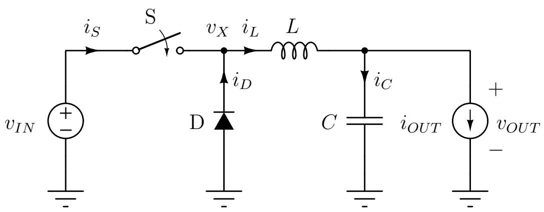
STABILITY ISSUES IN PEAK LIMITING CURRENT MODE CONTROLLED BUCK CONVERTER

Marija Glišić, Predrag Pejović

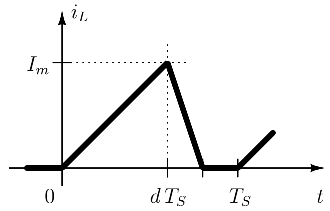
Introduction

- peak limiting current mode control ...
- known since 1978, C. W. Deisch, "Simple switching control method changes power converter into a current source," PESC'78 [2]
- revisited many times, e.g. in 2001 [6] and 2011 (!) [7]
- still something to say?
- CCM, DCM, stability, $D > 0.5$, chaos, ...
- artificial ramp ...
- purpose of the paper to clarify the issues ...
- and this presentation contains **more** than the paper does!

the circuit ... constant current load!

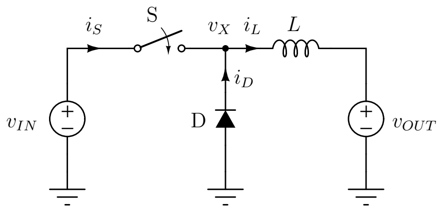


the waveform, ... DCM assumed!



$$\overline{i_L} = \frac{f_S\,L}{2}\,I_m^2\,\frac{v_{IN}}{v_{OUT}}\,\frac{1}{v_{IN} - v_{OUT}}$$

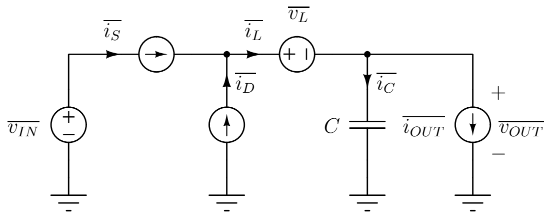
decoupling, switching cell ... also assumed, implicitly!



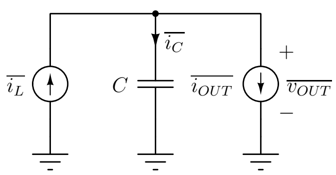
used to draw i_L, to compute $\overline{i_L}$ ...

v_{OUT} assumed constant over T_S ... implicitly!

decoupling, averaged model ...



decoupling, averaged model simplified ...



$$C\,\frac{d\,\overline{v_{OUT}}}{d\,t} = \overline{i_L} - \overline{i_{OUT}}$$

and $\overline{i_L}$ is given three slides above ...

and our story begins here ...

Averaging

$$C\,\frac{d\,\overline{v_{OUT}}}{d\,t} = \frac{f_S\,L}{2}\,I_m^2\,\frac{\overline{v_{IN}}}{\overline{v_{OUT}}}\,\frac{1}{\overline{v_{IN}} - \overline{v_{OUT}}} - \overline{i_{OUT}}$$

overline notation consistent?

$$\frac{d\,\overline{v_{OUT}}}{d\,t} = 0 \quad \Rightarrow \quad \text{fixed points}$$

two fixed points ... (overline notation dropped)

$$v_{OUT\,1,2} = \frac{v_{IN}}{2} \pm \sqrt{\frac{v_{IN}^2}{4} - \frac{f_S\,L\,I_m^2\,v_{IN}}{2\,i_{OUT}}}$$

and it is not a good practice to have two when you need only one ...

an example . . .

