Conduction Modes of a Peak Limiting Current Mode Controlled Buck Converter

Predrag Pejović, Marija Glišić

Introduction

- peak limiting current mode control ...
- known since 1978, C. W. Deisch, "Simple switching control method changes power converter into a current source," PESC'78 [2]
- revisited many times, e.g. in 2001 [6] and 2011 (!) [7]
- still something to say?
- CCM, DCM, stability, $D > 0.5$, chaos, ...
- artificial ramp ...
- purpose of the paper to clarify the issues ...
- continuation of our Ee 2013 paper, "Stability Issues in Peak Limiting Current Mode Controlled Buck Converter" ...
- initial plan turned out not to be ambitious enough ...
- since there is an infinity of DCMs!

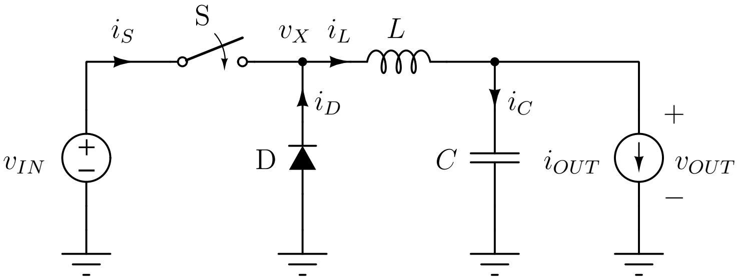
what is in the paper?

- nonlinear dynamics methods applied to analyze a peak limiting current mode controlled buck converter ...
- which required an iterated map model ...
- and resulted in an infinite number of the discontinuous conduction modes!
- regions where the modes occur identified
- clarification of notions of stability:
  - limit cycle stability
  - open loop (averaged model) stability
- just a homework assignment in nonlinear dynamics ...
- but haven't been done before!

what is not in the paper?

- this paper does not contain an algorithm that would earn you money ...
  - but would help you understand some phenomena you might observe in some circuits you build ...
  - or at least helped **me** understand what happened in some of my designs ...
  - and helped **me** understand phenomena I would rather avoid!
  - so, I would like to share that with you!

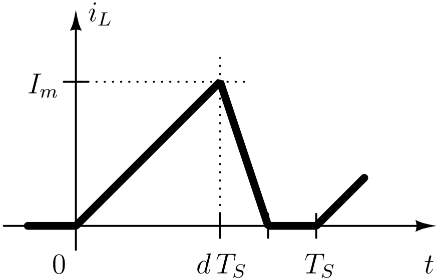
the circuit . . . constant current load!



The constant current load model affects the open loop (averaged model) stability!

And some people (RWE) prefer a resistor load . . .

and the control ...

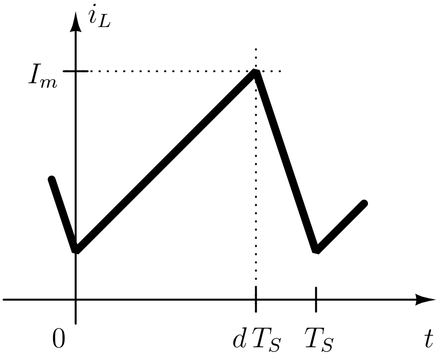


The discontinuous conduction mode (DCM) ...

Actually, period-1 DCM!

... where it all started! We just wanted to study open loop instability of the averaged model for $V_{OUT} > \frac{1}{2} V_{IN}$, and landed in nonlinear dynamics! We observed period-n DCM!

and the control . . .



The continuous conduction mode (CCM) . . .

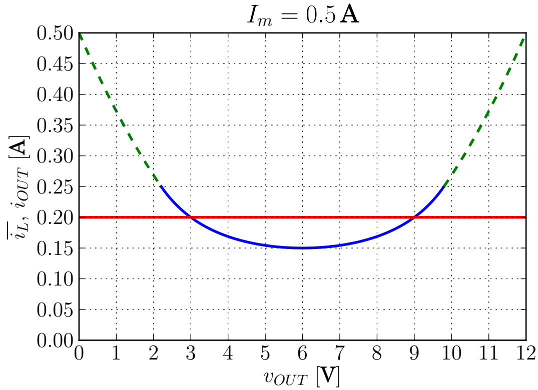
Actually, period-1 CCM!

. . . which is known to have limit cycle stability issues . . . and what is the averaged model then?

questions?

