

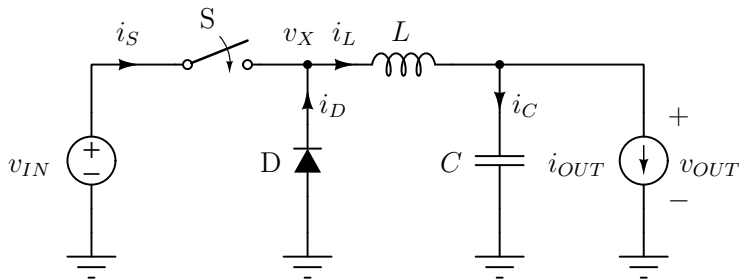
STABILITY ISSUES IN PEAK LIMITING CURRENT MODE CONTROLLED BUCK CONVERTER

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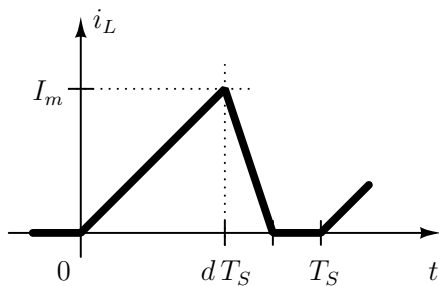
Introduction

- ▶ peak limiting current mode control ...
- ▶ known since 1978, C. W. Deisch, “Simple switching control method changes power converter into a current source,” PESC’78 [2]
- ▶ revisited many times, e.g. in 2001 [6] and 2011 (!) [7]
- ▶ still something to say?
- ▶ CCM, DCM, stability, $D > 0.5$, chaos, ...
- ▶ artificial ramp ...
- ▶ purpose of the paper to clarify the issues ...
- ▶ and this presentation contains **more** than the paper does!

the circuit ... constant current load!

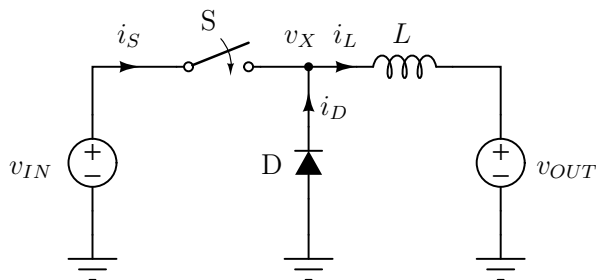


the waveform, ... DCM assumed!



$$\overline{i_L} = \frac{f_S L}{2} I_m^2 \frac{v_{IN}}{v_{OUT}} \frac{1}{v_{IN} - v_{OUT}}$$

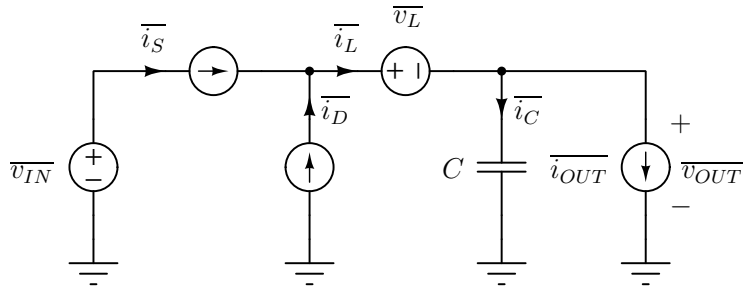
decoupling, switching cell ... also assumed, implicitly!



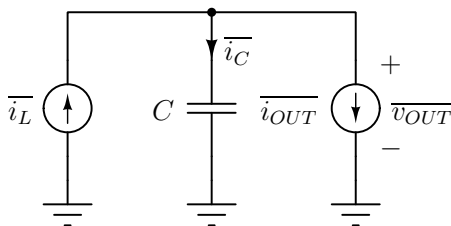
used to draw i_L , to compute $\overline{i_L}$...

v_{OUT} assumed constant over T_S ... implicitly!

decoupling, averaged model ...



decoupling, averaged model simplified ...



$$C \frac{d\overline{v_{OUT}}}{dt} = \overline{i_L} - \overline{i_{OUT}}$$

and $\overline{i_L}$ is given three slides above ...

and our story begins here ...

Averaging

$$C \frac{d\overline{v_{OUT}}}{dt} = \frac{f_S L}{2} I_m^2 \frac{\overline{v_{IN}}}{\overline{v_{OUT}}} \frac{1}{\overline{v_{IN}} - \overline{v_{OUT}}} - \overline{i_{OUT}}$$

overline notation consistent?

$$\frac{d\overline{v_{OUT}}}{dt} = 0 \quad \Rightarrow \quad \text{fixed points}$$

two fixed points ... (overline notation dropped)

$$v_{OUT\,1,2} = \frac{v_{IN}}{2} \pm \sqrt{\frac{v_{IN}^2}{4} - \frac{f_S L I_m^2 v_{IN}}{2 i_{OUT}}}$$

and it is not a good practice to have two when you need only one ...

an example ...