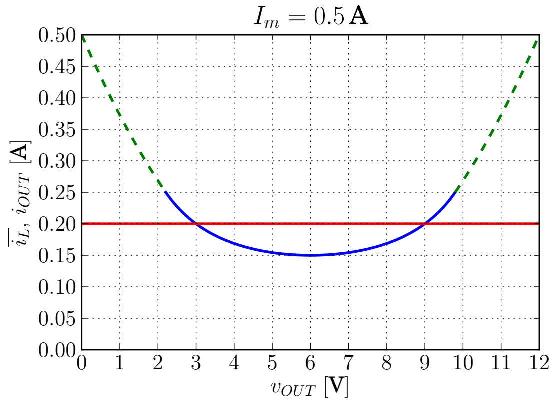
$v_{IN} = 12\,\mathrm{V}$, $f_S = 100\,\mathrm{kHz}$, $L = 36\,\mu\mathrm{H}$, $I_m = 0.5\,\mathrm{A}$, $i_{OUT} = 0.2\,\mathrm{A}$, $C = 200\,\mu\mathrm{F}$

$$\overline{i_L} = 0.45\,\mathrm{A}\,\frac{12\,\mathrm{V}}{v_{OUT}}\,\frac{1}{12\,\mathrm{V} - v_{OUT}}$$

$$\overline{i_L} = i_{OUT} = 0.2\,\mathrm{A}$$

$$\text{fixed points:}\quad v_{OUT} = \begin{cases} 3\,\mathrm{V} \\ 9\,\mathrm{V} \end{cases}$$

fixed points . . .



detour: normalization

$$m_X \triangleq \frac{v_X}{v_{IN}}$$

$$j_Y \triangleq \frac{f_S\,L\,i_Y}{v_{IN}}$$

$$\tau \triangleq \frac{t}{T_S} = f_S\,t$$

result:

$$L\,\frac{d\,i_L}{d\,t} = v_L \quad \Rightarrow \quad \frac{d\,j_L}{d\,\tau} = m_L$$

special: $v_{IN} \Rightarrow 1$, $v_{OUT} \Rightarrow M$, $I_m \Rightarrow J_m$, $i_{OUT} \Rightarrow j_{OUT}$

fixed points, normalized

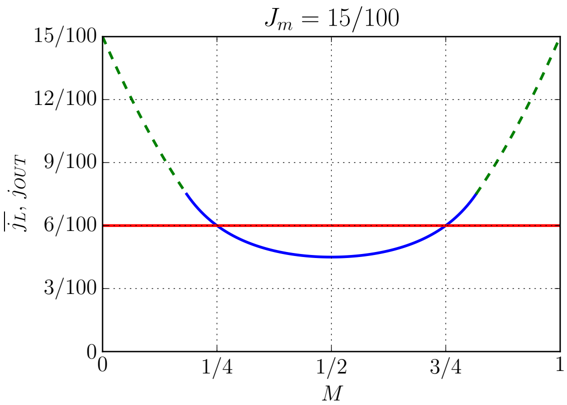
$V_{base} = 12\,\mathrm{V}$, $I_{base} = \frac{10}{3}\,\mathrm{A}$

$J_m = 0.15$, $j_{OUT} = 0.06$

$$M_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{J_m^2}{2\,j_{OUT}}}$$

$$\text{fixed ponts:}\quad M = \begin{cases} 1/4 \\ 3/4 \end{cases}$$

fixed points . . . normalized!



Linearization

$$C\,\frac{d\,\overline{v_{OUT}}}{d\,t} = \frac{f_S\,L}{2}\,I_m^2\,\frac{\overline{v_{IN}}}{\overline{v_{OUT}}}\,\frac{1}{\overline{v_{IN}} - \overline{v_{OUT}}}$$

$$s\,C\,\widehat{v}_{OUT} = g_{IN}\,\widehat{v}_{IN} + g_{OUT}\,\widehat{v}_{OUT} + \alpha_m\,\widehat{I}_m - \widehat{i}_{OUT}$$

$$g_{IN} = \frac{\partial\,\overline{i_L}}{\partial\,\overline{v_{IN}}} = -\frac{f_S\,L\,I_M^2}{2\,(V_{IN} - V_{OUT})^2}$$

$$g_{OUT} = \frac{\partial\,\overline{i_L}}{\partial\,\overline{v_{OUT}}} = \frac{f_S\,L\,I_M^2\,V_{IN}\,(2\,V_{OUT} - V_{IN})}{2\,V_{OUT}^2\,(V_{IN} - V_{OUT})^2}$$

$$\alpha_m = \frac{\partial\,\overline{i_L}}{\partial\,I_m} = \frac{f_S\,L\,I_M\,V_{IN}}{V_{OUT}\,(V_{IN} - V_{OUT})}$$

transfer functions . . .