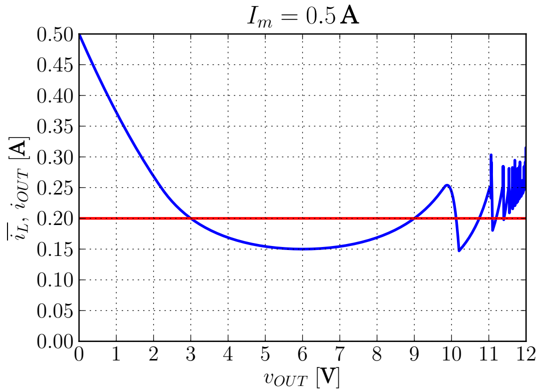
- limit cycle stability?
  1. limit cycles are stable in the DCM
  2. there are unstable limit cycles in the CCM, well known ...
  3. both are results of small perturbation analysis
- open loop (averaged model) stability?
  1. something quite different!
  2. depends on the load!
  3. analysis requires averaged circuit model
  4. averaged circuit models require periodicity
  5. well, at least in some sense ...
  6. the DCM might expose an open loop (averaged model) instability!!!
  7. which is a result of the previous paper
- periodicity?
  1. DCMs have periodic limit cycle
  2. CCM might have a periodic limit cycle
  3. but also, there are aperiodic attractors for the CCM

clarifications needed?



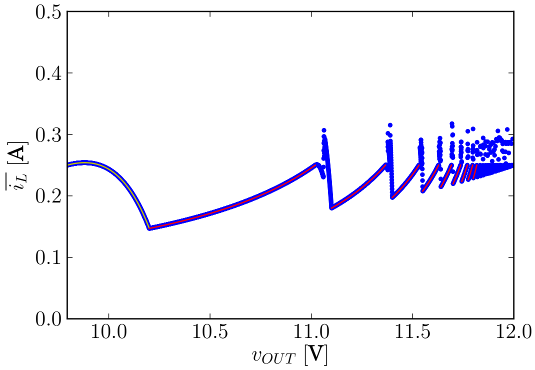
We expected this, since we assumed period-1 stable limit cycle operation ...

clarifications needed?



But, we obtained this, since in the CCM for $D > \frac{1}{2}$ the limit cycle is unstable, and we reach stable period-n stable limit cycle in the DCM; nothing to say about open loop (averaged model) stability!

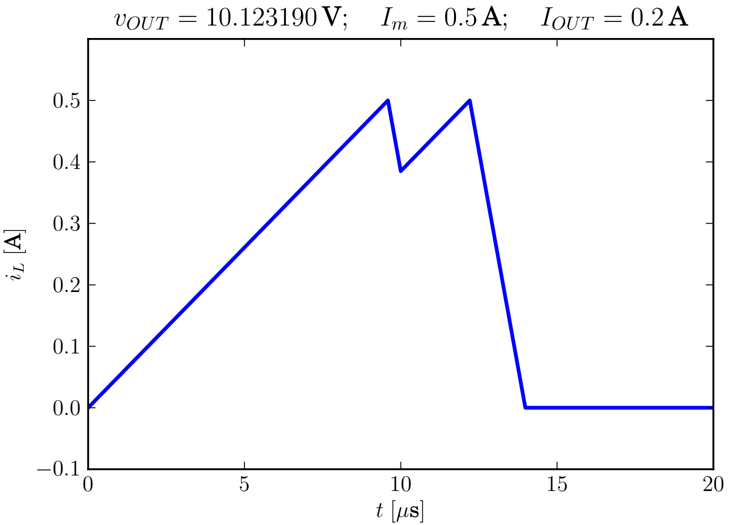
just a closer look . . .



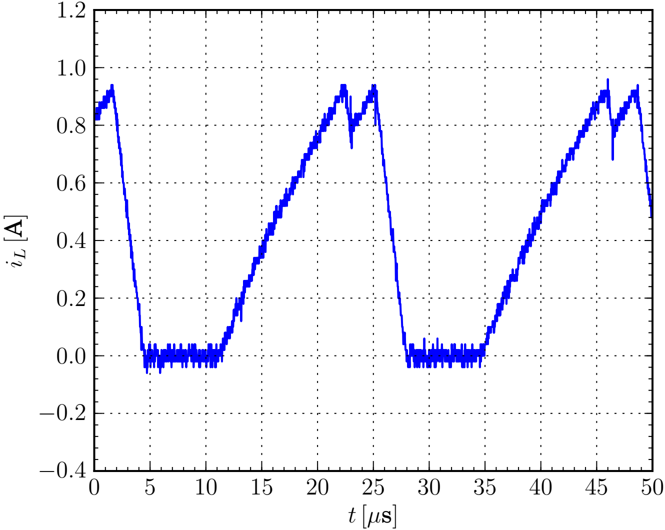
For the thin lines we have a closed-form solution . . .