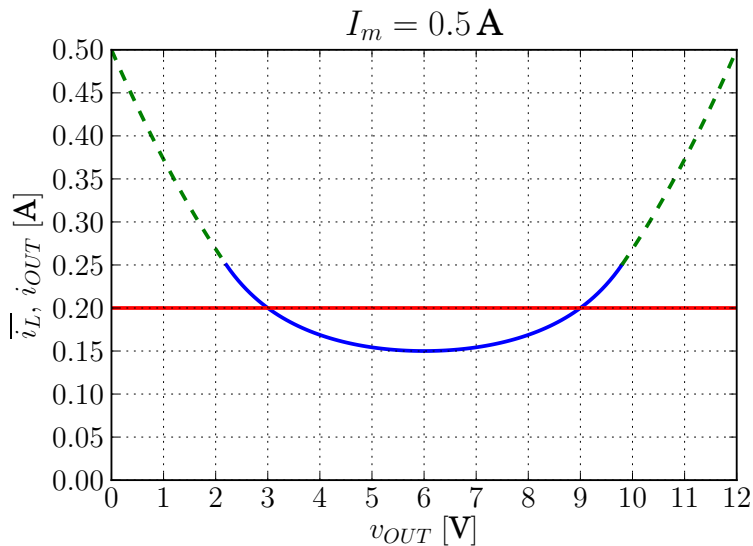
$$v_{IN} = 12 \text{ V}, f_S = 100 \text{ kHz}, L = 36 \mu\text{H}, I_m = 0.5 \text{ A}, \\ i_{OUT} = 0.2 \text{ A}, C = 200 \mu\text{F}$$

$$\overline{i_L} = 0.45 \text{ A} \frac{12 \text{ V}}{v_{OUT}} \frac{1}{12 \text{ V} - v_{OUT}}$$

$$\overline{i_L} = i_{OUT} = 0.2 \text{ A}$$

$$\text{fixed points: } v_{OUT} = \begin{cases} 3 \text{ V} \\ 9 \text{ V} \end{cases}$$

fixed points ...



detour: normalization

$$m_X \triangleq \frac{v_X}{v_{IN}}$$

$$j_Y \triangleq \frac{f_S L i_Y}{v_{IN}}$$

$$\tau \triangleq \frac{t}{T_S} = f_S t$$

result:

$$L \frac{d i_L}{d t} = v_L \quad \Rightarrow \quad \frac{d j_L}{d \tau} = m_L$$

special: $v_{IN} \Rightarrow 1$, $v_{OUT} \Rightarrow M$, $I_m \Rightarrow J_m$, $i_{OUT} \Rightarrow j_{OUT}$

fixed points, normalized

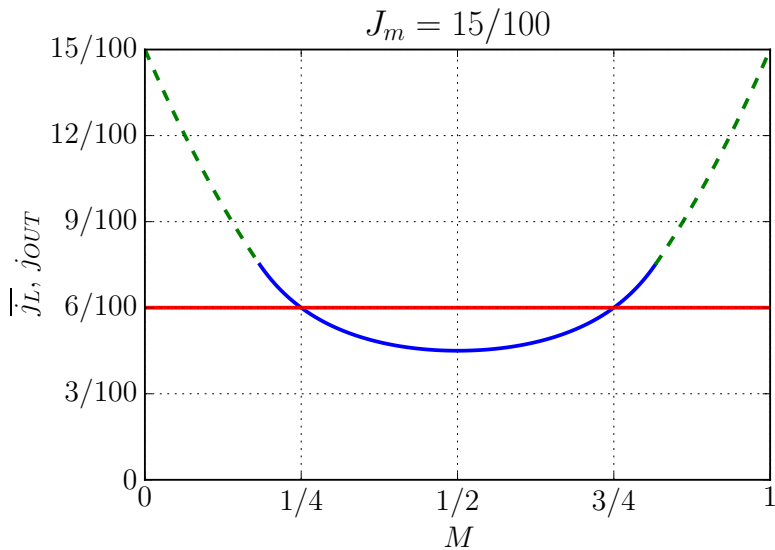
$$V_{base} = 12 \text{ V}, I_{base} = \frac{10}{3} \text{ A}$$

$$J_m = 0.15, j_{OUT} = 0.06$$

$$M_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{J_m^2}{2j_{OUT}}}$$

$$\text{fixed points: } M = \begin{cases} 1/4 \\ 3/4 \end{cases}$$

fixed points ... normalized!



Linearization

$$C \frac{d\overline{v_{OUT}}}{dt} = \frac{f_S L}{2} I_m^2 \frac{\overline{v_{IN}}}{\overline{v_{OUT}}} \frac{1}{\overline{v_{IN}} - \overline{v_{OUT}}}$$

$$s C \hat{v}_{OUT} = g_{IN} \hat{v}_{IN} + g_{OUT} \hat{v}_{OUT} + \alpha_m \hat{I}_m - \hat{i}_{OUT}$$

$$g_{IN} = \frac{\partial \overline{i_L}}{\partial \overline{v_{IN}}} = - \frac{f_S L I_M^2}{2 (V_{IN} - V_{OUT})^2}$$

$$g_{OUT} = \frac{\partial \overline{i_L}}{\partial \overline{v_{OUT}}} = \frac{f_S L I_M^2 V_{IN} (2 V_{OUT} - V_{IN})}{2 V_{OUT}^2 (V_{IN} - V_{OUT})^2}$$

$$\alpha_m = \frac{\partial \overline{i_L}}{\partial \overline{I_m}} = \frac{f_S L I_M V_{IN}}{V_{OUT} (V_{IN} - V_{OUT})}$$

transfer functions ...