$$\widehat{v}_{OUT} = H_{IN}\,\widehat{v}_{IN} + H_m\,\widehat{I}_m - H_{OUT}\,\widehat{i}_{OUT}$$

$$H_{IN} = \frac{g_{IN}}{s\,C - g_{OUT}}$$

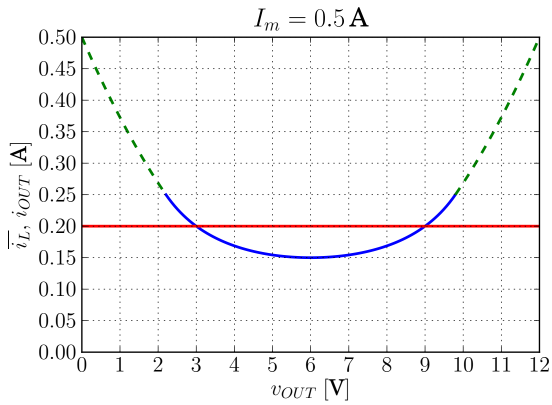
$$H_m = \frac{\alpha_m}{s\,C - g_{OUT}}$$

$$H_{OUT} = \frac{1}{s\,C - g_{OUT}}$$

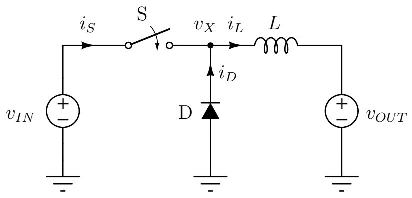
stability: $g_{OUT} < 0$, $\boxed{\frac{\partial\,\overline{i_L}}{\partial\,\overline{v_{OUT}}} < 0}$

. . . previous slide: $2\,V_{OUT} - V_{IN} < 0$, $V_{OUT} < V_{IN}/2$

fixed points, once again . . . what's going on?



Discrete Time Model



- v_{OUT} assumed constant over T_S
- want to know mapping $i_L(0) \to i_L(T_S)$
  … knowing I_m, v_{IN}, v_{OUT}, f_S, L …
  … or just J_m and M? $(5 \to 2)$
- $\overline{i_L}(n) \triangleq \frac{1}{T_S} \int_{(n-1)T_S}^{nT_S} i_L(t)\, dt$ is an auxiliary (but important!) result
- **normalization is useful here!!!**

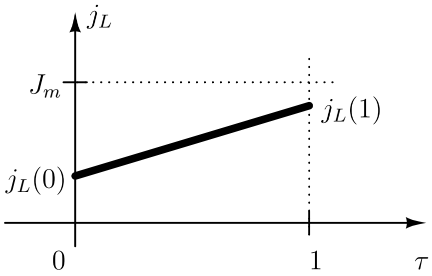
normalization, three cases …

- S-on, D-off: $\dfrac{d\, j_L}{dt} = 1 - M$
- S-off, D-on: $\dfrac{d\, j_L}{dt} = -M$
- S-off, D-off: $\dfrac{d\, j_L}{dt} = 0$

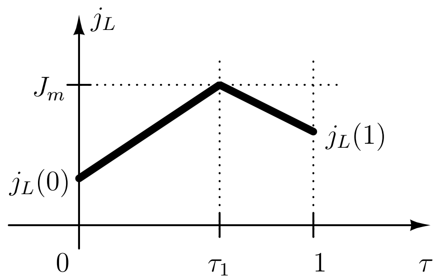
But only two parameters, J_m and M!

Look for $j_L(n-1) \to j_L(n)$ and $\overline{j_L}(n)$!

