

has to be formulated with a slightly more complicated procedure.

Let b_k, p_k – the amount and price of k raw material if it is purchased at less than the quantity discount volume; d_k, p_k' – the amount and price of k raw material if it is purchased at the quantity discount.

The new model, in that case, instead of one equation for k raw material has some more relations, and those are:

$$\begin{aligned} a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kn}x_n &= b_k + d_k & (3) \\ b_k - Q^* y_1 &\leq 0 \\ d_k - Q y_2 &\geq 0 \\ d_k - M y_2 &\leq 0 \end{aligned}$$

and, according to this, the budget constraint is:

$$p_1b_1 + p_2b_2 + \dots + p_kb_k + p_k'd_k + \dots + p_mb_m \leq B, \quad (4)$$

where M is a very large positive number ($M \gg 0$), or the upper limit for the procurement of the resource k , and Q^* is a number which is slightly lower than Q . Variables y_1 and y_2 are integer 0 - 1 variables, for which is:

$$y_1 + y_2 = 1 \quad (5)$$

In the above model, there are two 0-1 variables y_1 and y_2 , where only one of them always equals 1, and the other equals zero. Naturally, if the model comprises a number of resources that can be purchased at a discounted price then there are more 0-1 variables.

Since the same raw material has different price variable, the income from end product unit is not constant anymore. Therefore, maximizing the sum of $c_j x_j$, would not be an accurate measure of net income. Net income (1) should be recalculated as the difference between sales and total cost of materials, where the objective function will include materials at both prices. Consequently, if s_j is the sales price of j product, the objective function has the following form:

$$\text{Max } z = \sum_{j=1}^n s_j x_j - \sum_{i=1}^m p_i b_i - \sum_{k \in K} p_k' d_k \quad (6)$$

In that equation set, K presents the indices of raw materials that have increasing or discounted prices, and d_k ($k \in K$) stays for those materials where additional quantities can be bought only at a higher price (p_k'), or the quantities of raw materials if we bought them with quantity discounts.

In the budget equation, it is also necessary to introduce costs for additional quantities of raw materials, so that it now takes the following form:

$$\sum_{i=1}^m p_i b_i + \sum_{k \in K} p_k' d_k \leq B \quad (7)$$

There is no need to specify that b_i should reach the maximum value of Q first, before allowing d_i greater than zero. The optimization model ensures b_i reaching the maximum value of Q because of the lower penalty, i.e. lower price p_i .

III. PROBLEM SETTING

This paper analyses the production planning problem in one production company which produces stone souvenirs made from famous stone from the island of Brac. This company produces 13 different stone souvenirs, and these articles can be seen in Table I. In this table, there are lower bounds for the six-month production and selling prices of all the articles.

Table II presents the list of raw materials that are used in production of these articles. There are 26 different raw materials, and the purchasing prices for every of them are also presented in the table.

The amount of raw materials in one unit of articles (a_{ij}) is also used in production planning problem, and their values are presented in Tables III and IV.

TABLE I
 LIST OF ARTICLES PRODUCTS

Mark	Article name	Six-month amount production		Selling prices (kn)
		Lower bound		s_j
BRM (A1)	Bracelet - metal	900		55
BRSP (A2)	Bracelet – semiprecious stone	700		55
BRR (A3)	Bracelet – rope	6000		15
CRS (A4)	Crown - small	600		40
CRB (A5)	Crown - big	100		125
NECM (A6)	Necklace - metal	120		70
NECSP (A7)	Necklace - semiprecious stone	120		70
NECFM (A8)	Necklace full – metal	80		120
NECFSP (A9)	Necklace full - semiprecious stone	80		120
EMS (A10)	Earrings metal - small	500		20
ESPS (A11)	Earrings semiprecious stone - small	500		20
EMB (A12)	Earrings metal - big	220		30
ESPB (A13)	Earrings semiprecious stone - big	250		30