$$\hat{v}_{OUT} = H_{IN} \hat{v}_{IN} + H_m \hat{I}_m - H_{OUT} \hat{i}_{OUT}$$

$$H_{IN} = \frac{g_{IN}}{sC - g_{OUT}}$$

$$H_m = \frac{\alpha_m}{sC - g_{OUT}}$$

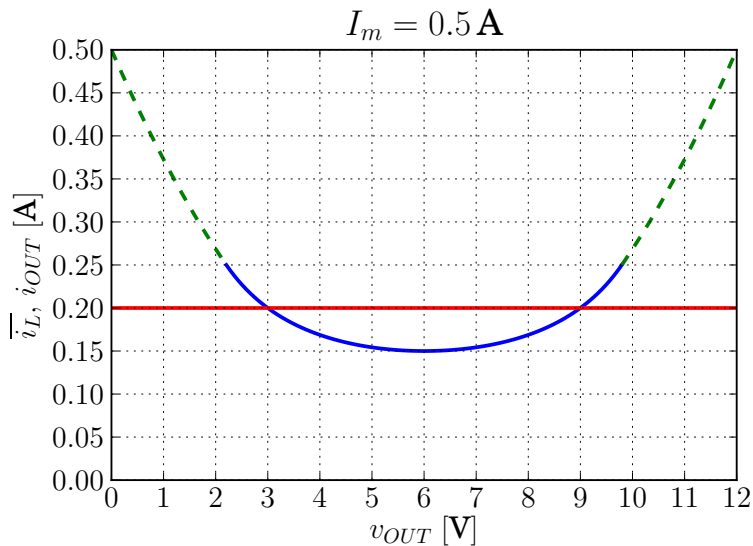
$$H_{OUT} = \frac{1}{sC - g_{OUT}}$$

stability: $g_{OUT} < 0$,

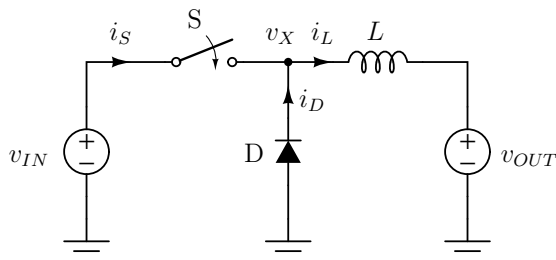
$$\boxed{\frac{\partial \overline{i_L}}{\partial \overline{v_{OUT}}} < 0}$$

...previous slide: $2V_{OUT} - V_{IN} < 0$, $V_{OUT} < V_{IN}/2$

fixed points, once again ... what's going on?



Discrete Time Model



- ▶ v_{OUT} assumed constant over T_S
- ▶ want to know mapping $i_L(0) \rightarrow i_L(T_S)$
... knowing I_m , v_{IN} , v_{OUT} , f_S , L ...
... or just J_m and M ? ($5 \rightarrow 2$)
- ▶ $\overline{i_L}(n) \triangleq \frac{1}{T_S} \int_{(n-1)T_S}^{nT_S} i_L(t) dt$ is an auxiliary (but important!) result
- ▶ **normalization is useful here!!!**

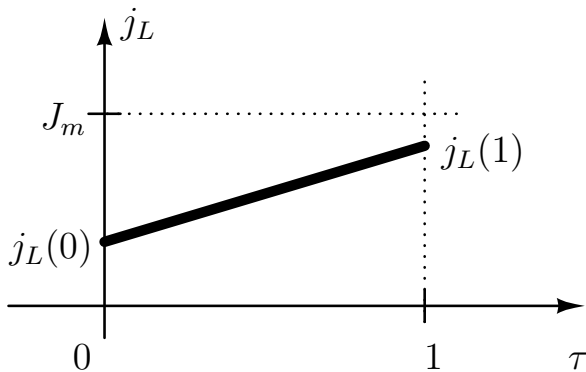
normalization, three cases ...

- ▶ S-on, D-off: $\frac{d j_L}{dt} = 1 - M$
- ▶ S-off, D-on: $\frac{d j_L}{dt} = -M$
- ▶ S-off, D-off: $\frac{d j_L}{dt} = 0$

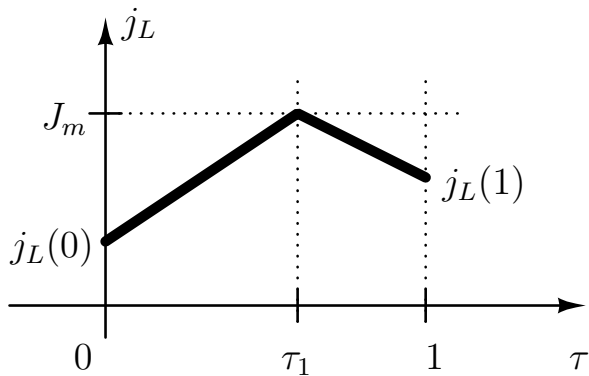
But only two parameters, J_m and M !

Look for $j_L(n-1) \rightarrow j_L(n)$ and $\overline{j_L}(n)$!

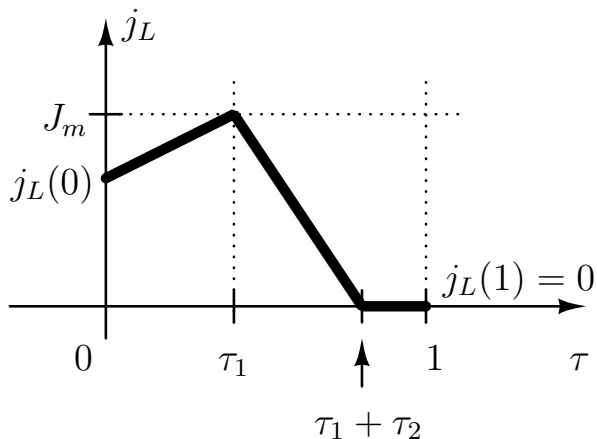
three cases, again ... case 1, no switching interval