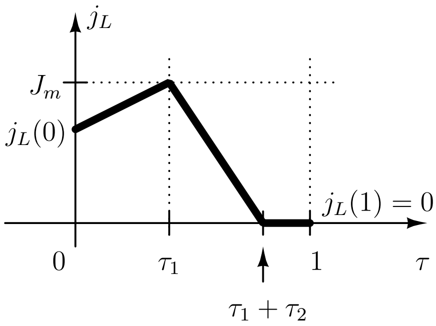
three cases, again … case 1, no switching interval



three cases, again … case 2, continuous conduction interval



three cases, again … case 3, discontinuous conduction interval

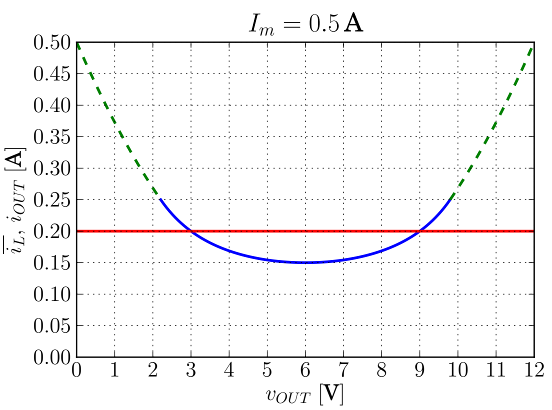


analytical …

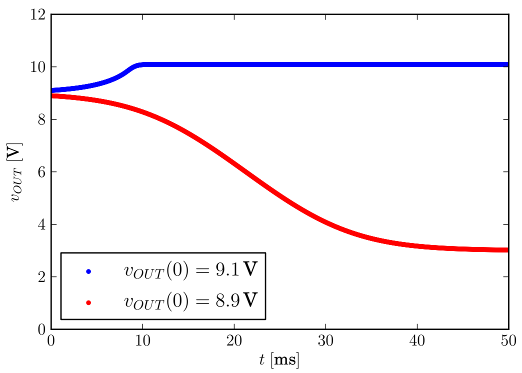
$$j_L(1)\,(j_L(0),\, M,\, J_m) = \begin{cases} 1 - M + j_L(0), & \text{if } j_L(0) < J_m + M - 1 \\ \dfrac{1}{1-M} J_m - M - \dfrac{M}{1-M} j_L(0), & \text{if } J_m + M - 1 < j_L(0) \text{ and } j_L(0) < \dfrac{J_m}{M} + M - 1 \\ 0, & \text{if } \dfrac{J_m}{M} + M - 1 < j_L(0) \end{cases}$$

… similar for j_{OUT} …

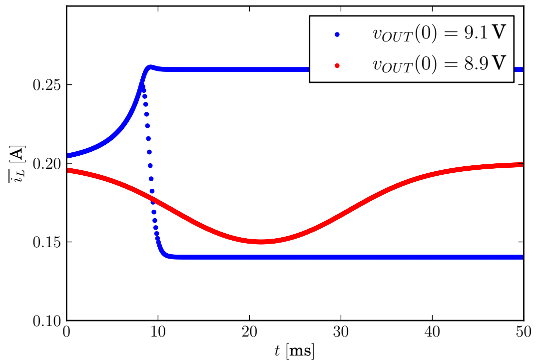
Basins of Attraction



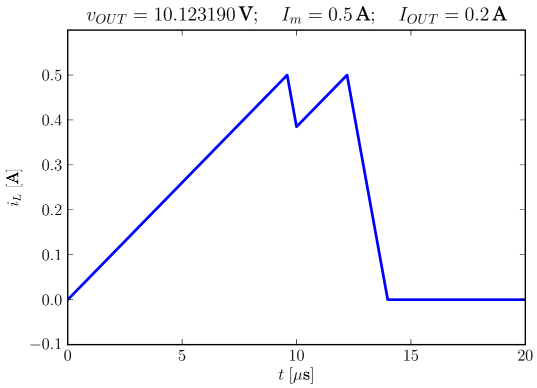
trajectory of v_{OUT} …



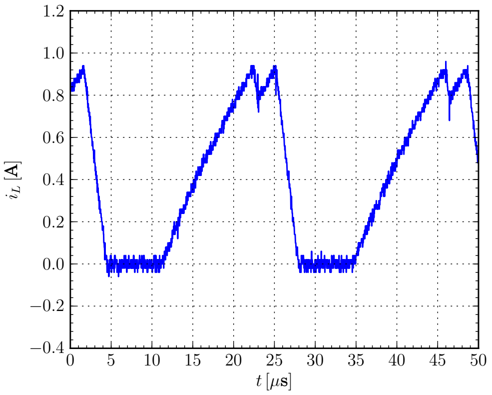
trajectory of i_L …



steady state waveform of i_L, … "twin peaks"



