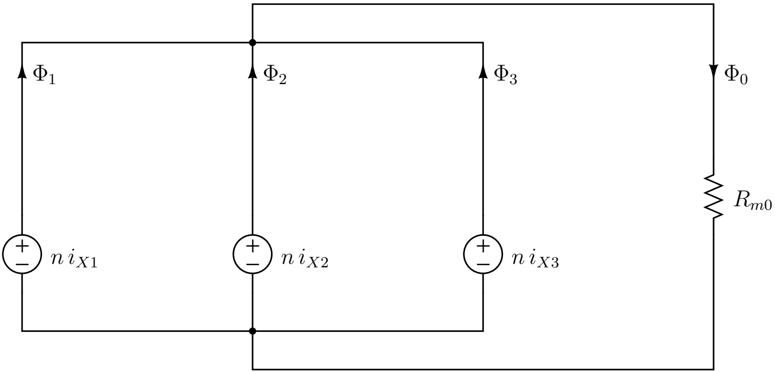
$$S_{\#4} = \frac{4\pi}{35\sqrt{3}} P_{IN} \approx 20.73\,\%\, P_{IN}$$

$$S_{\#4} = \frac{1}{2} S_{\#2}$$

an improvement over $S_{\#2}$ and even $S_{\#3}$ ...

but for what price?

"somewhat less" idealized . . .



element equations . . .

$$i_{X1} = i_{X2} = i_{X3} = i_X = \frac{1}{3}\, i_Y$$

$$\Phi_1 + \Phi_2 + \Phi_3 = \Phi_0 = \frac{n\, i_X}{R_{m0}}$$

$$n\,\frac{d\Phi_1}{dt} + n\,\frac{d\Phi_2}{dt} + n\,\frac{d\Phi_3}{dt} = n\,\frac{d\Phi_0}{dt} = \frac{n^2}{R_{m0}}\,\frac{di_X}{dt}$$

$$(v_N - v_1) + (v_N - v_2) + (v_N - v_3) = \frac{n^2}{R_{m0}}\,\frac{di_X}{dt}$$

element equations ...

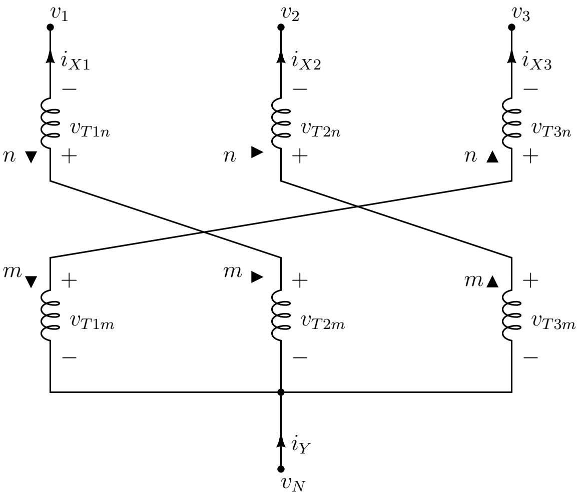
$$3\, v_N = \frac{n^2}{3\, R_{m0}} \frac{di_Y}{dt}$$

$$v_N = \frac{n^2}{9\, R_{m0}} \frac{di_Y}{dt} = L_N \frac{di_Y}{dt}$$

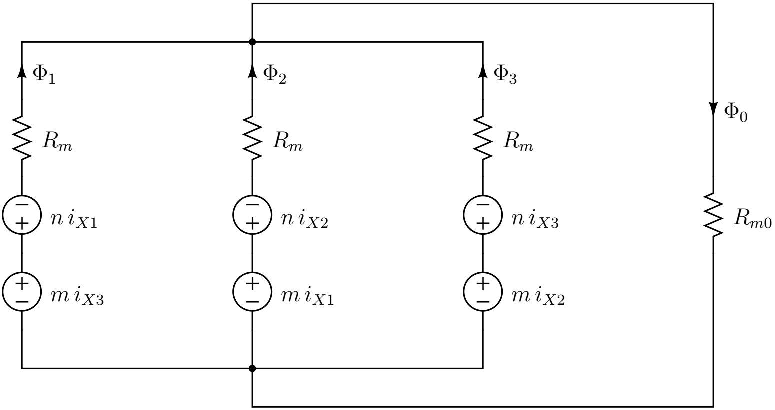
$$\boxed{L_N \triangleq \frac{n^2}{9\, R_{m0}}}$$

which might be a problem ...

a hybrid between #3 and #4



magnetic circuit of the hybrid ...



magnetomotive force equations, 3 items . . .