And we know what is going on there . . . twin-peaks (yellow) and triangular (red) DCM waveforms . . . infinity of DCMs . . .

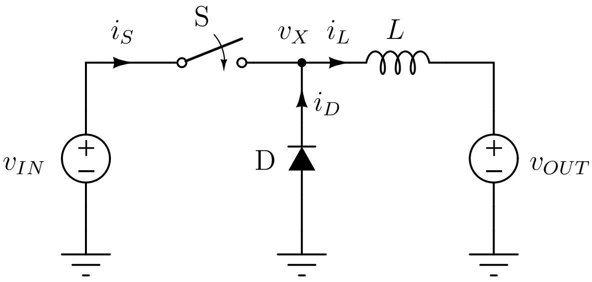
steady state waveform of i_L, ... "twin peaks"



actually happens ...



Reduction to a Switching Cell Model



used to draw i_L, to compute $\overline{i_L}$ ...

v_{IN}, v_{OUT} assumed constant over T_S ...

circuit equations . . .

$$L \frac{d\,i_L}{dt} = v_L$$

$$v_L = \begin{cases} V_{IN} - V_{OUT}, & \mathrm{S-on, D-off} \\ -V_{OUT}, & \mathrm{S-off, D-on} \\ 0, & \mathrm{S-off, D-off} \end{cases}$$

methods applied . . .

- numerical simulation of iterated maps
- Python, PyLab, lists . . .
- a way to generalize results and conclusions?
- normalization!
- I'm becoming boring!

normalization ...

$V_{base} = V_{IN}$:

$$m \triangleq \frac{v}{V_{IN}}$$

$$M_{IN} = 1$$

$$M \triangleq \frac{V_{OUT}}{V_{IN}}$$

$I_{base} = V_{IN} / (f_S\, L)$:

$$j \triangleq \frac{f_S\, L}{V_{IN}}\, i$$

$T_{base} = T_S$:

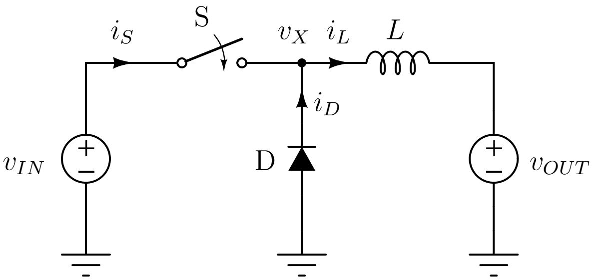
$$\tau \triangleq \frac{t}{T_S}$$

voilà!

$$\frac{d\,j_L}{d\tau} = m_L$$

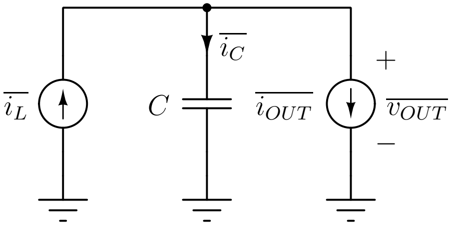
$$m_L = \begin{cases} 1 - M, & \mathrm{S-on, D-off} \\ -M, & \mathrm{S-off, D-on} \\ 0, & \mathrm{S-off, D-off} \end{cases}$$

Discrete Time Model of the Switching Cell



All we need is $\overline{j_L}$ as a function of J_m and M!

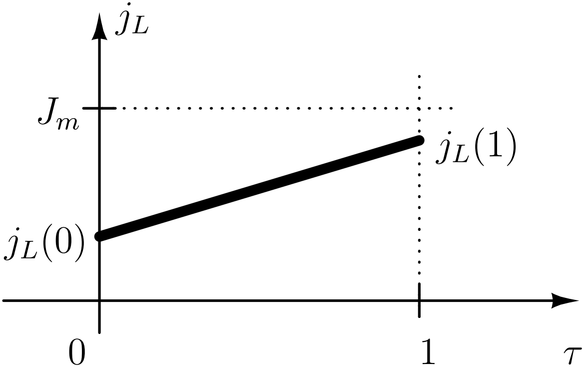
and when we get it ... averaged circuit model!