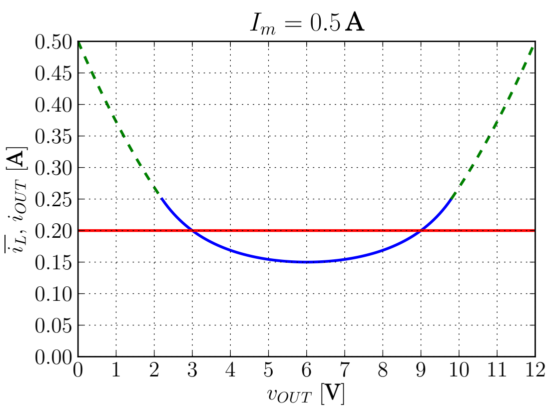
actually happens …



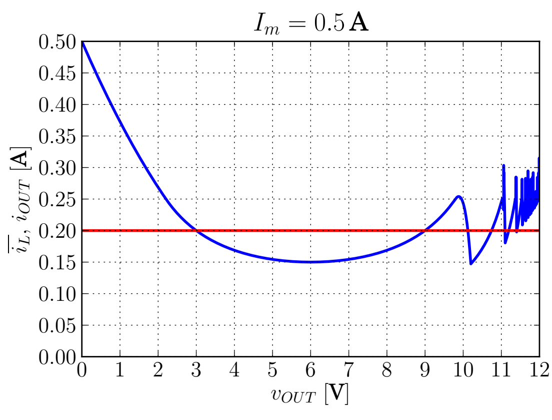
Limit Cycles

- the problem begins when the converter would enter CCM for $D > 1/2$
- **supposed limit cycle is unstable!**
- but the converter operates in **a** stable limit cycle, regardless our assumptions …
- … it happened to be period-2 DCM …
- … and here the mess starts …

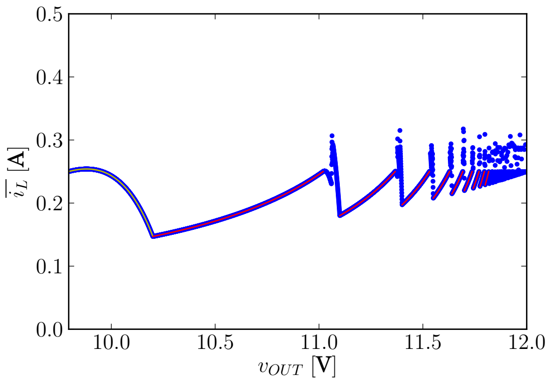
supposed …



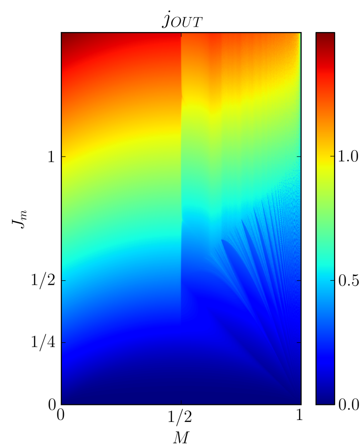
actual …



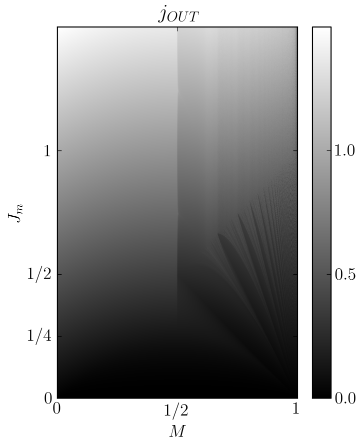
and we have it analytical … in the paper! (boring)



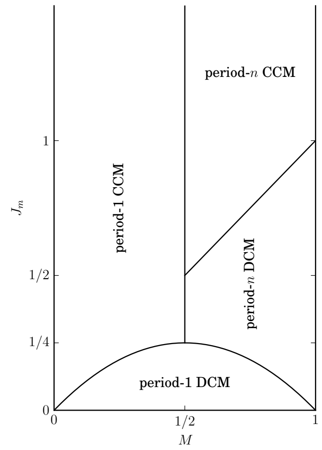
iterate over J_m … not in the paper!