three cases, again ... case 2, continuous conduction interval



three cases, again ... case 3, discontinuous conduction interval

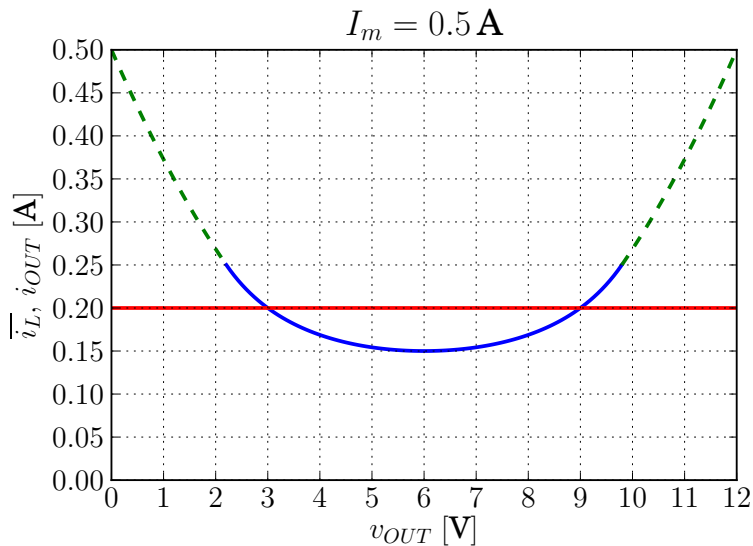


analytical ...

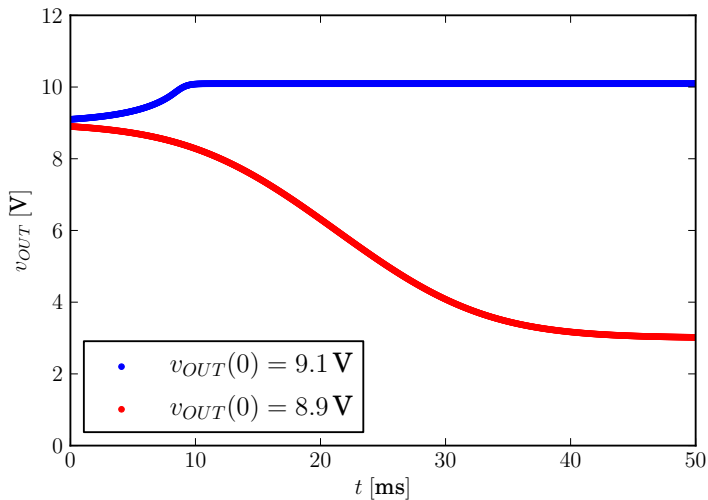
$$j_L(1)(j_L(0), M, J_m) = \left\{ \begin{array}{l} 1 - M + j_L(0), \\ \quad \text{if} \\ \quad j_L(0) < J_m + M - 1 \\ \\ \frac{1}{1 - M} J_m - M - \frac{M}{1 - M} j_L(0), \\ \quad \text{if} \\ \quad J_m + M - 1 < j_L(0) \\ \quad \text{and} \\ \quad j_L(0) < \frac{J_m}{M} + M - 1 \\ \\ 0, \\ \quad \text{if} \\ \quad \frac{J_m}{M} + M - 1 < j_L(0) \end{array} \right.$$

... similar for j_{OUT} ...

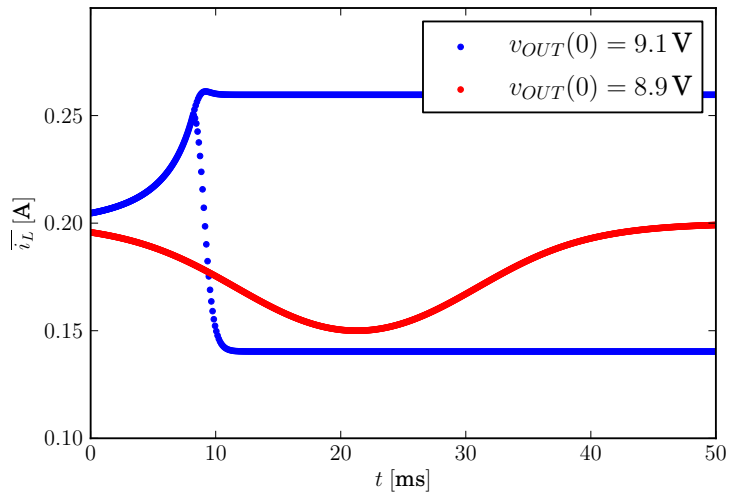
Basins of Attraction



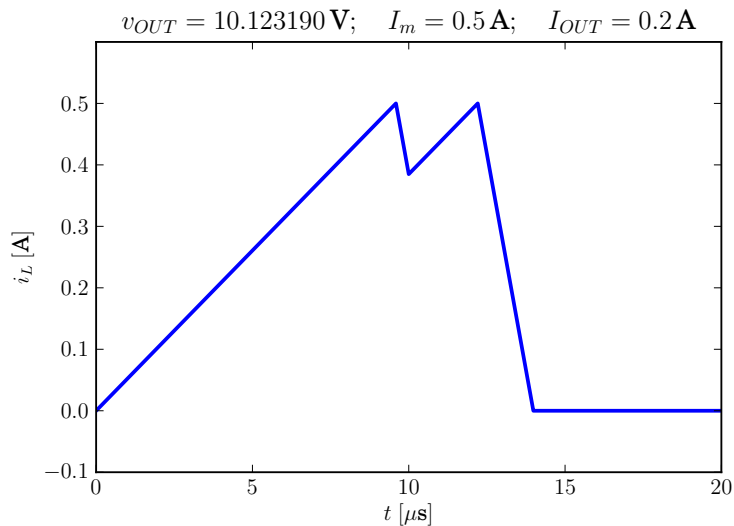
trajectory of v_{OUT} ...



