Peak Detector and/or Envelope Detector — A Detailed Analysis —

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Abstract

In this document, a simple circuit constructed using a diode, a resistor, and a capacitor, utilized as a peak detector and/or as an envelope detector is analyzed. The analysis is approached by applying approximate methods and by a mix of exact and numerical methods, aiming design guidelines and understanding of the circuit operation. Approximate and exact approaches are compared, and a region where the approximate analysis provides adequate answers is identified. Ability of the circuit to track the envelope variations is analyzed, and it is shown to depend both on the circuit time constant and the output voltage value, i.e. the modulation signal frequency and the modulation index. Relevant relations are derived and presented. Finally, distortion of the output voltage caused by the output voltage ripple is addressed, and averaged model of the circuit is derived. It is shown that average of the output voltage over the carrier period is increased about three times when filtering of the output voltage is applied. Transfer function for averaged waveforms of the envelope detector is derived, containing slight attenuation and a real pole at the double of the carrier frequency.

1 Introduction

Peak detector and/or envelope detector, built as a circuit of Fig. 1 is frequent in electrical engineering curricula and usually the first circuit built by enthusiasts in amateur radio clubs. The author of this document is no exception: still remembers his joy after this simple circuit demonstrated its operation. Analysis of the circuit could be found in many places, like [1] or [2], to mention a few. However, the author never found all the answers he asked himself in available literature, at least not arranged in the way he would like it to be. After student Miloš Nenadović asked a question during lab exercise, as stated in the Acknowledgement, the author realized that there are other people being bothered by the same questions, which initiated writing of this document. Peak and/or envelope detector is an old circuit, not being in the focus of leading edge research in electronics. However, it is a nice circuit, which might be used as an example to illustrate many complex circuit analysis techniques on a circuit containing only three elements: one linear resistive, one nonlinear resistive, and one accumulating. In such an example, the essence could be demonstrated without unnecessary burden of irrelevant details.

At this point, difference between the peak detector and the envelope detector should be clarified, since the circuit performing the tasks is the same. In both of the cases considered in this document, we would assume the same general form of the input voltage

$$v_{IN}(t) = v_A(t)\,\cos\left(\omega_0\, t\right) \tag{1}$$

where $v_A(t)$ is considered the signal envelope. In the case of a peak detector application, we assume constant envelope, $v_A(t) = V_C$, and the goal is to reconstruct the value of V_C. In the case of envelope detector, $v_A(t)$ is assumed as a variable signal, and reconstructing its waveform is the goal.

It is worth to mention that notion of a signal envelope [3] is much more complex than shown here. However, our aim is to construct a simple circuit for asynchronous demodulation of signals provided by standard amplitude modulation [4], and the definition applied here is sufficient for such application.

Aim of the analysis presented in this document is to obtain reasonably accurate and reasonably simple equations that model the output voltage, both in the sense of the output voltage ripple [5] and in the sense of the circuit ability to track the envelope variations. One of the goals was to provide a rule of thumb for choosing R and C. This task is essentially one parameter optimization problem, since R and C appear in equations merged in the time constant RC. The optimization could be experimentally performed just in a few steps, and requirement for such analysis did not emerge from practice. However, it might be nice to understand the circuit operation, modeling, and optimization from a theoretical viewpoint, also.

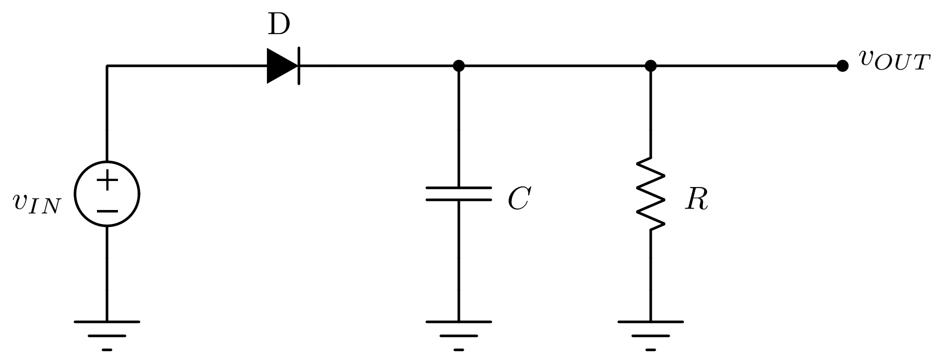

Figure 1: Peak detector and/or envelope detector, circuit diagram.

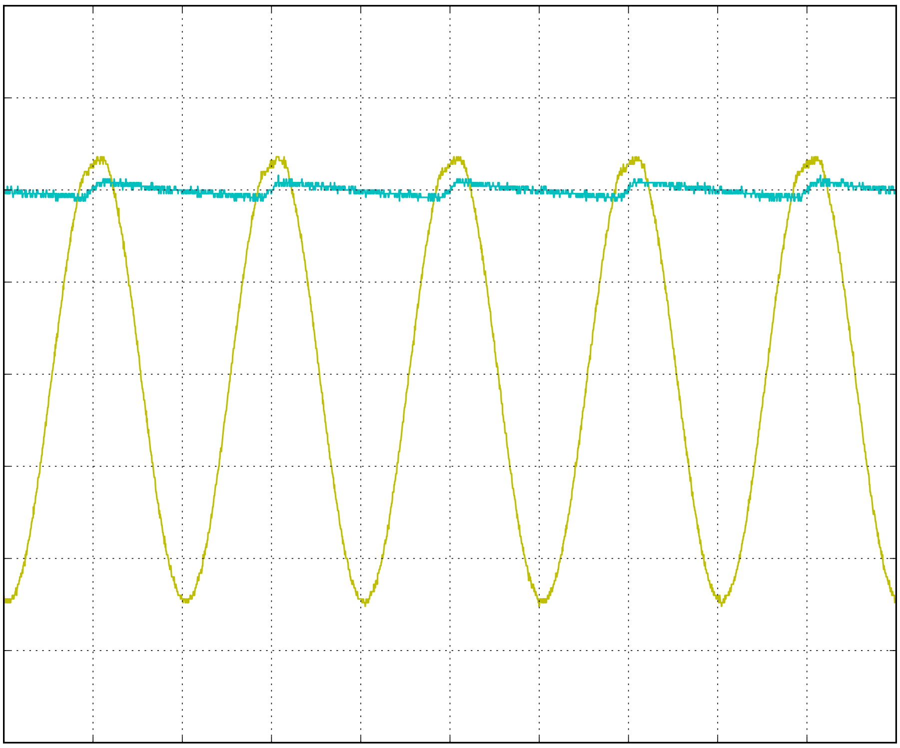

Figure 2: Peak detector, actual waveforms of v_{IN} and v_{OUT} in a situation of practical interest. Voltage scale: $2\,\mathrm{V/div}$; time scale: $0.5\,\mathrm{\mu s/div}$; parameters: $f_0 = 1\,\mathrm{MHz}$, $R = 10\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0\,R\,C = 10$, the diode is 1N4148.

This document will not help the readers to establish a startup company, nor to earn any money. The aim is to help the readers to analyze a simple circuit that does not succumb to simple and straightforward analytical techniques, in a hope that they would enjoy such a journey like the author did.

2 Small Ripple Approximation

At first, let us consider a peak detector application of the circuit of Fig. 1 assuming

$$v_{IN} = V_C\,\cos\left(\omega_0\,t\right) \tag{2}$$

aiming analysis of the circuit response and the waveform of the output voltage v_{OUT} in the periodic steady state.

To obtain appropriate model of a physical process, it is always useful to experimentally observe actual system with the parameters of practical interest. Actual waveforms of v_{IN} and v_{OUT} are presented in Fig. 2 for the experimental setup of Fig. 1 with $f_0 = 1\,\mathrm{MHz}$, $R = 10\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0\,R\,C = 10$, and the diode 1N4148. The waveforms are recorded using [6]. It is worth to mention at this point that in the v_{OUT} waveform of Fig. 2 effects caused by the diode forward voltage drop of about $V_D \approx 0.6\,\mathrm{V}$ could be observed. These effects will not be covered in any of the analyses shown in this document, since they can be included at the end of the analysis simply by reducing the v_{OUT} by V_D. Carrying V_D in the equations would complicate already complex analysis, without significant effect on gaining insight in the circuit operation. Thus, in the analyses the ideal diode model would be assumed.

Having in mind that the output voltage v_{OUT} is at the same time the capacitor voltage, $v_C = v_{OUT}$, it can be concluded that the output voltage waveform is characterized by long intervals of the capacitor discharge and short intervals of the capacitor charging through the diode. Such situation occurs when the output voltage ripple [5] is low, hence the name of the approximation proposed in this Section. However, the main feature of the approximation is small conduction angle $\alpha = \omega_0 \, t_\alpha$ of the diode, where t_α is the conduction time of the diode over the output voltage period. Assuming $\alpha \to 0$, the capacitor is instantly charged to the voltage

$$v_{OUT\,max} = V_C \tag{3}$$

($V_C - V_D$ if we had included the diode forward voltage drop in the model) at the time points when the input voltage reaches its maxima, and the capacitor is discharged during the remaining part of the period. Small ripple approximation therefore assumes $v_{OUT} \approx V_C$, resulting in the capacitor discharge equation

$$C \, \frac{dv_{OUT}}{dt} = -\frac{v_{OUT}}{R} \approx -\frac{V_C}{R} \tag{4}$$

which states that the capacitor is being discharged with an almost constant current, which corresponds to the experimentally observed almost linear decrease of the output voltage, shown in Fig. 2. This results in

$$\frac{dv_{OUT}}{dt} = -\frac{V_C}{R\,C} \tag{5}$$

which is integrated to obtain the waveform of v_{OUT}

$$v_{OUT} = V_C \left(1 - \frac{t}{R\,C}\right) \tag{6}$$

for $0 < t < T_0$, which in normalized form reduces to

$$\frac{v_{OUT}}{V_C} = 1 - \frac{\omega_0 \, t}{\omega_0 \, R\,C}. \tag{7}$$

This results in the output voltage minimum of

$$v_{OUT\,min\,app} = V_C \left(1 - \frac{T_0}{R\,C}\right) = V_C \left(1 - \frac{1}{f_0 \, R\,C}\right) \tag{8}$$

and the peak-to-peak ripple

$$\Delta v_{OUT\,p-p} = T_0 \, \frac{V_C}{R\,C} = \frac{V_C}{f_0 \, R\,C}. \tag{9}$$

Values of $v_{OUT\,max}$, $v_{OUT\,min}$, and $\Delta v_{OUT\,p-p}$ are proportional to the peak voltage V_C. This motivates normalization of the circuit voltages by V_C, resulting in an expression for relative, i.e. normalized ripple

$$\frac{\Delta v_{OUT\,p-p}}{V_C} = \frac{1}{f_0 \, R\,C} \tag{10}$$

where f_0, R, and C are glued together in a single term $f_0 \, R\,C$, without a physical dimension. This merge of quantities would occur frequently in the analyses that follow, and f_0, R, and C would frequently be treated as a single quantity represented by their product.