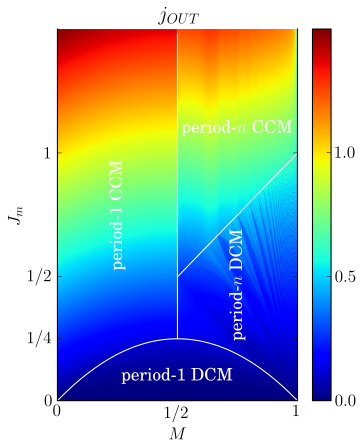
iterate over J_m ... not in the paper!



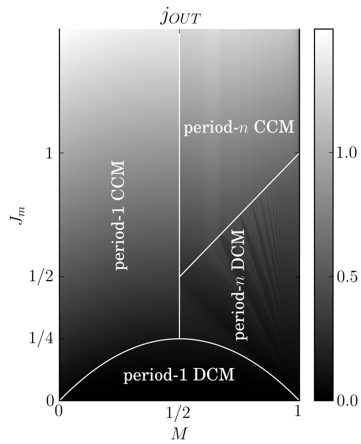
important: operating mode chart ... not in the paper!



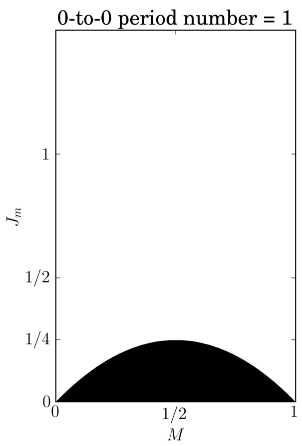
iterate over J_m ... not in the paper!



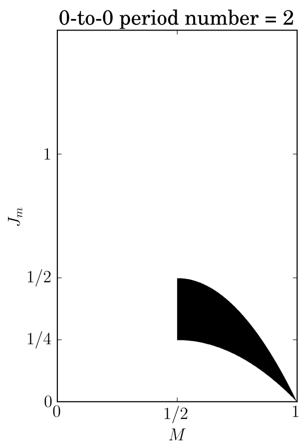
iterate over J_m ... not in the paper!



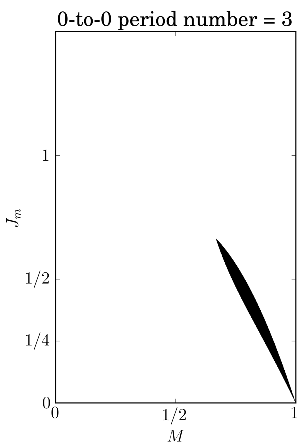
DCM, period number, $n = 1$, not in the paper!



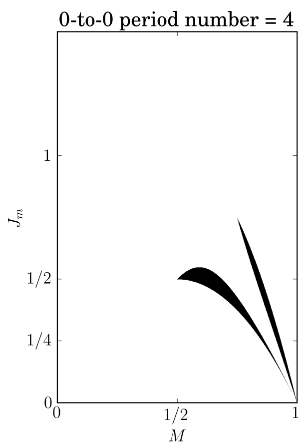
DCM, period number, $n = 2$, not in the paper!



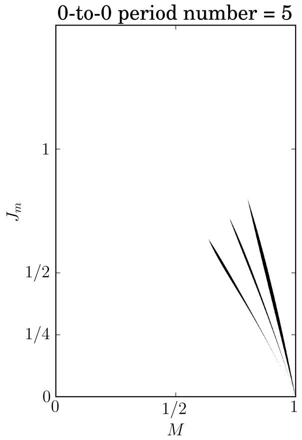
DCM, period number, $n = 3$, not in the paper!



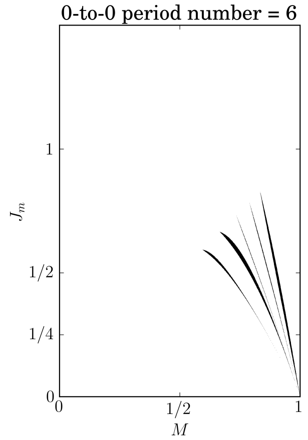
DCM, period number, $n = 4$, not in the paper!