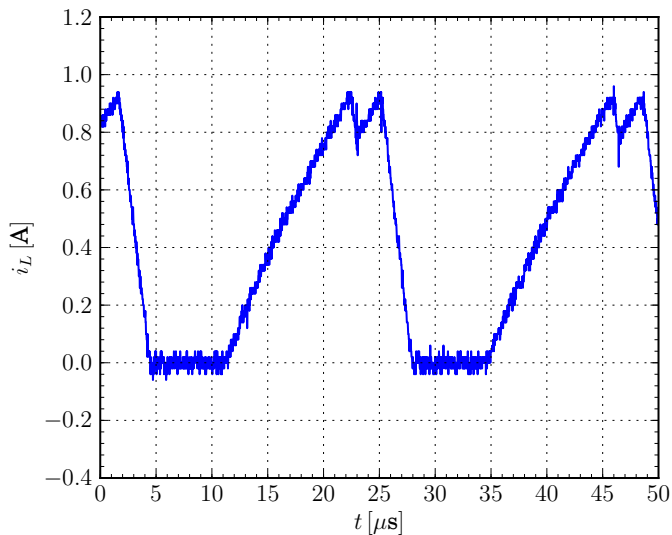
trajectory of i_L ...



steady state waveform of i_L , ... “twin peaks”



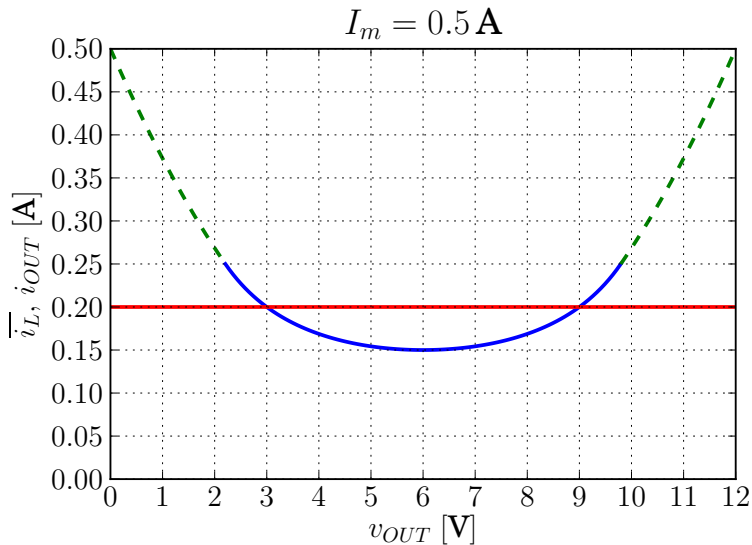
actually happens ...



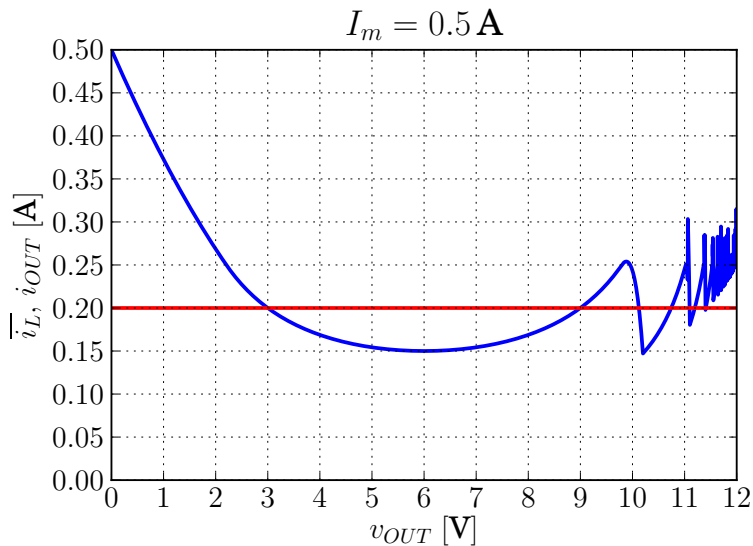
Limit Cycles

- ▶ the problem begins when the converter would enter CCM for $D > 1/2$
- ▶ **supposed limit cycle is unstable!**
- ▶ but the converter operates in **a** stable limit cycle, regardless our assumptions ...
- ▶ ... it happened to be period-2 DCM ...
- ▶ ... and here the mess starts ...

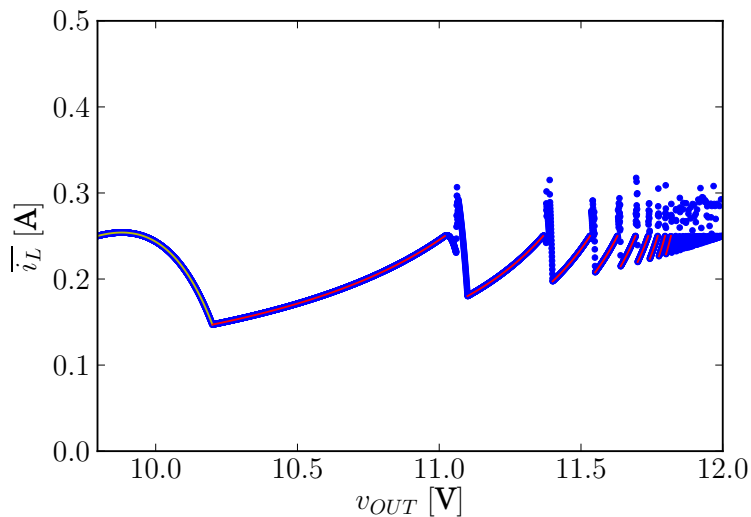
supposed ...