$$m\, i_{X3} - n\, i_{X1} - R_m\, \Phi_1 - R_{m0}\, \Phi_0 = 0$$

$$m\, i_{X1} - n\, i_{X2} - R_m\, \Phi_2 - R_{m0}\, \Phi_0 = 0$$

$$m\, i_{X2} - n\, i_{X3} - R_m\, \Phi_3 - R_{m0}\, \Phi_0 = 0$$

KΦL, 1 item . . .

$$\Phi_1 + \Phi_2 + \Phi_3 - \Phi_0 = 0$$

hint #1: we have four equations this far; eliminate Φ_0; three equations remain

hint #2: solve the remaining three equations over Φ_1, Φ_2, and Φ_3

hint #3: differentiate over time, $\frac{d}{dt}$)

why? we gonna need them that way
(teaching practice: put the parallel in a sequence)

voltage equations, 3 items, real KVL ...

$$u_1 = v_1 - v_N = n\,\frac{d\Phi_1}{dt} - m\,\frac{d\Phi_2}{dt}$$

$$u_2 = v_2 - v_N = n\,\frac{d\Phi_2}{dt} - m\,\frac{d\Phi_3}{dt}$$

$$u_3 = v_3 - v_N = n\,\frac{d\Phi_3}{dt} - m\,\frac{d\Phi_1}{dt}$$

and now you know why!

the result . . .

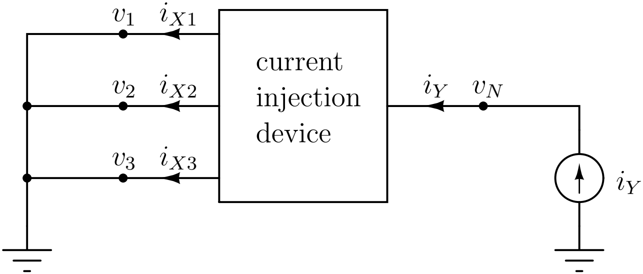
after having fun with wxMaxima . . .

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} L & -M & -M \\ -M & L & -M \\ -M & -M & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{X1} \\ i_{X2} \\ i_{X3} \end{bmatrix}$$

$$L = \frac{n^2 + m^2}{3R_0 + R} + \frac{2R_0\left(n^2 + nm + m^2\right)}{R_m\left(3R_0 + R_m\right)}$$

$$M = \frac{nm}{3R_0 + R} + \frac{R_0\left(n^2 + nm + m^2\right)}{R_m\left(3R_0 + R_m\right)}$$

a test . . .



and the final result ...

$$v_N = L_N \frac{di_Y}{dt}$$

$$L_N = \frac{L - 2M}{3}$$

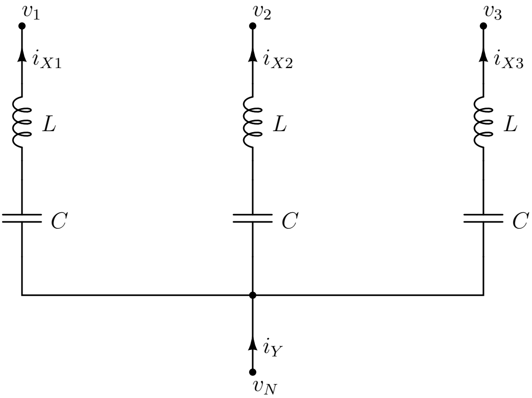
$$L_N = \frac{n^2 - 2nm + m^2}{3\left(3R_{m0} + R_m\right)}$$

which for $n = m$ ...

and for $n = 0$ ...

and for $m = 0$ ...

a disastrous solution ... (no #)

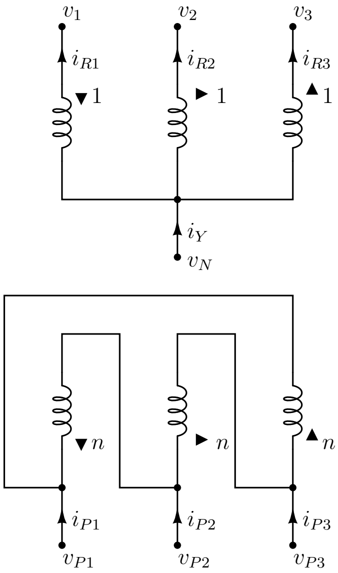


a disastrous solution, why is it such a disaster?