In Fig. 3, input voltage waveform is presented, as well as the output voltage waveforms obtained by applying small ripple approximation and by semi-numerical solution technique to be described in the following Section. Comparison of the output voltage waveforms yields conclusion that agreement between the small ripple approximation and the numerical solution of the circuit model is very good in time segment when the diode is off, which for the small ripple approximation and the resulting small conduction angle is the dominant part of the

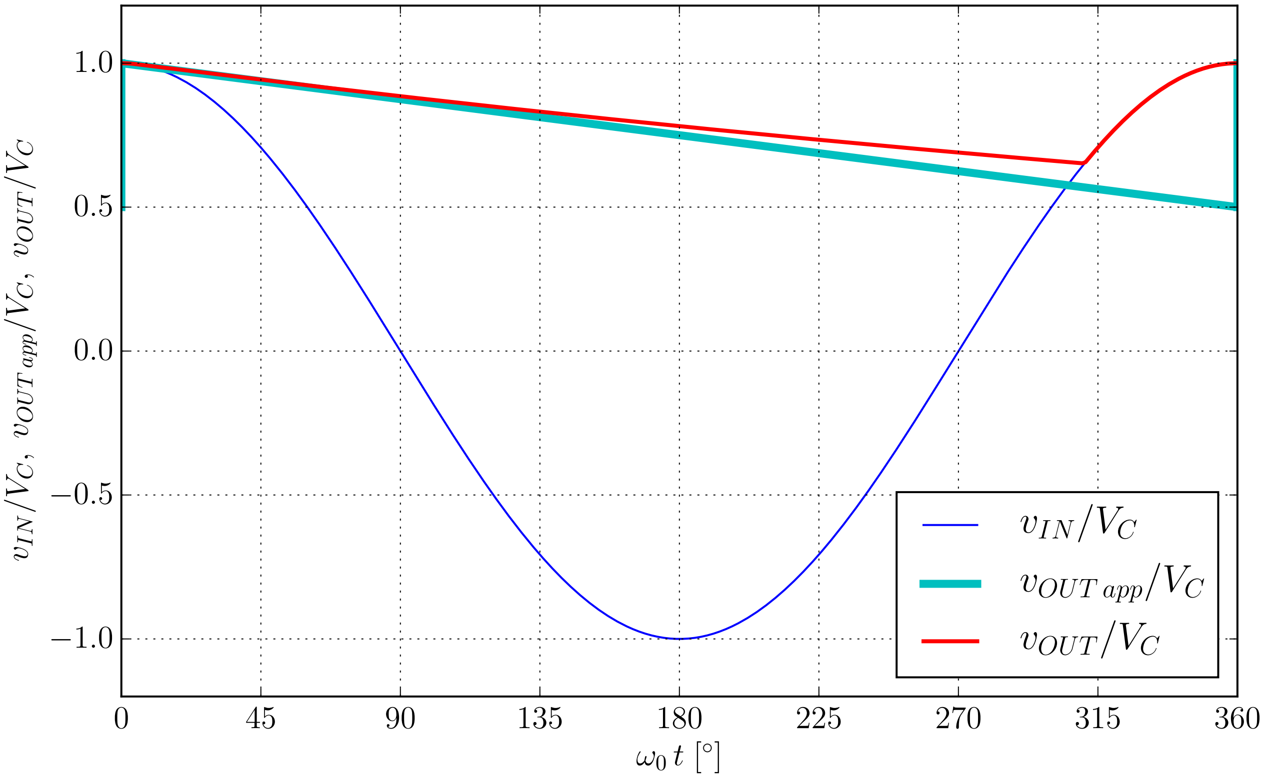

Figure 3: Small ripple approximation, v_{IN}, $v_{OUT\,app}$, and v_{OUT}, $f_0\,RC = 2$, ideal diode.

waveform period. Also, effects caused by the diode forward voltage drop, clearly observable in the waveform of Fig. 2, are absent from Fig. 3, since the ideal diode model is applied in the later case.

Analyzing the approximate output voltage waveform of Fig. 3, the average of the output voltage is found to be equal to the maximum of the input voltage reduced by one half of the peak-to-peak ripple,

$$v_{OUT\,mean\,app} = V_C - \frac{1}{2}\,\Delta v_{OUT\,p-p} = V_C\left(1 - \frac{1}{2\,f_0\,RC}\right). \tag{11}$$

According to this, if the peak voltage detection is the aim, the product $f_0\,RC$ should be as large as possible. On the other hand, this would affect the circuit dynamic response, forcing the designer to compromise between the peak voltage detection accuracy and the dynamic response, which is going to be discussed later.

3 Semi-Numerical Analysis

Previously described small ripple approximation assumes instantaneous charging of the capacitor and constant current discharge during the whole period. These assumptions provide a simple model, and its accuracy and applicability are challenged by the model proposed in this section. The aim is to provide exact solution for the circuit of Fig. 1 in which the diode is modeled as an ideal diode, and the input voltage is sinusoidal, as specified by (2). The waveform of the output voltage obtained by the analysis presented in this Section is shown in 4 and its quantitative parameters average of the output voltage, $v_{OUT\,mean}$, and minimum of the output voltage, $v_{OUT\,min}$ are being searched for.

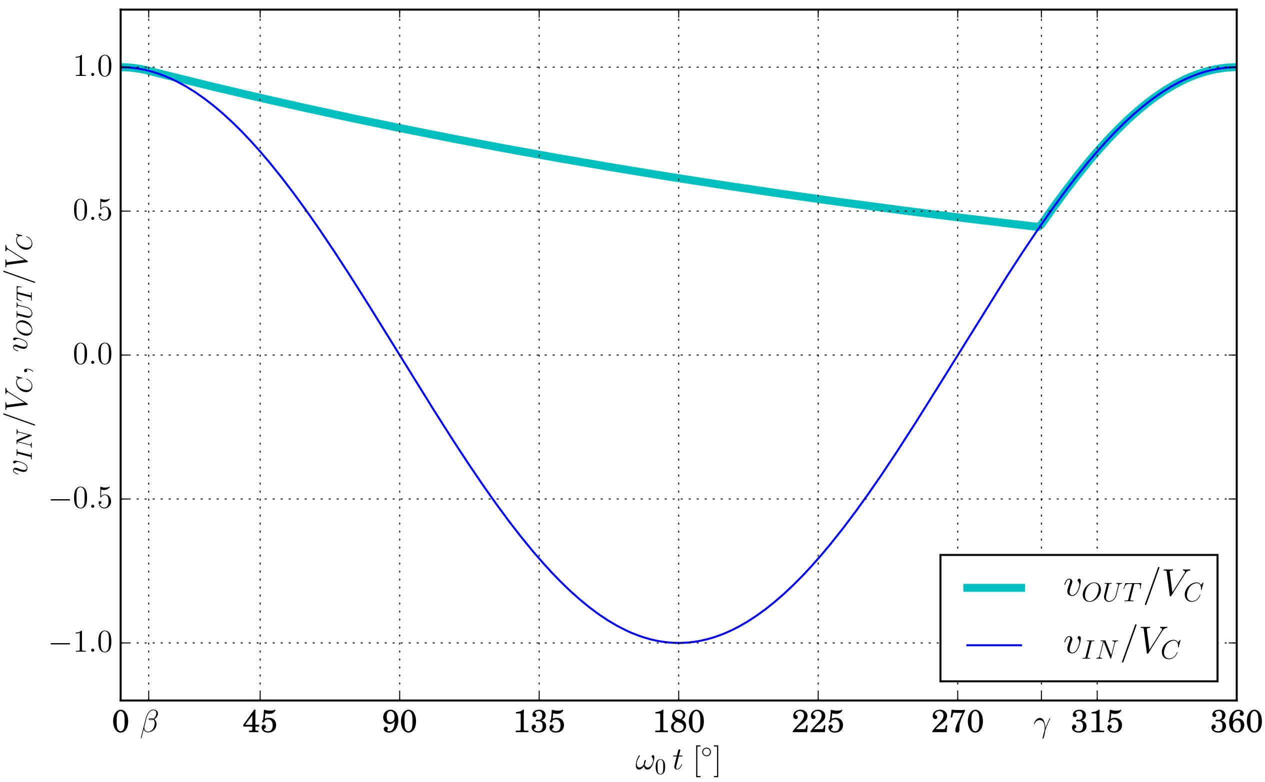

Figure 4: Semi-numerical solution, v_{IN} and v_{OUT}, $f_0\, R\, C = 1$.

3.1 First conducting interval

To start the analysis, let us note that the assumed ideal diode model results in $v_{OUT} = v_{IN}$ during the intervals when the diode is conducting. At $t = 0$ the diode is conducting since the capacitor is definitely being charged while the input voltage rises towards its maximum, and remains conducting while $i_D > 0$. Since

$$i_D = i_C + i_R = C\, \frac{dv_{OUT}}{dt} + \frac{v_{OUT}}{R} \tag{12}$$

condition $i_D = 0$ reduces to

$$\frac{dv_{OUT}}{dt} + \frac{v_{OUT}}{R\, C} = 0. \tag{13}$$

While the diode is conducting, $v_{OUT} = v_{IN} = V_C\, \cos(\omega_0\, t)$, and

$$\frac{dv_{OUT}}{dt} = -\omega_0\, V_C\, \sin(\omega_0\, t) \tag{14}$$

reducing the condition (13) to

$$-\,\omega_0\, \sin(\omega_0\, t) + \frac{\cos(\omega_0\, t)}{R\, C} = 0. \tag{15}$$

Equation (15) has an infinite number of solutions. To analyze the circuit, due to its periodic response it is sufficient to analyze the phase angle period $0 \le \omega_0\, t < 2\,\pi$. In this aim, let us define t_β as the time instant of the diode turn-off in this period, which occurs during the first quarter-period, $0 \le t_\beta < T_0/4$. Such solution of (15) results in $i_D(t_\beta) = 0$ and corresponds to the phase angle $\beta \triangleq \omega_0\, t_\beta$ given by

$$\beta = \arctan \frac{1}{\omega_0\, R\, C} \tag{16}$$

which satisfies the condition to be in the first quarter-period since $0 < \beta < \pi/2$ for positive $\omega_0\, R C$. In this manner, the first time segment of the circuit operation over considered period is determined by $0 \le \omega_0\, t < \beta$, where $v_{OUT} = v_{IN}$.