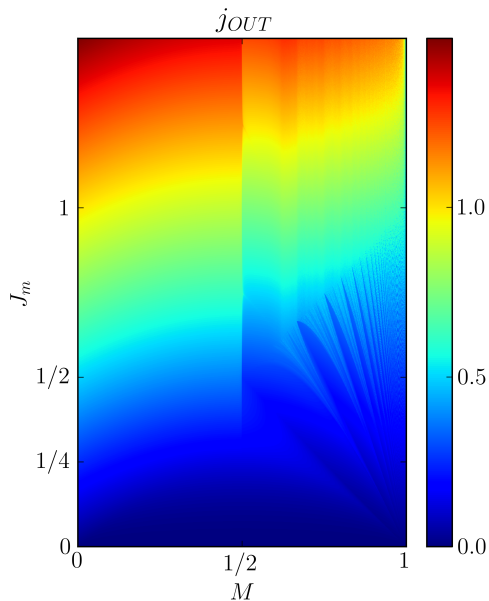
actual ...



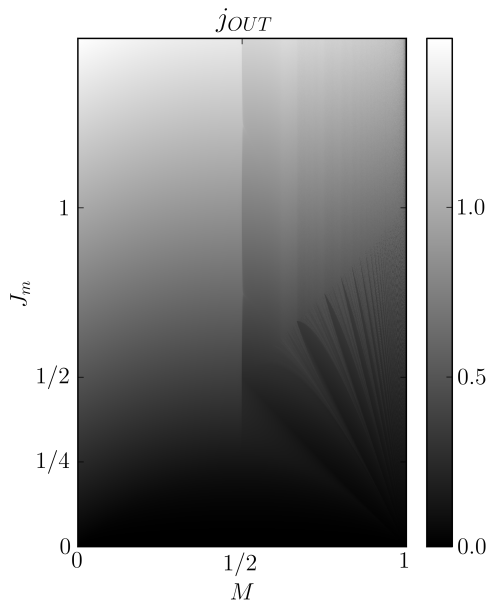
and we have it analytical ... in the paper! (boring)



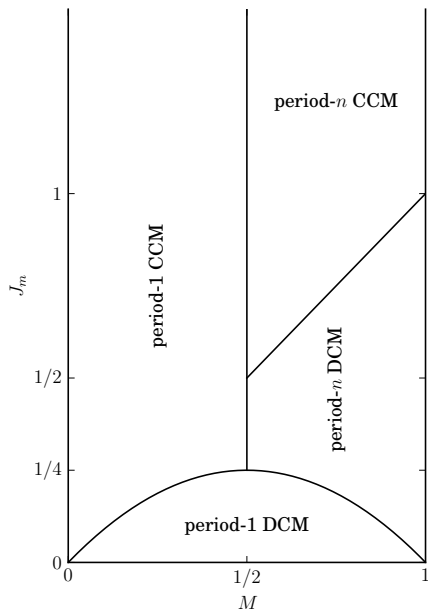
iterate over $J_m \dots$ not in the paper!



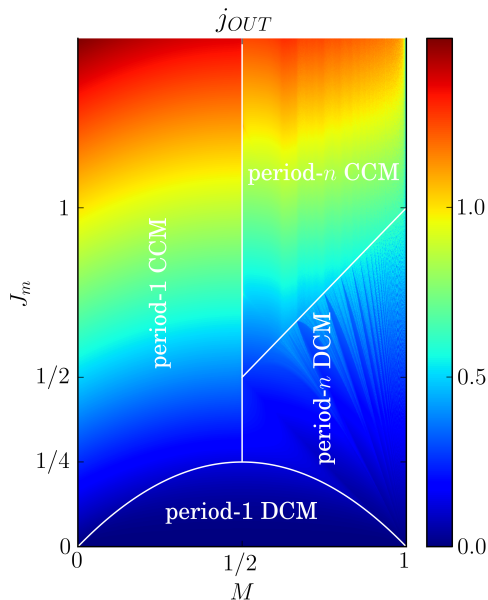
iterate over $J_m \dots$ not in the paper!



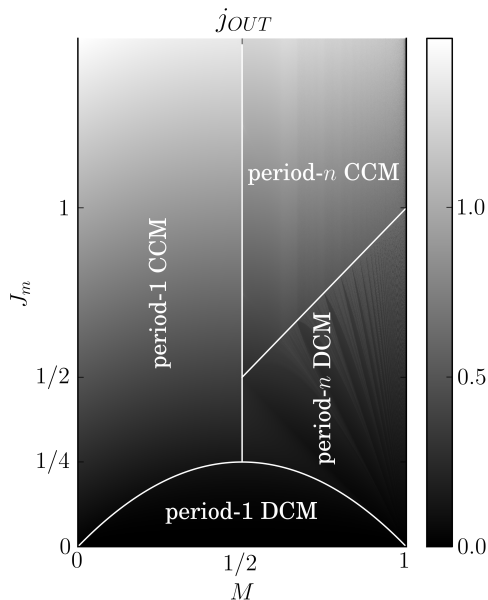
important: operating mode chart ... not in the paper!



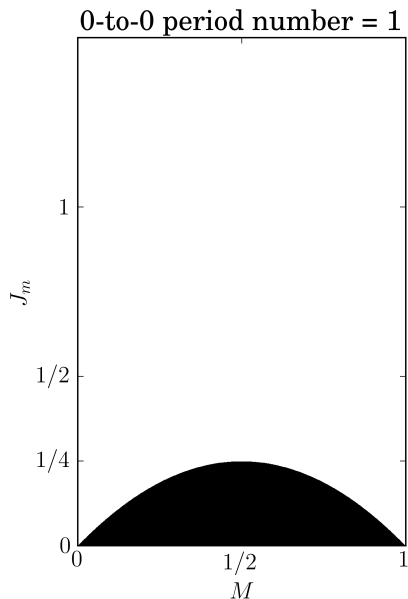
iterate over $J_m \dots$ not in the paper!