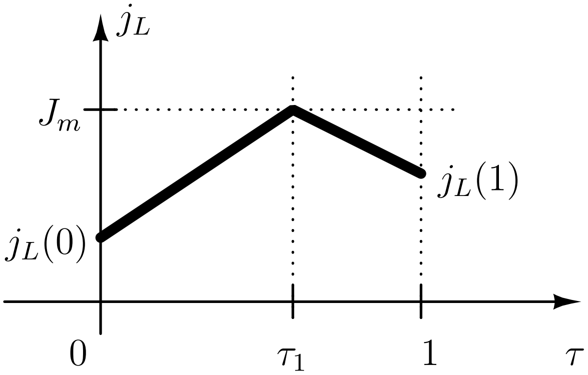
$$C\,\frac{d\,\overline{v_{OUT}}}{d\,t} = \overline{i_L} - \overline{i_{OUT}}$$

Decoupled!

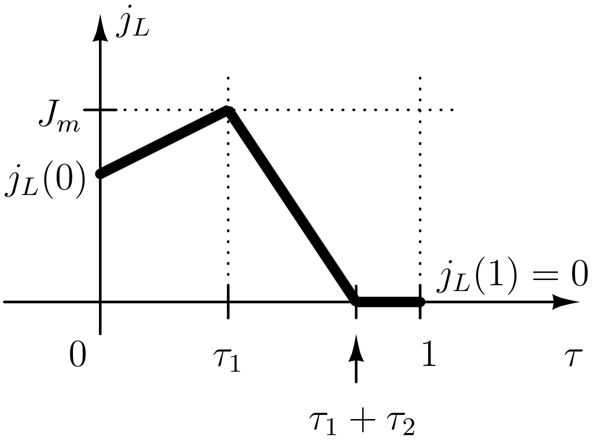
iterated map, case 1, no switching



iterated map, case 2, switch turn-off



iterated map, case 3, switch turn-off, diode turn-off



iterated map model . . .

- essentially $j_L(n)$ as a function of $j_L(n-1)$, M, and J_m
- auxiliary, compute the charge q_n carried over each period and store it
- equations are in the paper . . .
- simulation? numerical solution?
  - specify M and J_m
  - start from $j_L(0) = 0$, at least for the CCM
  - iterate till $j_L(k) = 0$; we got the periodicity!
  - sum all the charges, $Q = \sum_{n=1}^{k} q_k$
  - $\overline{j_L} = j_{OUT} = Q/k$
  - all the rest is the matter of presentation . . .

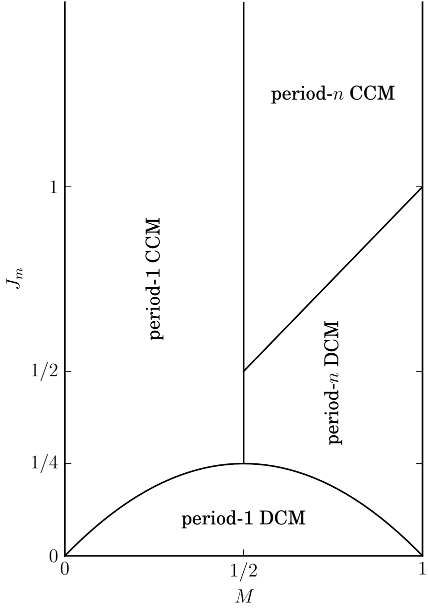
A Glimpse on the Period-1 Model

- assumed period-1 operation, regardless the limit cycle stability
- DCM occurs for $J_m < M\,(1 - M)$
- CCM occurs for $J_m > M\,(1 - M)$
- in DCM $j_{OUT} = \frac{J_m^2}{2\,M\,(1-M)}$
- in CCM $j_{OUT} = J_m - \frac{1}{2}\,M\;(1 - M)$
- open loop (averaged model) instability for $\frac{d\,j_{OUT}}{dM} > 0$
- in DCM $\frac{d\,j_{OUT}}{dM} = \frac{J_m^2\,(2M-1)}{2(M-1)^2 M^2}$
- in CCM $\frac{d\,j_{OUT}}{dM} = M - \frac{1}{2}$
- in **both** cases open loop (averaged model) instability for $M > \frac{1}{2}$
- elementary?