3.2 Nonconducting interval

The second segment of the circuit operation starts with the diode turn off at t_β, and lasts for $t_\beta \le t < t_\gamma$, where t_γ is the time instant when the diode turns on again, which corresponds to the phase angle $\gamma \triangleq \omega_0\, t_\gamma$. Within the considered phase angle scope, γ is located in the range $3\,\pi/2 \le \gamma < 2\,\pi$.

While the diode is off, the output voltage is

$$v_{OUT}(t) = v_{OUT}(t_\beta)\, e^{-\frac{t-t_\beta}{R\,C}} \tag{17}$$

where

$$v_{OUT}(t_\beta) = V_C\, \cos\beta = \frac{\omega_0\, R\, C}{\sqrt{1+(\omega_0\, R\, C)^2}}\, V_C. \tag{18}$$

In terms of the phase angle, which is normalized time, normalized output voltage during the interval when the diode is off is

$$\frac{v_{OUT}(\omega_0\, t)}{V_C} = \frac{\omega_0\, R\, C}{\sqrt{1+(\omega_0\, R\, C)^2}}\, e^{-\frac{\omega_0\, t-\beta}{\omega_0\, R\, C}} \tag{19}$$

The diode nonconducting interval ends at $t_\gamma = \gamma/\omega_0$ when v_{OUT} given by (19) and v_{IN} meet again, $v_{OUT}(\gamma) = v_{IN}(\gamma)$, which in expanded form results in

$$v_{OUT}(\gamma) = V_C\, \cos\gamma = \frac{\omega_0\, R\, C}{\sqrt{1+(\omega_0\, R\, C)^2}}\, V_C\, e^{-\frac{\gamma-\beta}{\omega_0\, R\, C}}. \tag{20}$$

After cancellation of V_C, determining γ reduces to solving

$$\cos\gamma = \frac{\omega_0\, R\, C}{\sqrt{1+(\omega_0\, R\, C)^2}}\, e^{-\frac{\gamma-\beta}{\omega_0\, R\, C}} \tag{21}$$

which is a transcendental equation [7] over γ. The equation does not have a closed form solution, and requires a numerical solution over γ. This requirement makes the analysis semi-numerical, since a closed form expression for γ would provide purely analytical solution.

To solve for γ numerically, a linear iterration proces is applied, designed to search a solution for γ in the range $3\,\pi/2 \le \gamma < 2\,\pi$ using the following iterration rule

$$\gamma_{k+1} = 2\,\pi - \arccos\left(\frac{\omega_0\, R\, C}{\sqrt{1+(\omega_0\, R\, C)^2}}\, e^{-\frac{\gamma_k-\beta}{\omega_0\, R\, C}}\right). \tag{22}$$

To design the iteration rule, special care is taken in inverting the cosine function, to provide solution in the desired range. This is the most delicate part of the analysis. The iteration is performed until the criterion

$$|\gamma_{k+1} - \gamma_k| < \varepsilon \tag{23}$$

is met. The results presented in this document are obtained with the exit parameter value $\varepsilon = 10^{-6}$.

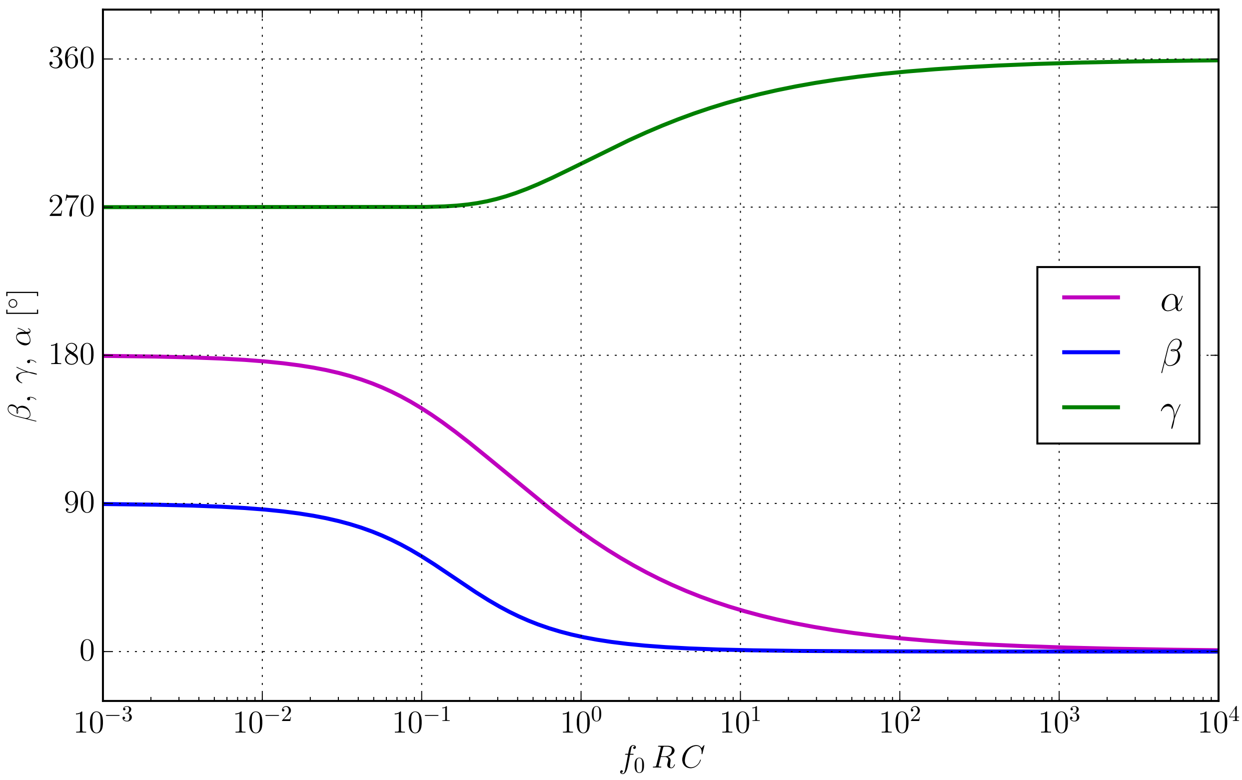

Figure 5: Angles: β, γ, and α.

3.3 Second conducting interval

The second conducting interval of the diode during the switching period occurs for $\gamma \leq \omega_0\,t < 2\,\pi$, which closes the period.

After angles β and γ are determined, the diode conduction angle is obtained as

$$\alpha = 2\,\pi - (\gamma - \beta) \tag{24}$$

and in the case of the small ripple approximation is considered to be negligibly small. Values of β, γ, and consequently α, according to (16), (21), and (24) depend on the combined parameter $f_0\,R\,C$ only. Dependence of α, β, and γ on $f_0\,R\,C$ is given in Fig. 5 in logarithmic scale over $f_0\,R\,C$. The figure illustrates that effects of filtering by the capacitor C are noticeable for $f_0\,R\,C > 10^{-2}$, when α starts to decrease from 180°. For $f_0\,R\,C > 10^{2}$ the filtering is perfect, and the conduction angle α approaches zero.

3.4 Parameters of the output voltage

Purpose of determining β analytically and γ numerically was to determine mean value of the output voltage

$$v_{OUT\,mean} = \frac{1}{2\,\pi} \int_0^{2\,\pi} v_{OUT}(\omega_0\,t)\,d(\omega_0\,t) \tag{25}$$

and its minimal value, $v_{OUT\,min}$. Both quantities are convenient to represent in normalized form, obtained by dividing with V_C, providing the result in V_C independent form. Average of the output voltage is after some symbolic computation obtained as

$$\frac{v_{OUT\,mean}}{V_C} = \frac{1}{2\,\pi} \left(\sin\beta - \sin\gamma + \omega_0\,R\,C\,\left(1 - e^{\frac{\beta-\gamma}{\omega_0\,R\,C}}\right)\cos\beta \right) \tag{26}$$

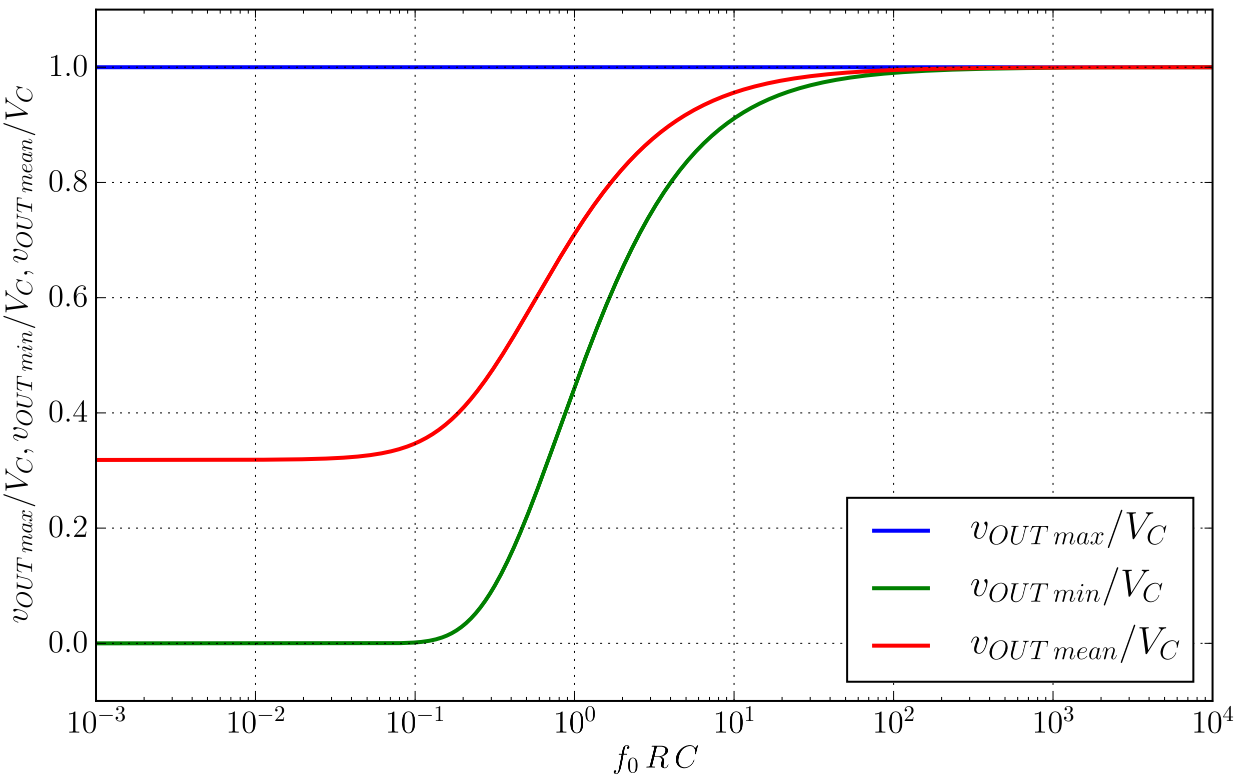

Figure 6: Semi-numerical solution, $v_{OUT\,max}$, $v_{OUT\,min}$, and $v_{OUT\,mean}$.