TABLE II
LIST OF RAW MATERIALS WITH INCREASING COSTS AND QUANTITY DISCOUNTS

Type of raw materials		Unit price		
b1	Stone 8 mm	0.6	kn/piece	
b2	Stone 10 mm	0.8	kn/piece	
b3	Stone 12 mm	1	kn/piece	
b4	Skullcap small	0.08	kn/piece	
b5	Skullcap big	0.08	kn/piece	
b6	Space big	0.06	kn/piece	
b7	Space small ≤ 30000	0,06	kn/piece	Increasing costs
d7	Space small > 30000	0.08	kn/piece	
b8	Bead metal 4 mm ≤ 25.000	0.04	kn/piece	Quantity discounts
d8	Bead metal 4 mm > 25.000	0.02	kn/piece	
b9	Bead metal 8 mm	0.4	kn/piece	
b10	Bead metal 10 mm	0.6	kn/piece	
b11	Cable	2	kn/m	
b12	Semiprecious stone 4 mm ≤ 3500	0.2	kn/piece	Increasing costs
d12	Semiprecious stone 4 mm > 3500	0.25	kn/piece	
b13	Semiprecious stone 8 mm	0.35	kn/piece	
b14	Semiprecious stone 10 mm	0.6	kn/piece	
b15	Semiprecious stone 12 mm ≤ 4.200	1.2	kn/piece	Quantity discounts
d15	Semiprecious stone 12mm > 4.200	1	kn/piece	
b16	Wire	2.5	kn/piece	
b17	Rope	1	kn/m	
b18	Small medal	0.3	kn/piece	
b19	Small cross	0.3	kn/piece	
b20	Big medal	2	kn/piece	
b21	Big cross	12	kn/piece	
b22	Needle	0.5	kn/piece	
b23	Needle	0.7	kn/piece	
b24	Crotchet	0.5	kn/piece	
b25	Buckle	0.3	kn/piece	
b26	Quoit	0.05	kn/piece	

TABLE III
THE RECIPES –PART I

	NM-NAR	NP-NAR	NK-NAR	KM-KRU	KV-KRU	OM-OG	OP-OG
Stone 8mm	8	6	2	11	59	16	12
Stone 10mm	2	2	0	0	0	4	4
Stone 12mm	3	0	0	0	0	6	0
Skullcap small	4	4	2	4	14	8	8
Skullcap big	6	6	0	0	0	12	12
Space big	2	2	0	0	0	4	6
Space small	6	6	2	0	0	12	12
Bead metal 4mm	0	0	0	22	81	0	0
Bead metal 8mm	0	2	0	0	0	4	4
Bead metal 10mm	2	0	1	0	0	3	3
Sajla	0,2	0,2	0	0,2	0,6	0,6	0,6
Cable	0	4	0	0	0	0	2
Semiprecious stone 4 mm	0	0	0	0	0	0	4
Semiprecious stone 8 mm	0	0	0	0	0	0	2
Semiprecious stone 10mm	0	3	0	0	0	0	6
Wire	0	0	0,08	0	0	0	0
Rope	0	0	0,3	0	0	0	0
Small medal	0	0	0	1	0	0	0
Small cross	0	0	0	1	0	0	0
Big medal	0	0	0	0	1	0	0
Big cross	0	0	0	0	1	0	0
Needle 20mm	0	0	0	0	0	0	0
Needle 30mm	0	0	0	0	0	0	0
Crotchet	0	0	0	0	0	0	0

According to these data, the production planning problem can be posted as the linear programming model with one or

more objective functions. Here, we will consider the production company which produces stone souvenirs where

we have the varying price of the same resource, i.e. increasing costs of the same raw material or quantity discounts for some of raw materials as can be seen in Table II.

Suppose that the available budget for six-month production is equal to the last year's costs for the raw materials, i.e. $B = 99\,430$ kn.