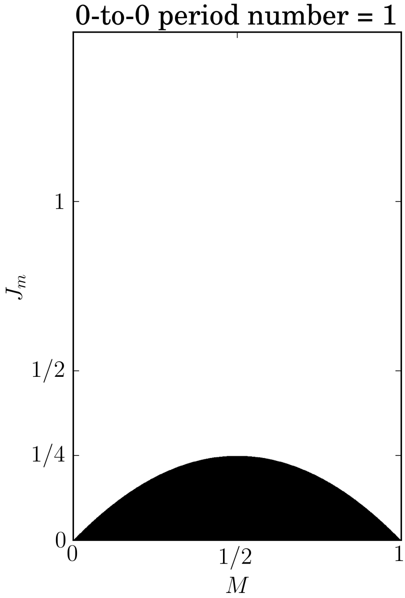
Conduction Modes

- effects caused by the period-1 limit cycle instability for supposed-to-be CCM
- where the phenomena occur?
  1. $M > \frac{1}{2}$ to ensure the period-1 limit cycle instability
  2. $J_m > M(1-M)$ to put period-1 mode in the continuous conduction
  3. $J_m < M$ to allow the inductor discharge over one period; **this is new**!
- what happens? **start from zero, instability, eventually return to zero; once returned, "it will start again, it won't be any different, will be exactly the same"**
- ... and this is period-n DCM!