while the minimum of the output voltage is given by

$$\frac{v_{OUT\,mean}}{V_C} = \cos\gamma. \tag{27}$$

Dependence of $v_{OUT\,max}$, $v_{OUT\,min}$, and $v_{OUT\,mean}$ on $f_0\,RC$ is presented in Fig. 6. Again, for $f_0\,RC > 100$ the filtering might be considered perfect. For $f_0\,RC < 0.1$ effects of the filtering are negligible.

4 Comparison of the Analyses

After the approximate solution and semi-numerical solution of the circuit model are obtained, it would be interesting to compare the predictions for the minimum of the output voltage, given by (8) and (27), as well as the predictions for the output voltage average, specified by (11) and (26). For the maximum of the output voltage, both methods provide the same result, V_C.

Solutions for $v_{OUT\,max}$, $v_{OUT\,min}$, and $v_{OUT\,mean}$ as they depend on $f_0\,RC$ are presented in Fig. 7 in logarithmic scale over $f_0\,RC$. The diagram indicates that for $f_0\,RC > 10$ the solutions are in a good agreement.

To provide more detailed visualization of the error introduced by the approximate solution, let us define the errors as

$$\Delta v_{OUT\,min} = v_{OUT\,min\,app} - v_{OUT\,min} \tag{28}$$

and

$$\Delta v_{OUT\,mean} = v_{OUT\,mean\,app} - v_{OUT\,mean}. \tag{29}$$

Dependence of $\Delta v_{OUT\,min}/V_C$ and $\Delta v_{OUT\,mean}/V_C$ on $f_0\,RC$ is presented in Fig. 8. The diagrams of Fig. 8 indicate that the approximate method always provides lower quantities, since the error

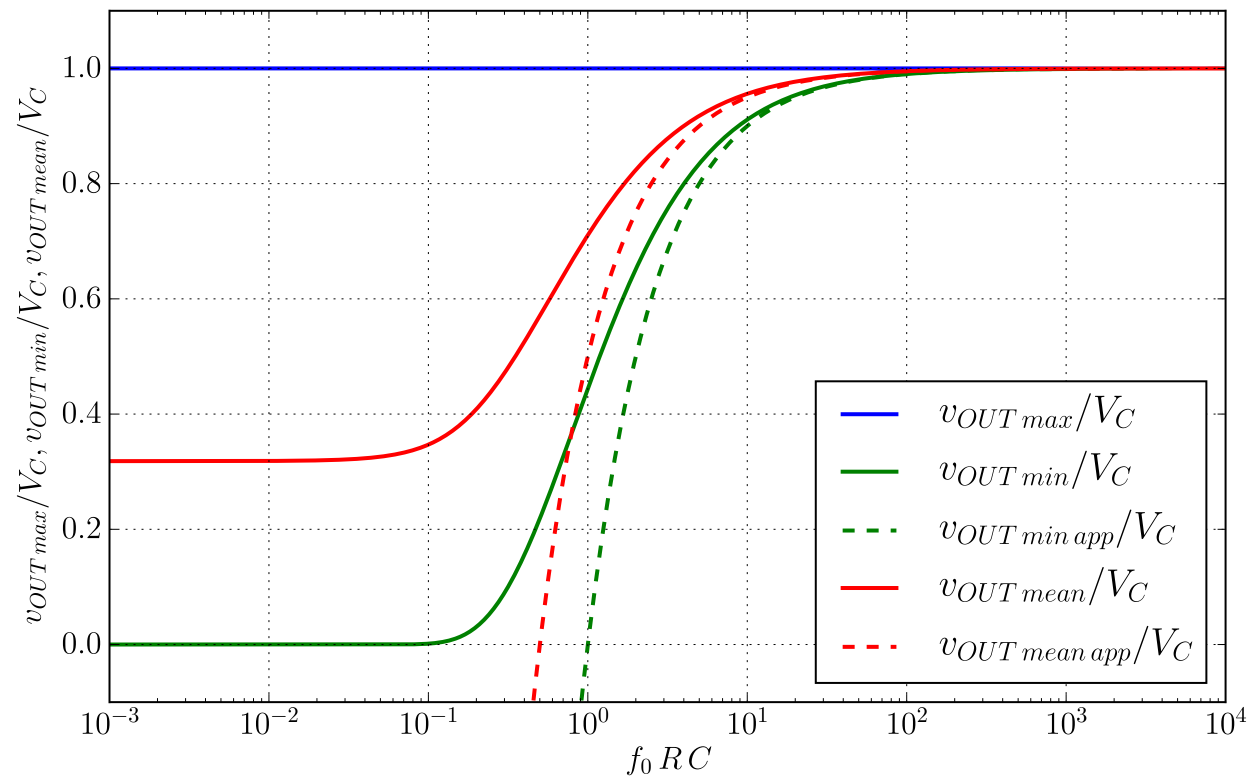

Figure 7: Comparison of the solutions.

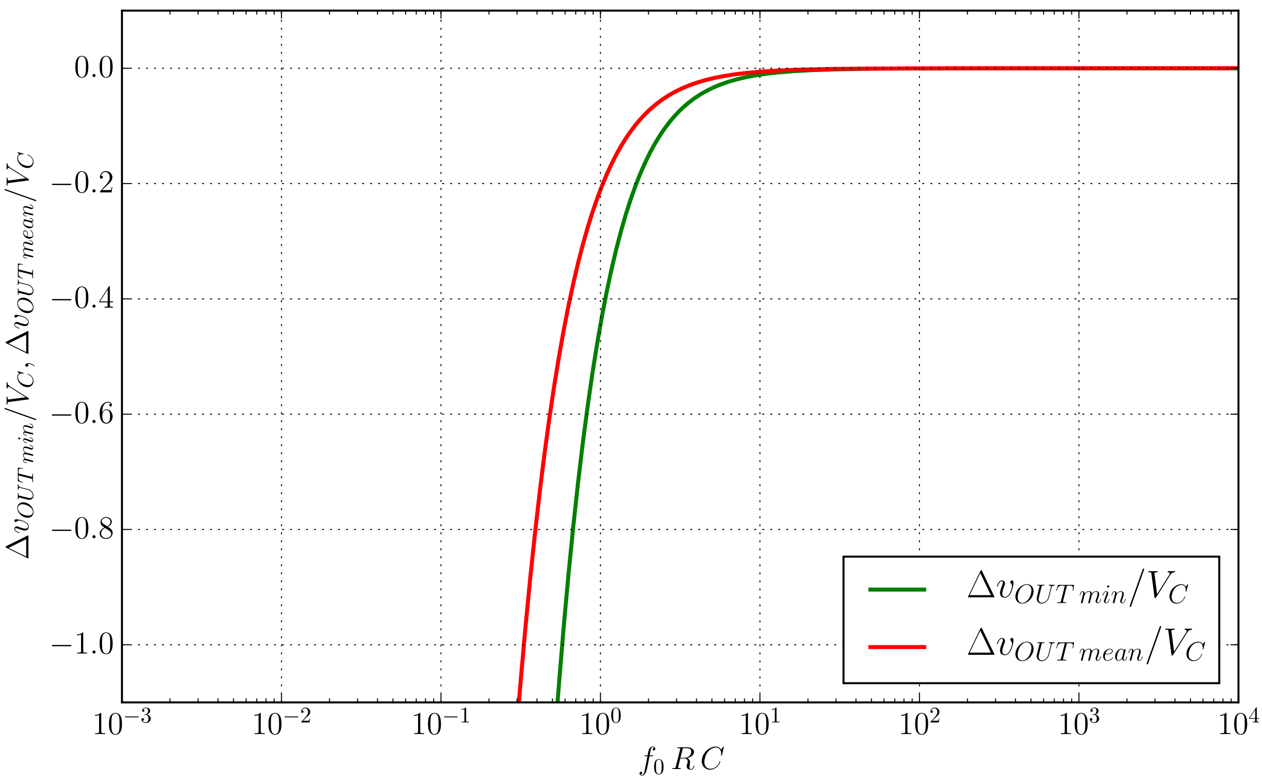

Figure 8: Difference from the exact solution.