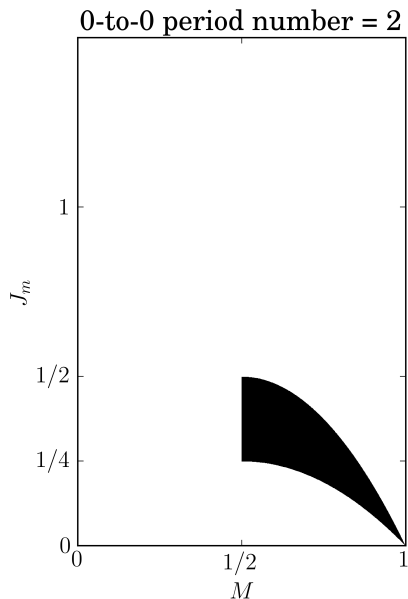
iterate over $J_m \dots$ not in the paper!



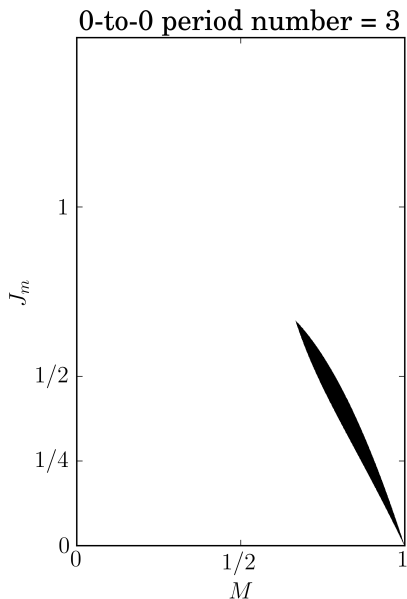
DCM, period number, $n = 1$, not in the paper!



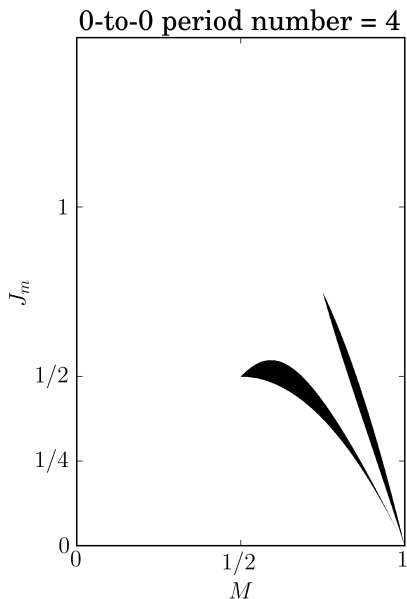
DCM, period number, $n = 2$, not in the paper!



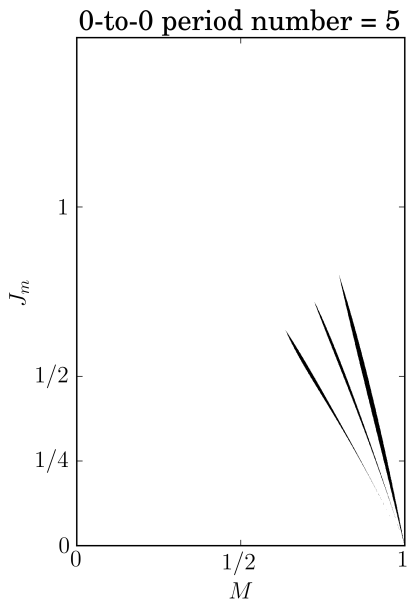
DCM, period number, $n = 3$, not in the paper!



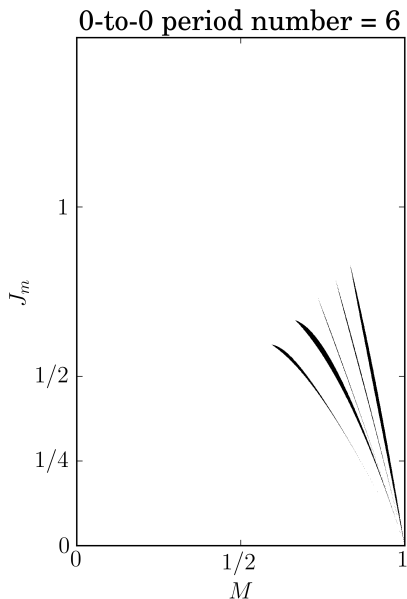
DCM, period number, $n = 4$, not in the paper!



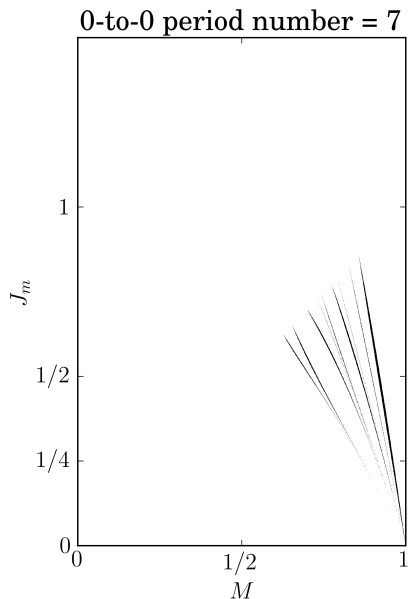
DCM, period number, $n = 5$, not in the paper!