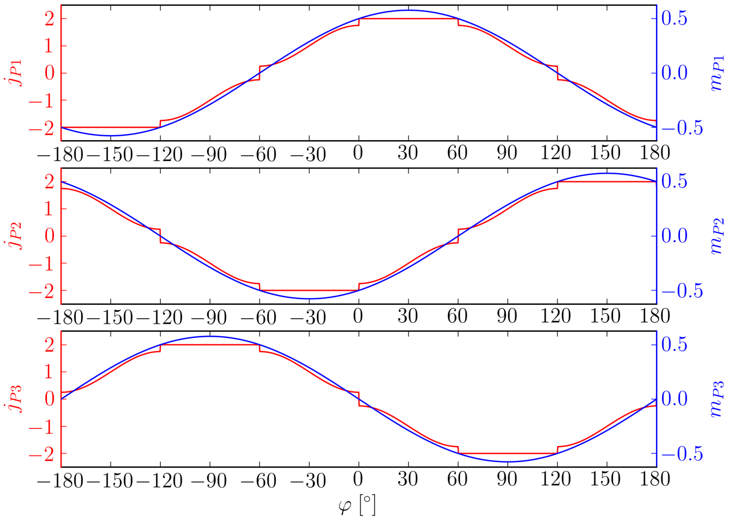
$$3\,\omega_0 = \frac{1}{\sqrt{L\,C}}$$

- current sharing depends on parasitic resistance
- resonance constraints to be met
- tolerances?
- transient response?
- bipolar capacitors?
- leakage at ω_0?
- Q-factor?
- VA-rating?

a solution: nice one (CID T#1)



waveforms, primary, 1:1 assumed . . .



VA rating, T#1

the waveforms are different, but $THD = \sqrt{\frac{32\,\pi^2}{315} - 1} \approx 5.12\,\%$

$$S_{T\#1} = \frac{3\omega_0}{2\sqrt{2}} \times \frac{V_m}{\omega_0} \times 2 \times \sqrt{\frac{41}{48}}\, I_{OUT}$$

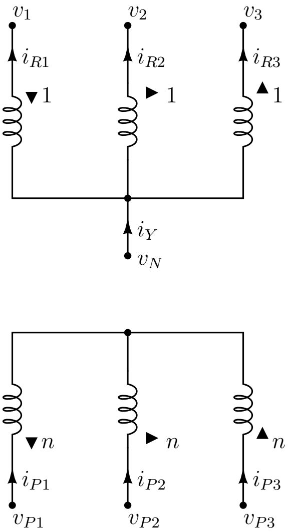
$$S_{T\#1} = \sqrt{\frac{123}{32}}\, V_m\, I_{OUT}$$

$$S_{T\#1} = \frac{\pi\sqrt{41}}{12\sqrt{2}}\, P_{OUT} \approx 1.1853\, P_{OUT}$$

$$S_{T\#1} = \frac{8\pi\sqrt{41}}{105\sqrt{2}}\, P_{IN} \approx 1.0837\, P_{IN}$$

no problem with L_N; proof?

a solution: not so nice (CID T#2)



element equations . . .

$$i_{R1} - n\, i_{P1} = F_0$$

$$i_{R2} - n\, i_{P2} = F_0$$

$$i_{R3} - n\, i_{P3} = F_0$$

$$i_{P1} + i_{P2} + i_{P3} = 0$$

four equations, seven variables, solve for i_{P1}, i_{P2}, i_{P3}, and F_0 (not that we really care about F_0)

element equations . . .

note that $i_{R1} + i_{R2} + i_{R3} = i_Y = 3\, i_X$

$$i_{P1} = \frac{1}{n}\,(i_{R1} - i_X)$$

$$i_{P2} = \frac{1}{n}\,(i_{R2} - i_X)$$

$$i_{P3} = \frac{1}{n}\,(i_{R3} - i_X)$$

looks like this is what we need, but L_N is the problem . . .

try to determine L_N . . .

VA-rating, T#2

$$S_{T\#2} = \frac{3\omega_0}{2\sqrt{2}} \times \frac{V_m}{\omega_0} \times \left(\sqrt{\frac{41}{48}} + \frac{\sqrt{105}}{12} \right) I_{OUT}$$

$$S_{T\#2} = \frac{3}{2\sqrt{2}} \left(\frac{\sqrt{41}}{4\sqrt{3}} + \frac{\sqrt{105}}{12} \right) V_m \, I_{OUT}$$

$$S_{T\#2} = \frac{\pi}{2\sqrt{6}} \left(\frac{\sqrt{41}}{4\sqrt{3}} + \frac{\sqrt{105}}{12} \right) P_{OUT} \approx 1.1403 \, P_{OUT}$$

$$S_{T\#2} = \frac{16\pi}{35\sqrt{6}} \left(\frac{\sqrt{41}}{4\sqrt{3}} + \frac{\sqrt{105}}{12} \right) P_{IN} \approx 1.0425 \, P_{IN}$$

conclusions

- several current injection devices introduced
- all of the devices are magnetic devices
- volt-ampere rating introduced as a measure
- magnetic circuits used to analyze
- zigzag transformer based CID, CID#3, of interest (23.94 %)
- delta-star (D-Y) transformer based CID, CID T#1, of interest (+8.37 %)
- problems with zero-sequence inductance . . .
- problems?