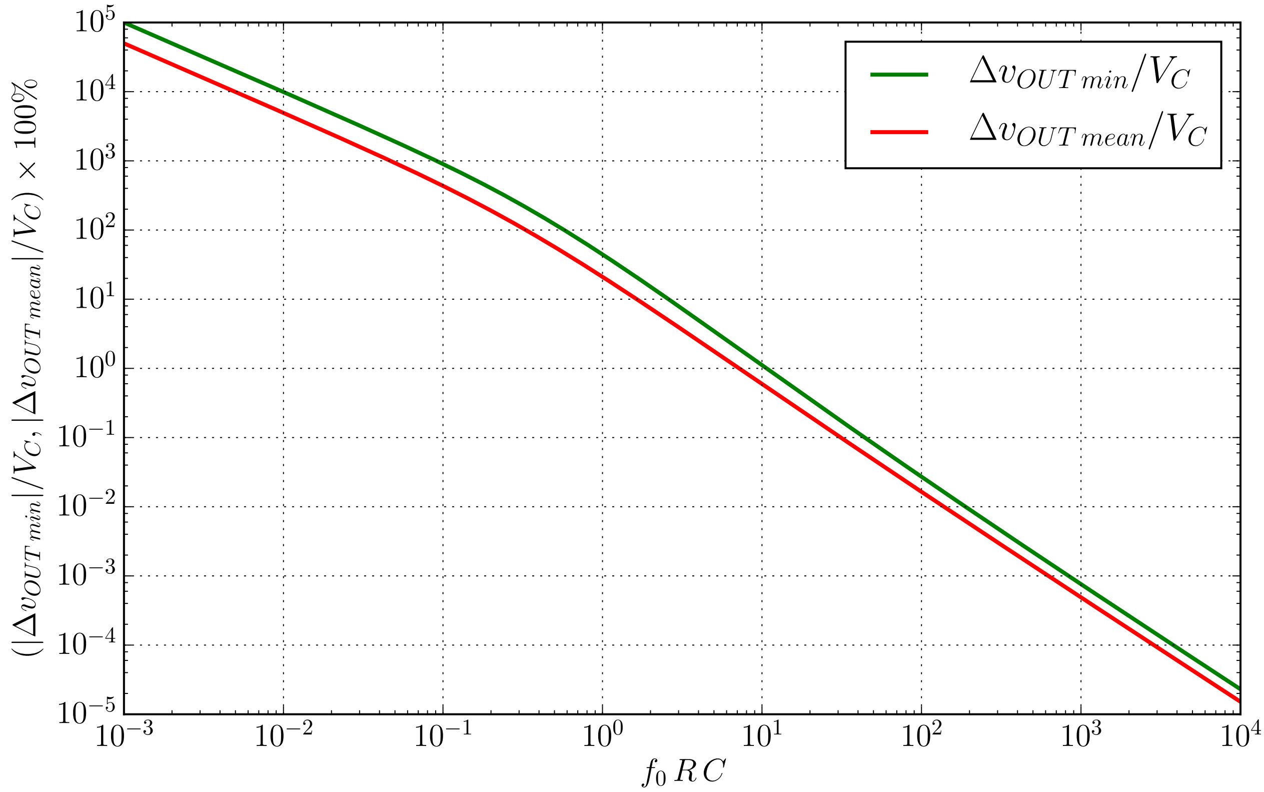

Figure 9: Difference from the exact solution, logarithmic scale.

is negative. Above $10\,f_0\,R\,C$ the error seems negligible, but seems to grow rapidly in magnitude below this value, in an exponential-like fashion.

Exponential-like variation is an indication to apply logarithmic scale, and in Fig. 9 absolute value of the normalized error expressed in percent is presented in logarithmic scale. As expected, the variation is linear-like, and it can be concluded that for $f_0\,R\,C > 10$ the error is in the range below 1%. Thus, in this range small ripple approximation should definitely be considered as acceptable.

5 Scaling

In the case the diode conducts, normalized output voltage is

$$\frac{v_{OUT}}{V_C} = \cos\left(\omega_0\,t\right) \tag{30}$$

while in the case the diode is off normalized output voltage is determined by (7) in the case small ripple approximation is applied, or by (19) in the case it is not. This yields a conclusion that modification of V_C just scales the waveforms, not affecting the conduction angles, nor the the output waveform shape. This yields conclusion that the output voltage ripple, of interest in analyzing performance of the circuit of Fig. 1, is proportional to the input voltage amplitude, V_C. Furthermore, all the waveforms depend on normalized time $\omega_0\,t$ and parameter $f_0\,R\,C$ in which the input voltage frequency, resistance of the applied resistor, and capacitance of the applied capacitor are merged in a single value without a physical dimension [8].

To illustrate the effects of scaling, in Fig. 10 waveforms obtained applying semi-numerical analysis are presented for V_C, $V_{C1} = 0.5\,V_C$, and $V_{C2} = 1.5\,V_C$. In all of the cases, normalization is performed taking V_C as the base quantity. The output voltage waveforms are scaled by the

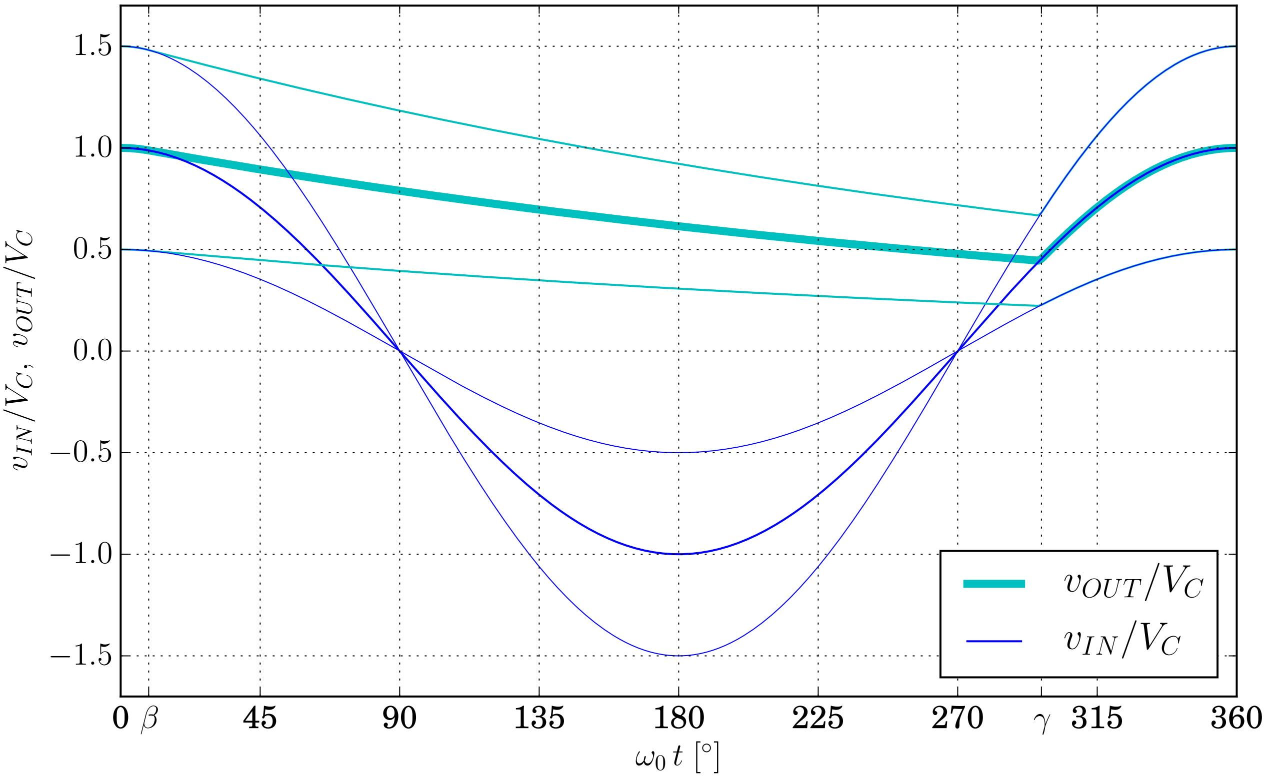

Figure 10: Semi-numerical solution, scaling, $\omega_0\, RC = 1$.

same factor the input voltage amplitude is scaled, while the conduction angles remained the same.

6 Analysis of Transients, Envelope Detection

Finally, after a lot of preparation, we reached the central point that motivated writing of this document: envelope detection. The goal is to provide an analytical guide to choose $f_0\, RC$ in order to provide good envelope detection. Addressing the issue is motivated by a student question, as stated in the Acknowledgement.

At first, let us consider two results of two experiments: for $f_0\, RC = 10$, when the envelope detection is performed successfully, as depicted in Fig. 11, and for $f_0\, RC = 100$ when it is not, as depicted in Fig. 12. Two important conclusions could be drawn: in the first case the ripple is higher, while the tracking is better. In the second case the ripple is lower, but the envelope tracking is poor. Choice for $f_0\, RC$ is constrained by a compromise between the tracking performance and the ripple.

What characterizes good envelope detection? To perform tracking, in each period of the carrier, T_0, the diode should be turned on. On the other hand, the ripple should be as low as possible.

Consider the case of unsuccessful envelope reconstruction shown in Fig. 12. Significant intervals of capacitor discharge without diode conduction to recharge could be observed. This is caused by the envelope decrease faster than the decrease of the output voltage that given choice of R and C could provide at a certain value of v_{OUT}. The fastest decrease of the output voltage that the envelope detector could perform is

$$\left.\frac{dv_{OUT}(t)}{dt}\right|_{min} = -\frac{v_{OUT}(t)}{RC}. \tag{31}$$

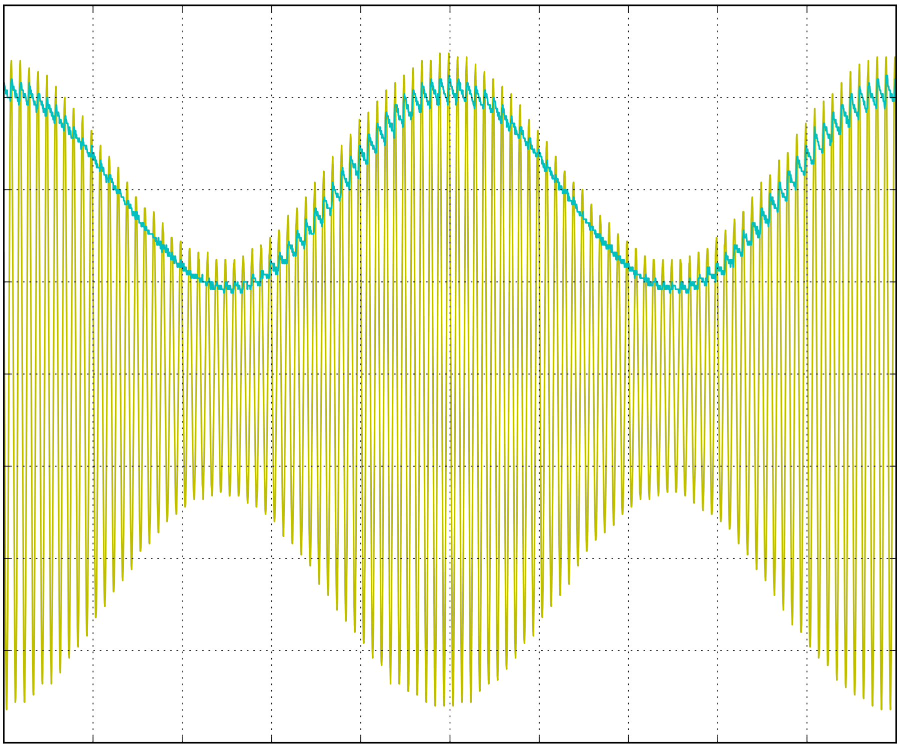

Figure 11: Successful envelope detection. Parameters: $R = 10\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0 = 1\,\mathrm{MHz}$, $f_0\,RC = 10$, $f_m = 20\,\mathrm{kHz}$, $m = 0.5$. Time scale: $10\,\mathrm{\mu s/div}$; voltage scale: $2\,\mathrm{V/div}$.