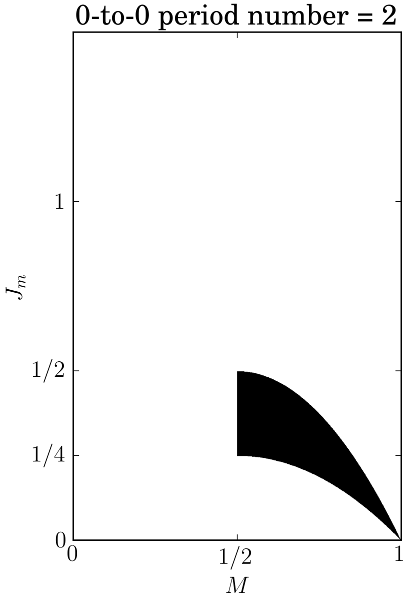
chart of modes



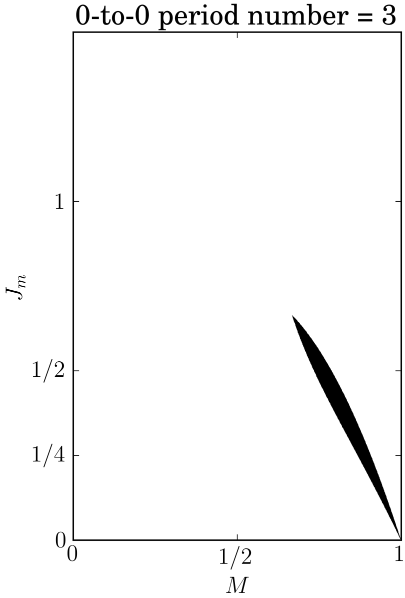
DCM-1



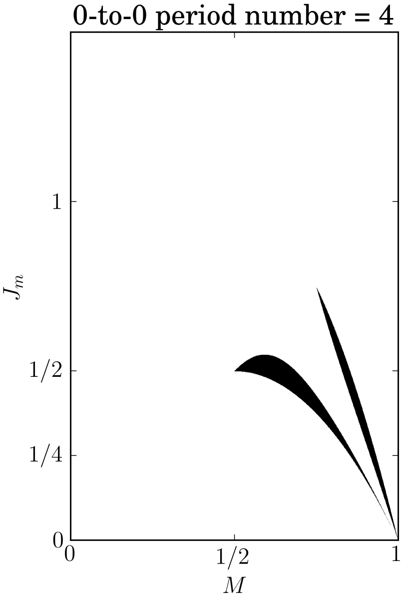
DCM-2



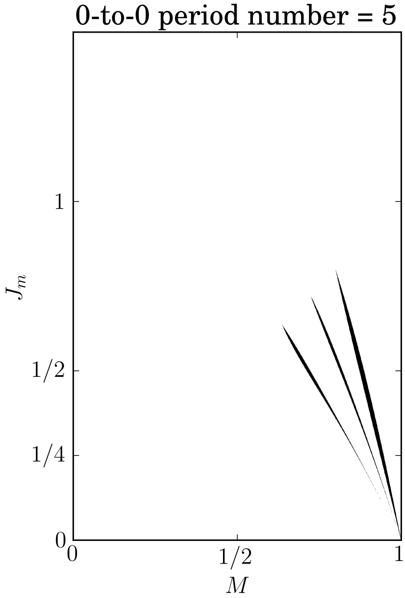
DCM-3



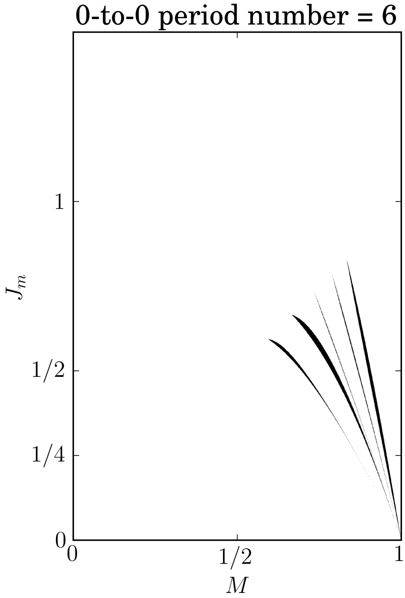
DCM-4



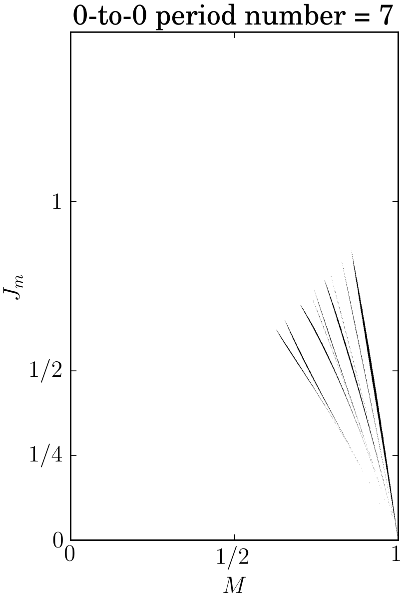
DCM-5



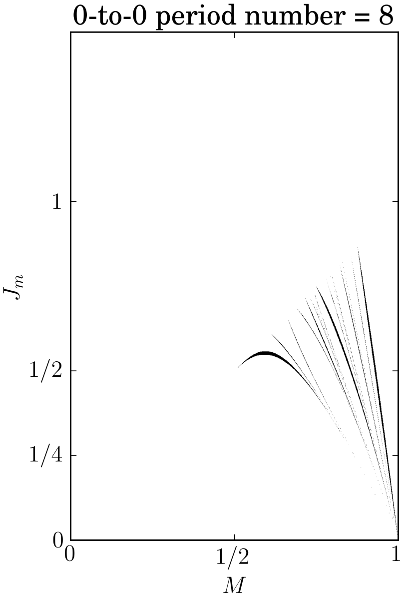
DCM-6



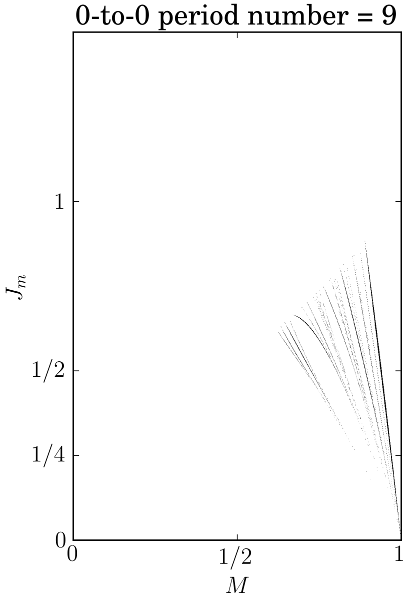
DCM-7