TABLE IV
 THE RECIPES – PART II

	OPM-OG	OPP-OG	NMM-NAU	NPM-NAU	NMV-NAU	NPV-NAU
Stone 8mm	32	24	0	0	2	0
Stone 10mm	8	8	2	2	2	4
Stone 12mm	12	0	0	0	0	0
Skullcap small	16	16	4	4	8	8
Skullcap big	24	24	0	0	0	0
Space big	8	12	2	2	4	4
Space small	24	24	0	0	0	0
Bead metal 4mm	0	0	0	0	0	0
Bead metal 8mm	8	8	2	0	0	0
Bead metal 10mm	6	6	0	0	2	0
Sajla	0,6	0,6	0	0	0	0
Cable	0	4	0	0	0	0
Semiprecious stone 4 mm	0	8	0	0	0	0
Semiprecious stone 8 mm	0	4	0	2	0	2
Semiprecious stone 10mm	0	12	0	0	0	0
Wire	0	0	0	0	0	0
Rope	0	0	0	0	0	0
Small medal	0	0	0	0	0	0
Small cross	0	0	0	0	0	0
Big medal	0	0	0	0	0	0
Big cross	0	0	0	0	0	0
Needle 20mm	0	0	2	2	0	0
Needle 30mm	0	0	0	0	2	2
Crochet	0	0	2	2	2	2

Since raw materials are now of different costs, variable prices of end product are not constant any more. Therefore, maximizing the sum of $c_j x_j$, where c_j is the unit profit for article A_j , would not be an accurate measure of profit. Rather, profit equation should be recalculated as sales income less total cost of materials.

If x_j is the production quantity of i stone product, the model which will take this increasing costs and quantity discounts into consideration is as follows:

Objective function (total contribution) which has to be maximized is:

$$\text{Max } z = \sum_{j=1}^{13} s_j x_j - \sum_{i=1}^{26} p_i b_i - \sum_{k \in K} p_k' d_k, K = \{7,8,12,15\} \quad (8)$$

In that equation set, K presents the indices of raw materials that have increasing or discounted prices. In our case, that happens for S_7, S_8, S_{12} and S_{15} . Let us consider such situation for our souvenirs production model. The eighth and fifteenth raw material (S_8 - Bead metal 4 mm and S_{15} - Semiprecious stone 12 mm) can be purchased at a discounted price if the bought quantity is $Q_1 > 2500$ pieces and $Q_2 > 4200$ pieces, and this reduced price is valid for the entire quantity supplied, i.e. $p_1' = 0.02$ kn and $p_2' = 1$ kn. In addition to this, let us assume increasing costs for Small space (S_7) and Semiprecious stone 4 mm (S_{12}) in this way:

The limit of S_7 purchased at a lower price is 30000 pieces, while this limit in S_{12} is 3500 pieces. The purchasing price of the additional quantity of S_7 is $p_7' = 0.08$, and of S_{12} $p_{12}' = 0.25$ currency units. Assuming that the budget level is $B = 99\,430$ kn, and selling prices as in Table I, the constraints in the production model are:

Raw material constraints:

$$\sum_{j=1}^{13} a_{ij} x_j - b_i - d_i = 0, i = 1, \dots, 26 \quad (9)$$

where $d_i = 0$ except for $i \in K$, and a_{ij} are the amount of raw materials in one unit of specific article. Constraints for the discounted prices for the eighth and fifteenth raw material:

$$b_8 - 24990 y_1 \leq 0, d_8 - 25000 y_2 \geq 0, d_8 - M y_2 \leq 0 \quad (11)$$

$$b_{15} - 4190 y_3 \leq 0, d_{15} - 4200 y_4 \geq 0, d_{15} - M y_4 \leq 0 \quad (12)$$

where M is a very large positive number ($M \gg 0$), or the upper limit for the procurement of the specific resource k . Variables y_1, y_2, y_3 and y_4 are integer 0 - 1 variables, for which $y_1 + y_2 = 1$ and $y_3 + y_4 = 1$ is valid. In the above model there are four 0-1 variables, where due to the upper relations only one of them in each pair always equals 1, and the other equals zero. Naturally, if the model comprises a number of resources