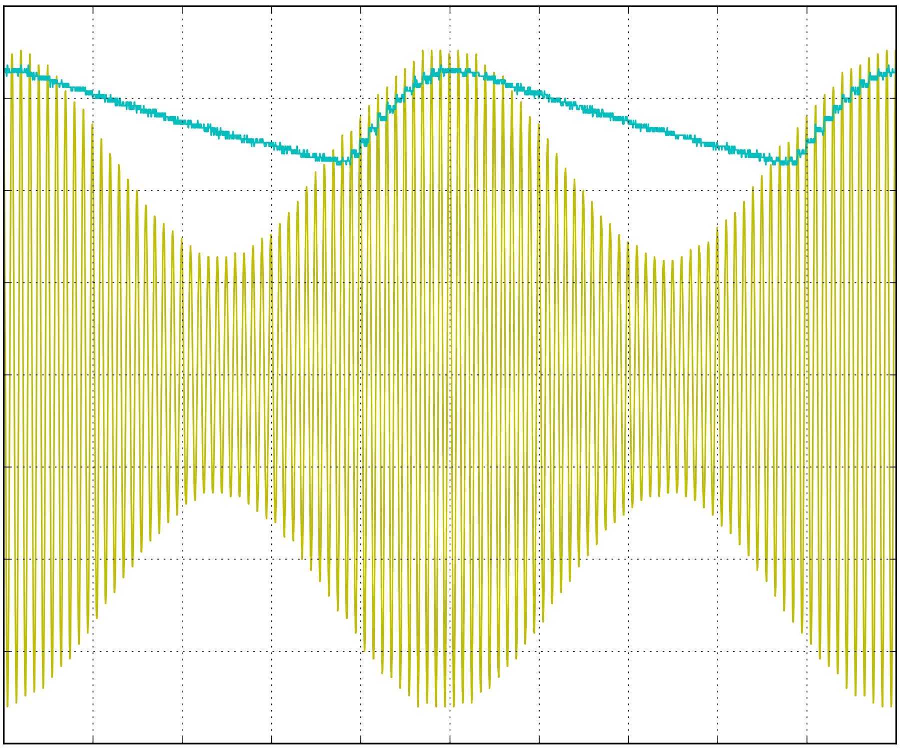

Figure 12: Unsuccessful envelope detection. Parameters: $R = 100\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0 = 1\,\mathrm{MHz}$, $f_0\,RC = 100$, $f_m = 20\,\mathrm{kHz}$, $m = 0.5$. Time scale: $10\,\mathrm{\mu s/div}$; voltage scale: $2\,\mathrm{V/div}$.

Good tracking assumes $v_{OUT}(t) \approx v_A(t)$. Thus, to provide good tracking of a given envelope $v_A(t)$, values of R and C should satisfy

$$\frac{dv_A(t)}{dt} \geq -\frac{v_A(t)}{R\,C} \tag{32}$$

for $\forall t$, which provides diode conduction in each period of the carrier. Separation of variables yields a general rule for the choice of the time constant $R\,C$

$$-\frac{1}{v_A(t)}\,\frac{dv_A(t)}{dt} \leq \frac{1}{R\,C}. \tag{33}$$

Let us consider a typical test situation, when the envelope carries a sinusoidal message signal of the frequency f_m

$$v_A(t) = V_C + V_m\,\cos\left(\omega_m\,t\right) = V_C\,\left(1 + m\,\cos\left(\omega_m\,t\right)\right). \tag{34}$$

To simplify the notation, parameter m defined as

$$m \triangleq \frac{V_m}{V_C} \tag{35}$$

is introduced, being normalized amplitude of the message signal and named "modulation index". For $|m| < 1$ overmodulation does not occur.

Performing technical tasks, derivative of the envelope is obtained as

$$\frac{dv_A(t)}{dt} = -\omega_m\,V_m\,\sin\left(\omega_m\,t\right) = -m\,\omega_m\,V_C\,\sin\left(\omega_m\,t\right) \tag{36}$$

reducing the condition of (33) to

$$\frac{m\,\omega_m\,\sin\left(\omega_m\,t\right)}{1 + m\,\cos\left(\omega_m\,t\right)} \leq \frac{1}{R\,C}. \tag{37}$$

Dividing by ω_m to remove physical dimensions from the analysis

$$\frac{m\,\sin\left(\omega_m\,t\right)}{1 + m\,\cos\left(\omega_m\,t\right)} \leq \frac{1}{\omega_m\,R\,C} \tag{38}$$

is obtained. To simplify the notation further, it is convenient to define a quantity without physical dimension

$$k \triangleq \frac{1}{\omega_m\,R\,C} \tag{39}$$

reducing (38) to

$$\frac{m\,\sin\left(\omega_m\,t\right)}{1 + m\,\cos\left(\omega_m\,t\right)} \leq k. \tag{40}$$

Since it is assumed that the signal is not overmodulated,

$$1 + m\,\cos\left(\omega_m\,t\right) > 0 \tag{41}$$

and (40) reduces to

$$\sin\left(\omega_m\,t\right) - k\,\cos\left(\omega_m\,t\right) \leq \frac{k}{m} \tag{42}$$

which is satisfied for

$$\sqrt{1 + k^2} \leq \frac{k}{m} \tag{43}$$

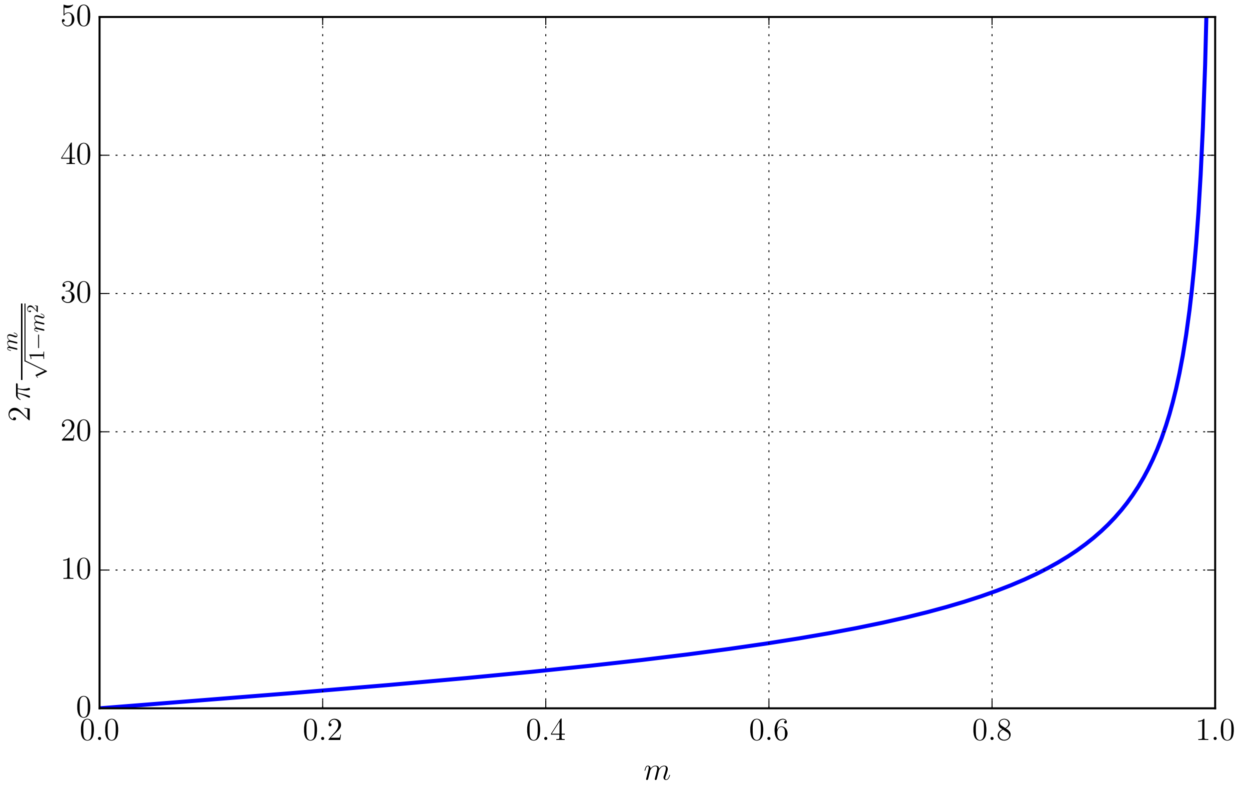

Figure 13: Relative ripple versus m.

or

$$\frac{1}{k} \leq \frac{\sqrt{1-m^2}}{m}. \tag{44}$$

Restoring (39), choice of the time constant RC is reduced to

$$RC \leq \frac{1}{\omega_m} \frac{\sqrt{1-m^2}}{m} \tag{45}$$

in order to provide the envelope recovery.