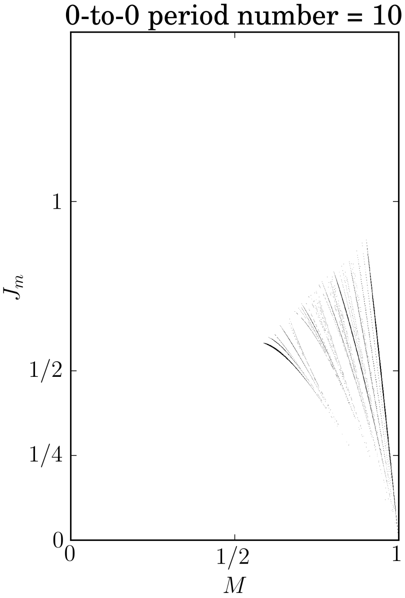
DCM-8



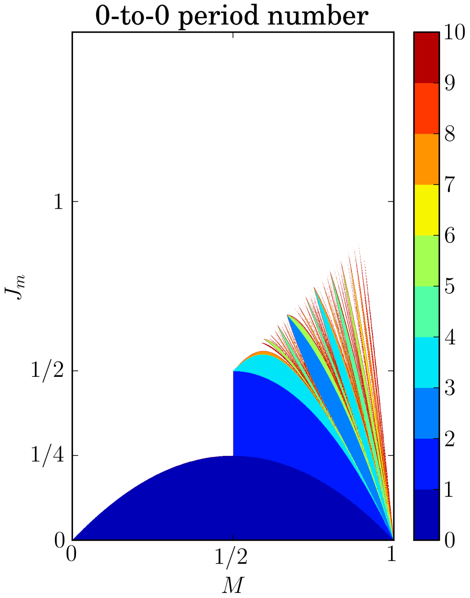
DCM-9



DCM-10



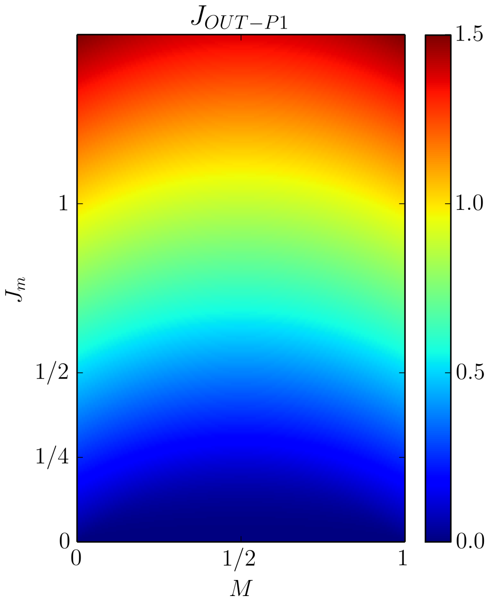
dependence of the period number on M and J_m



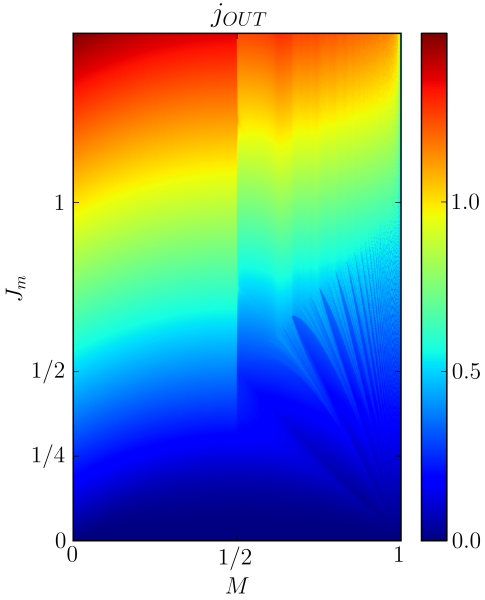
Output Current and Stability

- $j_{OUT} = \overline{j_L}$
- interested in $j_{OUT}(M, J_m)$
- open loop (averaged model) stability for $\frac{d\,j_{OUT}}{d\,M} < 0$
- comparison to period-1 model