that can be purchased at a discounted price then there are more 0-1 variables. Last is the budget constraint:

$$\sum_{i=1}^{26} p_i b_i + \sum_{k \in K} p_k 'd_k \leq 99430 \quad (13)$$

In addition to that the model has 13 integer variables (units of articles - x_j). Of course due to the data from Table I, all articles have the lower bounds which are 13 more constraints. There are two more constraints for the raw materials which have increasing costs:

$$b_7 \leq 30000, b_{12} \leq 3500 \quad (14)$$

In addition to that, we have some production lines (capacities) constraints and that is:

$$x_1 + x_2 \leq 1800 \quad (\text{bracelet production line}) \quad (15)$$

$$x_6 + x_7 \leq 300 \quad (\text{necklace 1 production line}) \quad (16)$$

$$x_8 + x_9 \leq 200 \quad (\text{necklace 2 production line}) \quad (17)$$

$$x_{10} + x_{11} \leq 1000 \quad (\text{earrings 1 production line}) \quad (18)$$

$$x_{12} + x_{13} \leq 500 \quad (\text{earrings 2 production line}) \quad (19)$$

TABLE V
OPTIMAL SOLUTION

Variables	Optimal solution	Variables	Optimal solution
X1	900	b1	48860
X2	900	b2	9900
X3	6000	b3	5220
X4	600	b4	36880
X5	120	b5	19200
X6	180	b6	10960
X7	120	b7	30000
X8	120	d7	1200
X9	80	b8	22920
X10	500	d8	0
X11	500	b9	5600
X12	250	b10	10400
X13	250	b11	852
b12	3500	b23	1000
d12	660	b24	3000
b13	1120	b25	8900
b14	2060	b26	18760
b15	0	y1	1
d15	4380	y2	0
b16	480	y3	0
b17	1800	y4	1
b18	600		
b19	600		
b20	120		
b21	120		
b22	2000		
			z* = 228 682.20

It should be remarked that it is the mixed integer programming problem with thirteen integer variables (x_j), 30 continuous variables (26 b_i and 4 d_i), and four binary variables (y_j). Its optimal solution is obtained by MATLAB and is presented in Table V. Optimal value of the objective function is: $z^* = 228682.20$ and for that production the available budget is not completely spent. The reason for that is the production lines constraints that do not allow higher production. In this table, we can see the required quantities of raw materials. For the fifteenth raw material (S15), we have the quantity discount because we purchase this raw material (semiprecious stone of 12 mm) over the limited quantity ($Q_{15} = 4200$ pieces). In our model, of course, the binary variables y_1 and y_4 are equal to 1. The seventh raw material (Small space) have to be purchased over the limited quantity and so the quantity over the limit ($Q_7 = 30000$) is purchased at a higher price. The same happens for the twelfth raw material (Semi precious stone 4 mm), and we purchase additional quantity at the higher price (0.25 kn/piece).

IV. CONCLUSION

In this paper, the De Novo programming model in the production plan optimization of the production of stone souvenirs is considered. The efficiency of the proposed model is investigated on the case of a company that produces 13 various souvenirs. The De Novo approach does not limit the resources as most of the necessary resource quantities can be obtained at certain prices. Resources, of course, are actually limited because their maximum quantity is controlled by the budget, which is an important element of De Novo.

The obtained results indicate a high application efficiency of the proposed model by using the De Novo programming in solving the production plan optimization problem in various production companies. Using the De Novo approach, most varied cases can be handled more effectively than by the standard programming models and in this paper increasing costs of raw materials and quantity discounts for some raw materials in souvenirs production are investigated.

The future work on this issue will investigate the possibilities of introducing new objective functions in the model, and solving this production problem as the multi-criteria ones [4]-[6].

ACKNOWLEDGMENT

This work has been fully supported by Croatian Science Foundation under the project Innovative Smart Enterprise – INSENT (1353).

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