Analysis of the envelope tracking up to this point did not involve carrier frequency, since the carrier frequency is related only to the output voltage ripple, the higher the carrier frequency the lower the ripple. However, there is a relation between the output voltage ripple and the envelope tracking capability, which is going to be derived here. First, consider a result from small ripple approximation analysis that relates the output voltage ripple and the time constant RC, derived from (10)

$$RC = \frac{2\pi}{\omega_0} \frac{v_{OUT}}{\Delta v_{OUT\,p-p}}. \tag{46}$$

Substituting in (45) yields

$$\frac{2\pi}{\omega_0} \frac{v_{OUT}}{\Delta v_{OUT\,p-p}} \leq \frac{1}{\omega_m} \frac{\sqrt{1-m^2}}{m} \tag{47}$$

which can be transformed to

$$\frac{\Delta v_{OUT\,p-p}}{v_{OUT}} \geq 2\pi \frac{\omega_m}{\omega_0} \frac{m}{\sqrt{1-m^2}} \tag{48}$$

or

$$\frac{\Delta v_{OUT\,p-p}}{v_{OUT}} \geq 2\pi \frac{f_m}{f_0} \frac{m}{\sqrt{1-m^2}}. \tag{49}$$

Relations (48) and (49) present fundamental relation between the normalized output voltage ripple, ratio of the modulation frequency and the carrier frequency, and the modulation index m, since the normalized output voltage ripple is greater or equal to the product of f_m/f_0 and a function of m, $2\,\pi\, m/\sqrt{1-m^2}$, depicted in Fig. 13. The relations indicate that the ripple is lower for higher ratios of f_0/f_m, and that for high values of the modulation index, when m approaches 1, the envelope tracking could be provided only at the expense of high output voltage ripple. The later is caused by low values of v_{OUT}, insufficient to provide required decrease rate of the output voltage.

7 Distortion

AC component of the envelope detector output voltage should reconstruct the message sent applying standard amplitude modulation [4]. However, the output voltage ac component contains some distortion even if the requirements of Section 6 are met. The cause of distortion is incomplete reduction of the output voltage ripple, the more present the lower f_0/f_m ratio is. As an example, consider an experimental result for the output voltage AC component corresponding to the case of Fig. 11, presented in Fig. 14. In Fig. 14 reconstructed sinusoidal message signal could be observed, as well as a superimposed sawtooth signal. To measure distortion of a signal $x(t)$, total harmonic distortion defined as

$$THD \triangleq \sqrt{\frac{X_{RMS}}{X_{1\,RMS}} - 1} \tag{50}$$

is used, where X_{RMS} is the signal root-mean-square value, and $X_{1\,RMS}$ is the root-mean-square value of its first harmonic. This definition is slightly computationally more convenient than the one of [9]. After five repeated waveform captures, by digital signal post-processing the mean value of $THD_{exp} = 6.13\%$ is obtained, with the standard deviation estimated as $\sigma_{THD\,exp} = 0.05$.

Extending the ideas of small ripple approximation, derived in Section 2, from the constant envelope to the modulated envelope case, the approximate model could be designed such that at $t = n\,T_0$, $n \in \mathbb{N}$

$$v_{OUT}(n\,T_0) = v_{IN}(n\,T_0) \tag{51}$$

and that for $n\,T_0 \le t < (n+1)\,T_0$

$$v_{OUT}(t) = v_{IN}(n\,T_0) - \frac{v_{IN}(n\,T_0)}{R\,C}\,(t - n\,T_0)\,. \tag{52}$$

This is nothing more than superimposing the sawtooth waveform with proper amplitude and frequency to the envelope. Such waveform is computationally obtained, and presented in Fig. 15 in the same scale and for the same parameter values as the waveform of Fig. 14. The waveforms agree to a great extent, and the main difference are pronounced discontinuities in the waveform of Fig. 15, caused by neglected intervals of diode conduction. Total harmonic distortion of the waveform of Fig. 15 is $THD_{sim} = 6.44\%$ which is in pretty good agreement with the experimental result.

To conclude, the ripple caused distortion could be reduced by increasing $f_0\,R\,C$ parameter. This can either be done by increasing f_0, which is rarely a parameter available to play with, or by increasing the time constant $R\,C$, but this would decrease the envelope tracking capability.

8 Averaged Model

The material presented in this section is inspired by the method of averaging, frequently used in power electronics, as pioneered in [10], and described in a tutorial fashion in [11]. Essence of

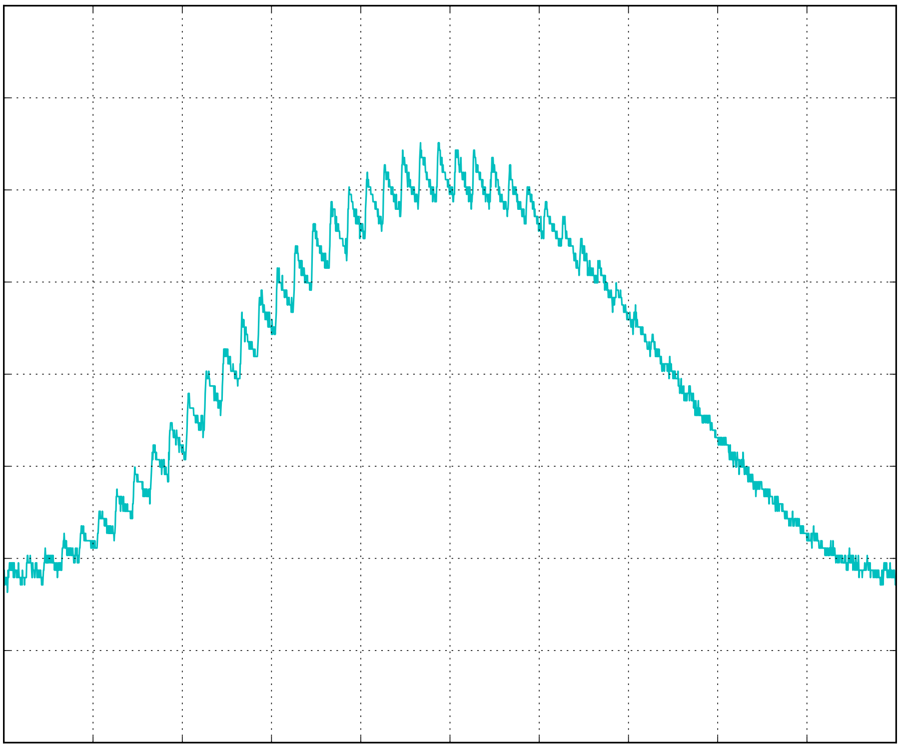

Figure 14: The output voltage AC component, experimental result. Parameters: $R = 10\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0 = 1\,\mathrm{MHz}$, $f_0\,RC = 10$, $f_m = 20\,\mathrm{kHz}$, $m = 0.5$. Time scale: $5\,\mu\mathrm{s/div}$; voltage scale: $1\,\mathrm{V/div}$.

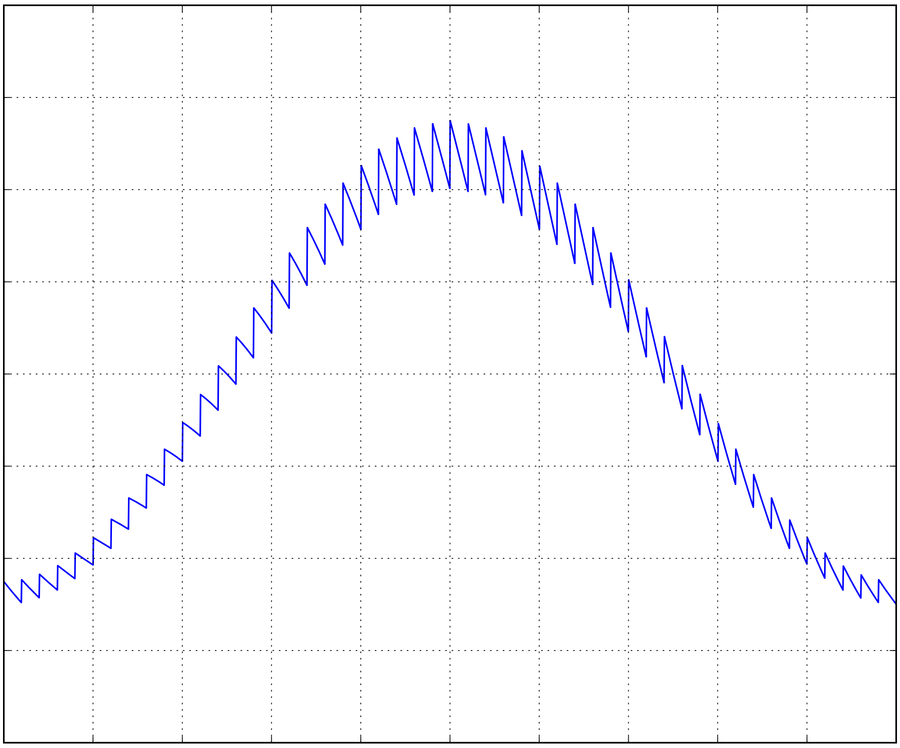

Figure 15: The output voltage AC component, simulation. Parameters: $R = 10\,\mathrm{k\Omega}$, $C = 1\,\mathrm{nF}$, $f_0 = 1\,\mathrm{MHz}$, $f_0\,RC = 10$, $f_m = 20\,\mathrm{kHz}$, $m = 0.5$. Time scale: $5\,\mu\mathrm{s/div}$; voltage scale: $1\,\mathrm{V/div}$.

the method is to remove the ripple by averaging. In power electronics, the procedure continues to linearization and to determining transfer functions, in order to apply linear control system theory to system design. Some of these concepts would be applied here.

Physical justification to apply the averaging method in the case analyzed in this document is in the fact that the envelope reconstructed by the detector is going to be post-processed by a low-pass system, even if it is only a headphone set connected directly to the detector, as in amateur radio.

Let us define the averaged waveform samples for a signal $x(t)$ over T_0 interval as

$$\langle x(n\,T_0)\rangle \triangleq \frac{1}{T_0}\int_{(n-1)\,T_0}^{n\,T_0} x(t)\,dt. \tag{53}$$

In the case of small ripple approximation, according to (52), the averaged output voltage waveform is

$$\langle v_{OUT}\,(n\,T_0)\rangle = v_A\,((n-1)\;T_0)\left(1-\frac{1}{2\,f_0\,R\,C}\right) \tag{54}$$

assuming that $v_A\,(n\,T_0) = v_{IN}\,(n\,T_0)$. This means that samples of the averaged output voltage waveform are the envelope waveform samples delayed for one period of the carrier and scaled down by a factor $1 - 1/\,(2\,f_0\,R\,C)$.

Having in mind that the envelope detector output voltage is going to be filtered anyway, a question that naturally arises is why any filtering of the output voltage had been applied at all? Applying the averaging technique in the case $C = 0$, the average of the output voltage is obtained as

$$\langle v_{OUT}\,(n\,T_0)\rangle = \frac{1}{\pi}\,v_A\,((n-1)\;T_0) \tag{55}$$

which is about three times lower than the value obtained in the case of filtering. Thus, immediate filtering of the signal at the envelope detector output increases the signal amplitude for about three times.