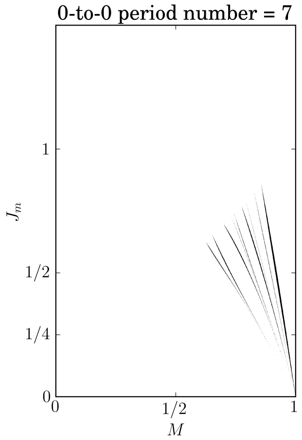
DCM, period number, $n = 5$, not in the paper!



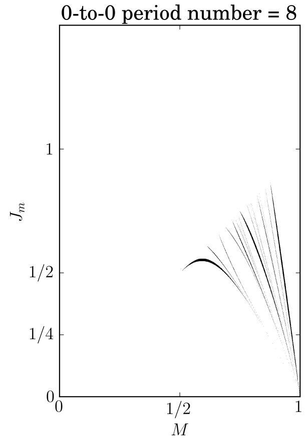
DCM, period number, $n = 6$, not in the paper!



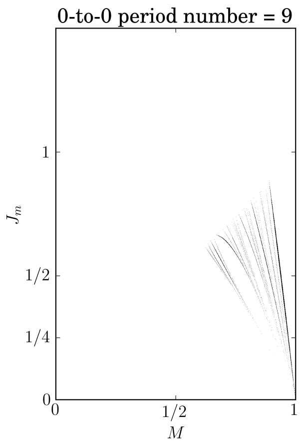
DCM, period number, $n = 7$, not in the paper!



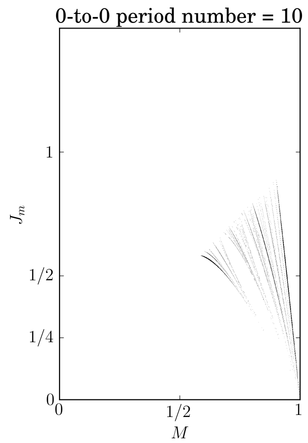
DCM, period number, $n = 8$, not in the paper!



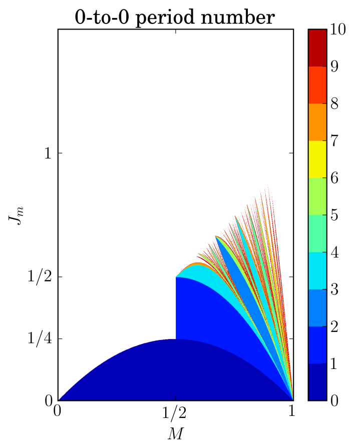
DCM, period number, $n = 9$, not in the paper!



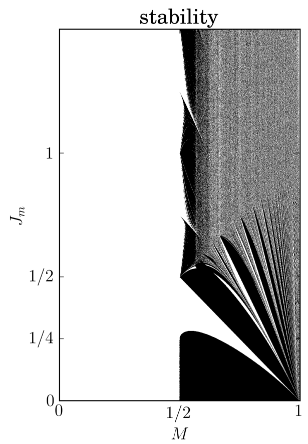
DCM, period number, $n = 10$, not in the paper!



period number, in color, … not in the paper!



stability … not in the paper!



Conclusions, 1

- buck converter analyzed, PLCMC applied
- decoupling (in **reversed** order):
  1. “averaged” model, linearized later on ...
  2. “discrete time” model
- stability:
  1. **stability of the averaged model**
  2. **limit cycle stability** (stability of the discrete time model)
- limit cycle instability:
  1. occurs in would-be CCM for $D > 1/2$
  2. results in sensitive small-signal parameters
  3. **affects averaged model stability!**
- analytical techniques, models, normalization ...

Conclusions, 2

- **in the paper**, case study for $J_m = 0.15$
- analytical techniques developed, discrete time model
- detailed study of the discrete time model
- identification of modes
- pretty good analytical description ...
- analysis of stability

Conclusions, 3

- **in this presentation**, generalized over J_m, the remaining degree of freedom, along with M, completeness achieved
- **important:**
  1. occurrence of period-n modes when assumed period-1 CCM has unstable limit cycle, for $D > 1/2$
  2. both period-n CCM and period-n DCM exist
- **charts:**
  1. **chart of modes**
  2. **chart of periodicity (chart of n)**
  3. **chart of stability**

Conclusion

avoid period-n modes!