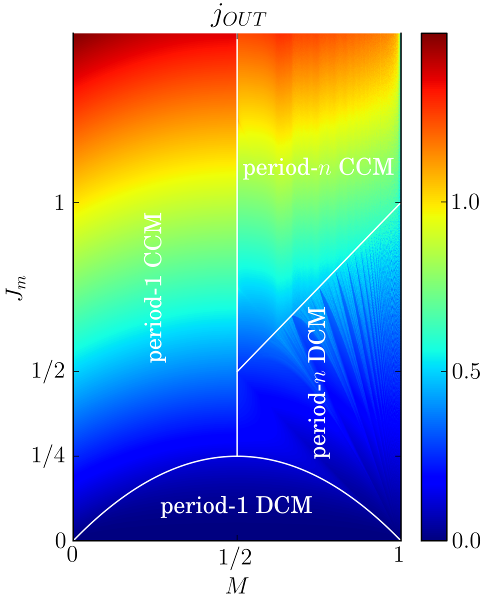
$j_{OUT}(M, J_m)$, period-1 assumed



$j_{OUT}(M, J_m)$, actual

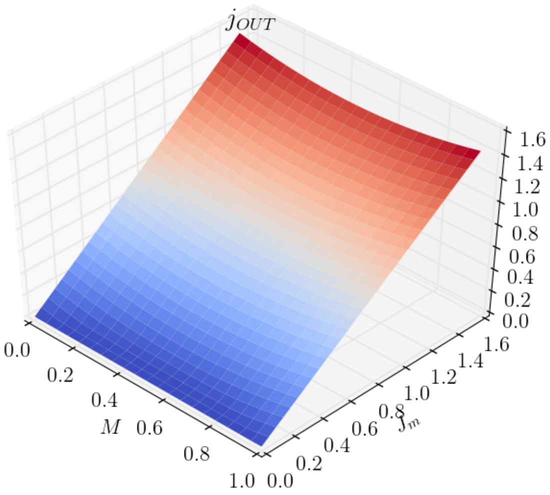


$j_{OUT}(M, J_m)$

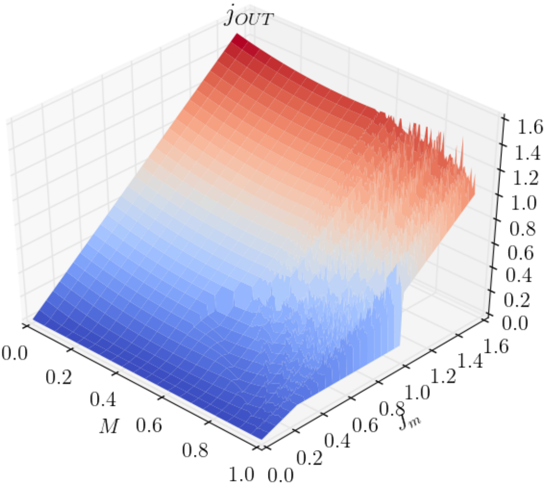


j_{OUT}
period-n CCM
period-1 CCM
period-n DCM
period-1 DCM
J_m
1
1/2
1/4
0
0
1/2
1
M
1.0
0.5
0.0

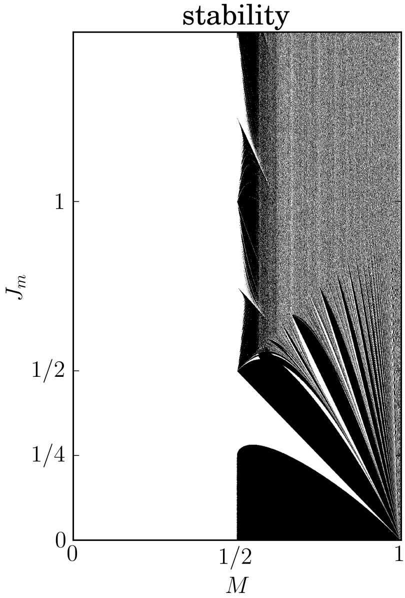
$j_{OUT}(M, J_m)$ in 3D, period-1



$j_{OUT}(M, J_m)$ in 3D



stability, different than 50 : 50



Conclusions 1

- analysis of a PLCMC buck converter
- reduction to a switching cell
- discrete-time model
- normalized discrete time model
- limit cycle instability (CCM) causes all the problems ...
- where the period-1 limit cycle is stable everything is as predicted by the averaged model obtained assuming the period-1 mode
- unstable limit cycle does not hold for long, does not repeat, not suitable for average modelling

Conclusions 2

- the converter might end up in period-n discontinuous conduction mode, with stable limit cycle, an infinite number of such modes
- region where period-n DCM occurs identified
- regions for DCM-1, DCM-2, DCM-3, DCM-4, $\ldots$ identified, some of them numerically
- complex behavior observed

Conclusions 3

- out of the DCM region, the converter might reach some sort of period-n CCM, where n might be ∞, not so easy to model in that case
- for averaged models it is nice to have periodic behavior . . .
- but long-term statistics seem to converge . . . (?)

Conclusions 4

- control model of the switching cell “derived”, $j_{OUT}(M, J_m)$
- … applying simulation of the normalized discrete-time model
- open loop stability analyzed, averaged model
- instability of the period-1 limit cycle in CCM modifies the assumed model …
- resulting in a complex behavior …
- where small-signal models are of limited value
- conclusion: avoid period-n modes
- which we knew before
- but we understand it better now!