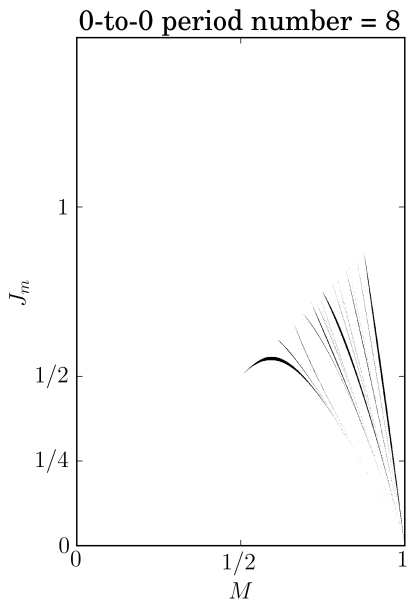
DCM, period number, $n = 6$, not in the paper!



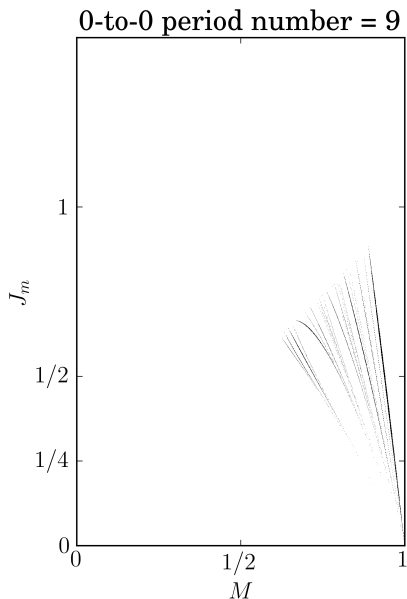
DCM, period number, $n = 7$, not in the paper!



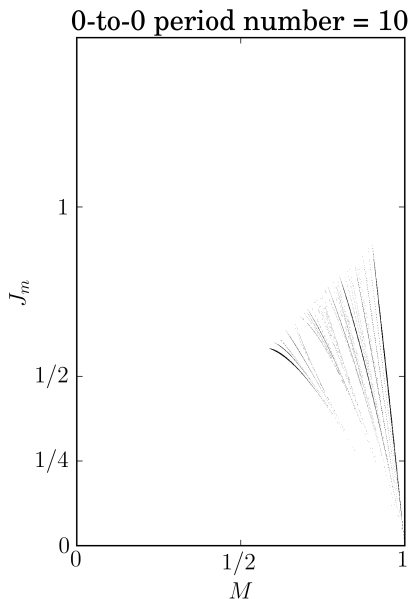
DCM, period number, $n = 8$, not in the paper!



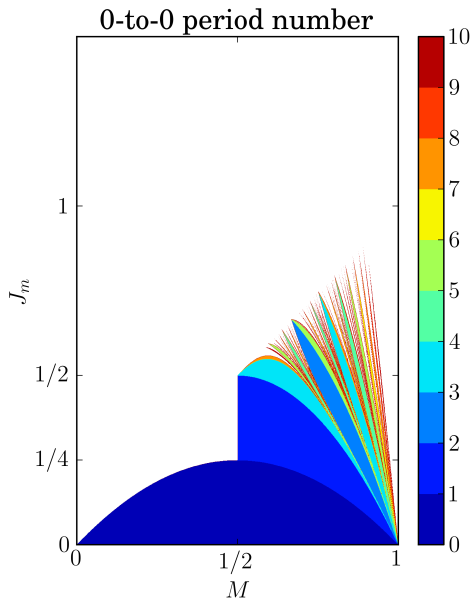
DCM, period number, $n = 9$, not in the paper!



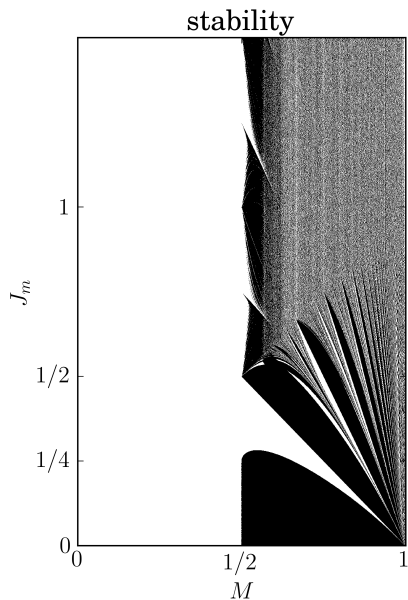
DCM, period number, $n = 10$, not in the paper!



period number, in color, ... not in the paper!



stability ... not in the paper!



Conclusions, 1

- ▶ buck converter analyzed, PLCMC applied
- ▶ decoupling (in **reversed** order):
 1. “averaged” model, linearized later on ...
 2. “discrete time” model
- ▶ stability:
 1. **stability of the averaged model**
 2. **limit cycle stability** (stability of the discrete time model)
- ▶ limit cycle instability:
 1. occurs in would-be CCM for $D > 1/2$
 2. results in sensitive small-signal parameters
 3. **affects averaged model stability!**
- ▶ analytical techniques, models, normalization ...

Conclusions, 2

- ▶ **in the paper**, case study for $J_m = 0.15$
- ▶ analytical techniques developed, discrete time model
- ▶ detailed study of the discrete time model
- ▶ identification of modes
- ▶ pretty good analytical description ...
- ▶ analysis of stability

Conclusions, 3

- ▶ **in this presentation**, generalized over J_m , the remaining degree of freedom, along with M , completeness achieved
- ▶ **important:**
 1. occurrence of period- n modes when assumed period-1 CCM has unstable limit cycle, for $D > 1/2$
 2. both period- n CCM and period- n DCM exist
- ▶ **charts:**
 1. chart of modes
 2. chart of periodicity (chart of n)
 3. chart of stability

Conclusion

avoid period- n modes!