9 Transfer Function

After the averaged output voltage waveform is obtained by (54), the result might be applied to determine transfer function of the envelope detector as it applies to AC components of averaged waveforms, which is a procedure common in power electronics.

Let us assume that the envelope takes form

$$v_A\,(t) = V_C\;(1 + m\,\cos\,(\omega_m\,t))\,. \tag{56}$$

According to (54), averaged value of the envelope detector output voltage would be

$$\langle v_{OUT}\,(n\,T_0)\rangle\,(t) = V_C\;(1 + m\,\cos\,(\omega_m\;((n-1)\;T_0)))\left(1-\frac{1}{2\,f_0\,R\,C}\right). \tag{57}$$

From here, AC component could be extracted and represented using the phasor transform [12], resulting in

$$\underline{V}_{OUT\,ac} = m\,V_C\left(1-\frac{1}{2\,f_0\,R\,C}\right). \tag{58}$$

On the other hand, the same averaging technique might be applied to the envelope, resulting in

$$\langle v_A\,(n\,T_0)\rangle = v_A\,((n-1)\;T_0) + \frac{T_0}{2}\,\frac{d\,v_A(t)}{d\,t}\bigg|_{t=(n-1)\,T_0} \tag{59}$$

which for the assumed envelope waveform reduces to

$$\langle v_A\left(n\,T_0\right)\rangle = V_C\,\left(1 + m\,\left(\cos\left(\omega_m\,\left(n-1\right)\,T_0\right) - \pi\,\frac{f_m}{f_0}\,\sin\left(\omega_m\,\left(n-1\right)\,T_0\right)\right)\right). \tag{60}$$

Extracting the AC component and performing the phasor transform [12],

$$\underline{V}_{A\,ac} = m\,V_C\,\left(1 + j\,\pi\,\frac{f_m}{f_0}\right) = m\,V_C\,\left(1 + j\,\pi\,\frac{\omega_m}{\omega_0}\right) = m\,V_C\,\left(1 + j\,\frac{\omega_m}{2\,f_0}\right) \tag{61}$$

is obtained.

Defining the transfer function of the envelope detector as

$$H\left(j\omega_m\right) \triangleq \frac{\underline{V}_{OUT\,ac}}{\underline{V}_{A\,ac}} \tag{62}$$

the transfer function is obtained as

$$H\left(j\omega_m\right) = \frac{1 - \frac{1}{2\,f_0\,R\,C}}{1 + j\,\frac{\omega_m}{2\,f_0}}. \tag{63}$$

Obtained transfer function consists of a scaling factor $1 - \frac{1}{2\,f_0\,R\,C}$ and a single real pole at $\omega_p = 2\,f_0$, which should be a pretty high frequency to provide significant effects on the output voltage waveform.

10 Conclusions

In this document, a thorough analysis of the peak and/or envelope detector circuit is performed. The analysis is first approached using a small ripple or small conduction angle approximation, and relevant equations are derived for this simplified model. To validate the results obtained using the approximate techniques, a semi-numerical analysis of the circuit model is performed, aiming exact analysis wherever possible and resorting to numerical techniques only where transcendental equations prohibited closed form solution. The results of the two approaches are compared, and it is shown that for $f_0\,R\,C > 10$ the small ripple approximation provides the results with an error bellow 1%. Next, an application related topic of scaling is covered, indicating that the output voltage ripple, the output voltage average, and the output voltage waveform are proportional to the input voltage amplitude for a given value of $f_0\,R\,C$, while the conduction angles are not affected by the input voltage amplitude variations.

Main issue discussed in the paper is the ability of the circuit to accurately track the envelope variations, which was the essence of the question posed by a curious student, which initiated publishing of this document. After the analysis, it is shown that ability of the envelope detector to follow the envelope variations depends both on the circuit time constant $R\,C$, as well as the output voltage itself. Author of the document is not aware of such a conclusion clearly stated in available literature. In the case of sinusoidal modulating signal, this reduces to a conclusion that ability of the circuit to track variations of the envelope depend both on the modulating signal frequency and the modulation index. This is the main result presented in the document, and the dependence is analyzed in detail, relating the envelope tracking ability to the output voltage ripple, indicating that there is a trade-off between the two.

Finally, the distortion caused by the output voltage ripple is analyzed, and the experimental results are compared to the low ripple approximation model, showing good agreement. Averaged model of the envelope detector is derived, showing that average of the output voltage during the carrier period follows the envelope with a delay of one carrier period and with some linear downscaling caused by the output voltage ripple. A case when filtering of the output voltage has not been applied is addressed, showing that in that case the averaged output voltage would be about three times lower than in the case filtering is applied. Transfer function of the envelope detector defined on the level of averaged waveforms of the output voltage and the envelope is derived, containing scaling factor $1 - \frac{1}{2\, f_0\, R\, C}$ and a single real pole at $\omega_p = 2\, f_0$, which should be a frequency high enough not to cause significant effects on the output voltage waveform.

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References

[1] Wikipedia, The Free Encyclopedia, "Envelope detector" [Online]. Available: https://en.wikipedia.org/wiki/Envelope_detector [Accessed: February 15, 2018]

[2] Anant Agarwal, Jeffrey H. Lang, *Foundations of Analog and Digital Electronic Circuits*, Morgan Kaufmann Publishers is an imprint of Elsevier. San Francisco, CA, 2005.

[3] Wikipedia, The Free Encyclopedia, "Envelope (mathematics)" [Online]. Available: https://en.wikipedia.org/wiki/Envelope_(mathematics) [Accessed: February 15, 2018]

[4] Wikipedia, The Free Encyclopedia, "Amplitude modulation" [Online]. Available: https://en.wikipedia.org/wiki/Amplitude_modulation [Accessed: February 15, 2018]

[5] Wikipedia, The Free Encyclopedia, "Ripple (electrical)" [Online]. Available: https://en.wikipedia.org/wiki/Ripple_(electrical) [Accessed: February 15, 2018]

[6] Predrag Pejović, "oscusb.py, A Class for USB Communication, Supports Tektronix TBS 1052B-EDU" [Online]. Available: http://tnt.etf.bg.ac.rs/ oe2em/oscusb.py [Accessed: February 15, 2018]

[7] Wikipedia, The Free Encyclopedia, "Transcendental equation" [Online]. Available: https://en.wikipedia.org/wiki/Transcendental_equation [Accessed: February 15, 2018]

[8] Wikipedia, The Free Encyclopedia, "Dimensional analysis" [Online]. Available: https://en.wikipedia.org/wiki/Dimensional_analysis [Accessed: February 15, 2018]

[9] D. Shmilovitz, "On the definition of total harmonic distortion and its effect on measurement interpretation," *IEEE Transactions on Power Delivery*, vol. 20, no. 1, pp. 526–528, Jan. 2005. doi: 10.1109/TPWRD.2004.839744

[10] Slobodan Ćuk, *Modelling, analysis, and design of switching converters*. Dissertation (Ph.D.), California Institute of Technology, 1977. [Online]. Available: http://resolver.caltech.edu/CaltechETD:etd-03262008-110336 [Accessed: February 15, 2018]

[11] Robert W. Erickson, Dragan Maksimović, *Fundamentals of power electronics*. Springer Science & Business Media, 2007.

[12] Predrag Pejović, "Phasor Transform" [Online]. Available: http://tnt.etf.bg.ac.rs/~oe3ee/phasor-transform.pdf, [Accessed: February 25, 2018]

Appendix

```
from pylab import *

f0RC = logspace(-3, 4, 701)
w0RC = 2 * pi * f0RC

n = len(w0RC)
beta = empty(n)
gamma = empty(n)
alpha = empty(n)

def f(gamma):
    f = k * exp(- gamma / w0RC) - cos(gamma)

for i in range(n):

    beta[i] = arctan(1.0 / w0RC[i])
    k = cos(beta[i]) * exp(beta[i] / w0RC[i])

    b0 = 7 * pi / 4
    cmax = 1e5
    c = 0
    eps = 1e-6
    while c < cmax:
        b1 = 2 * pi - arccos(k * exp(-b0 / w0RC[i]))
        if abs(b1 - b0) < eps:
            break
        b0 = b1
    gamma[i] = b1
    alpha[i] = 2 * pi - (gamma[i] - beta[i])


close('all')

rc('text', usetex = True)
rc('font', family = 'serif')
rc('font', size = 16)
rcParams['text.latex.preamble'] = [r'\usepackage{amsmath}']

figure(1, figsize = (10, 6))
semilogx(f0RC, degrees(alpha), 'm', label = r'$\alpha$', linewidth = 2)
semilogx(f0RC, degrees(beta), 'b', label = r'$\beta$', linewidth = 2)
semilogx(f0RC, degrees(gamma), 'g', label = r'$\gamma$', linewidth = 2)
ylim(-30, 390)
yticks([0, 90, 180, 270, 360])
xlabel(r'$f_0 \, R \, C$')
ylabel(r'$\beta, \, \gamma, \, \alpha \; [^\circ]$')
legend(loc = 'center right')
grid()
savefig('fbetagammaalpha.pdf', bbox_inches = 'tight')

data = array([w0RC / 2.0 / pi, beta, gamma, alpha]).transpose()
np.save('fbetagammaalphadata.npy', data)


voutmean = (sin(beta) - sin(gamma) +
cos(beta) * (w0RC * (1 - exp((beta - gamma) / w0RC)))) / (2 * pi)

voutmin = cos(gamma)
```

```
voutmax = ones(n)

figure(2, figsize = (10, 6))
semilogx(f0RC, voutmax, 'b', label = r'$v_{OUT\,max} / V_C$', linewidth = 2)
semilogx(f0RC, voutmin, 'g', label = r'$v_{OUT\,min} / V_C$', linewidth = 2)
semilogx(f0RC, voutmean, 'r', label = r'$v_{OUT\,mean} / V_C$', linewidth = 2)
ylim(-0.1, 1.1)
xlabel(r'$f_0 \, R \, C$')
ylabel(r'$v_{OUT\,max} / V_C, v_{OUT\,min} / V_C, v_{OUT\,mean} / V_C$')
legend(loc = 'lower right')
grid()
savefig('fmaxminmean.pdf', bbox_inches = 'tight')
```