Thinned Spectrum Radar Waveforms

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***Abstract*—Small phase perturbations applied to a stepped-frequency or linear-frequency-modulated radar waveform have been proposed as a means to generate frequency nulls in the spectrum of the transmitted waveform. By nulling selected frequencies (or thinning) in the spectrum of transmitted pulses, the interference between the transmitted and received signals is reduced, which facilitates the spectral cohabitation of multiple RF users. Preliminary experimental results of two methods of spectral nulling are presented with consideration to in-band and out-of-band interference. Range processing is examined to determine the effects of the phase perturbation on radar operations.**

I. Introduction

The available RF spectrum for radar operations is becoming increasingly crowded as the number of users, and thus the required bandwidth, continues to increase. Much work has been performed in the area of spatial nulling of antenna patterns by using phase-only coding [1-6]. These approaches have also been adapted to the spectral domain, resulting in thinned-spectrum constant-modulus radar waveforms [7,8]. Thinned-spectrum waveforms allow narrowband systems to operate near or even within the radar's frequency band. The method in [7] applies to stepped-frequency (SF) waveforms [9-11] which utilize multiple narrowband pulses, each with a different center frequency, to synthesize a wider bandwidth. Each waveform is augmented to create a stepped-frequency polyphase code (SFPC) that exhibits a spectral null at the desired frequency. A method for spectral nulling of linear frequency modulated (LFM) waveforms is presented in [8]. Similar to [7], the phase of the radar pulse is modified to produce a null at the desired frequency.

In this paper, preliminary experimental results using the SFPC and phase modulated LFM techniques are examined to assess the effectiveness of these methods. Two cases are considered for each spectral nulling technique, one with a null in the operating bandwidth of the radar waveform and another with the null located just outside the spectral mainlobe. Waveforms were generated on a vector signal generator (VSG) and were recorded on a real-time spectrum analyzer (RSA) for offline analysis. The thinned spectrum waveforms are compared to unperturbed waveforms to determine the performance of each approach.

In addition, the effect of spectral nulling on radar performance is considered by applying range processing to the recorded waveforms. In particular, IFFT processing [9] is used for the SFPC waveform, and matched filtering is performed for the LFM waveform. Each example is compared to a standard, non-nulled waveform with no phase perturbations.

II. Thinned Spectrum Algorithms

A. Stepped-Frequency Polyphase Codes

The SFPC waveform is composed as a burst of M pulses where the frequency of each pulse is denoted as

$$f_m = f_0 + (m-1)\Delta f \, , \tag{1}$$

where $m = 1,2,\ldots,M$, f_0 is the starting frequency, and Δf is the frequency step. The duration of each pulse is

$$T_m = 1/\Delta f \, , \tag{2}$$

which has an instantaneous bandwidth of Δf. The range resolution of the burst is proportional to $1/(M\Delta f)$. Each pulse is divided into N equal length chips with ϕ_{mn} being the phase of the n^{th} chip of the m^{th} pulse. The amplitude of each chip may also be manipulated to control the frequency response further. However, to maintain power efficiency, only constant modulus chips are considered.

Generation of phase values is based upon a method used for implementing nulls in the sidelobe pattern of an antenna array [5,7] under the assumption that the optimal phase values are small. This assumption justifies a linear approximation for the constraint equations, allowing the optimization problem to be solved in closed form. To generate optimal phase values from [7], form the $N\times K$ matrix C_m with the nk^{th} element defined by

$$c_{nk}^{(m)} = \exp\left\{ j(n-1)\tilde{\omega}_k T_m / N \right\} , \tag{3}$$

where $\tilde{\omega}_k$ is the angular frequency of the k^{th} desired null, $k = 1,2,\ldots,K$. The matrix C_m is then separated into its real and imaginary components,

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$$C_m^{(r)} = \mathrm{Re}\{C_m\} \tag{4}$$

and

$$C_m^{(i)} = \mathrm{Im}\{C_m\}, \tag{5}$$

to form the $N\times 2K$ matrix

$$\tilde{C}_m = \left[C_m^{(r)} \quad C_m^{(i)} \right]. \tag{6}$$

Finally, the solution is obtained as

$$\hat{\phi}_m = \tilde{C}_m \left(\tilde{C}_m^T \tilde{C}_m \right)^{-1} \begin{bmatrix} -C_m^{(i)^T} & \mathbf{1} \\ C_m^{(r)^T} & \mathbf{1} \end{bmatrix}, \tag{7}$$

where $\mathbf{1}$ is an $N\times 1$ vector of ones. The result in (7) produces N phase values, or chips, for each of the M pulses. The values of the phase perturbations for each pulse vary according to the location of the null relative to the center frequency of a given pulse. The phase values are then modulated such that the m^{th} pulse is

$$s_m(t) = \cos\left(2\pi f_m t + \phi_m(t)\right), \qquad 0 \le t \le T_m \tag{8}$$

where

$$\phi_m(t) = \phi_{m,n} \quad \text{for} \quad n\frac{T_m}{N} \le t \le (n+1)\frac{T_m}{N} \tag{9}$$

for n = 0,1,…,N–1. The resulting phase values are small, allowing standard SF range processing to be used.

B. Phase-Coded LFM

The generation of the phase perturbation for the Phase-Coded LFM (PC-LFM) waveform follows much in the same manner as for the SFPC waveform. The Fourier transform of the waveform at the chosen spectral null(s) is desired to be zero. Further, the phase offsets are required to be small such that the original phase modulation is maintained as much as possible. The algorithm for finding the phase offsets from [8] is highlighted below.

The length-L discretized LFM waveform is defined as

$$x(\ell) = e^{j\psi(\ell)}, \tag{10}$$

for l = 0,1,…,L–1 where $\psi(l)$ is the LFM phase progression. The desired location of the spectral nulls are represented by the collection of K length-L sinusoids with the l^{th} sample of the k^{th} sinusoid expressed as

$$p_{\ell,k} = e^{-j2\pi \tilde{f}_k \ell}, \tag{11}$$

where $\tilde{f}_k$ is the normalized frequency of the k^{th} null. Formulate an $L\times K$ matrix Z with elements defined as

$$z_{\ell,k} = x(\ell)\cdot p_{\ell,k}. \tag{12}$$

Decomposing Z into its real and imaginary components yields the $L\times 2K$ matrix

$$\tilde{Z} = [C \quad S], \tag{13}$$

where $C = \mathrm{Re}\{Z\}$ and $\mathbf{S} = \mathrm{Im}\{Z\}$. Now define a matrix A as

$$A = \tilde{Z}^T \tilde{Z} \tag{14}$$

and vector $\mathbf{b}$ as

$$\mathbf{b} = \begin{bmatrix} -S^T\mathbf{1} \\ C^T\mathbf{1} \end{bmatrix}, \tag{15}$$

Where $\mathbf{1}$ is an $L\times 1$ vector of ones. The solution for the set of L optimal phase values is then

$$\boldsymbol{\theta} = \tilde{Z}A^T \left(A^T A\right)^{-1} \mathbf{b}. \tag{16}$$

The resulting phase perturbed length-L discretized LFM waveform is

$$\tilde{\mathbf{x}} = e^{j\boldsymbol{\theta}+\boldsymbol{\psi}}. \tag{17}$$

III. Experimental Results

This section details the results from waveforms created using the above algorithms. Both the SFPC and LFM waveforms are created with a total bandwidth of 10 MHz. Complex samples are generated at baseband with a sampling frequency of 80 MS/s. Next, an analog signal is excited using a Rohde & Schwartz SMBV100A VSG at a start frequency of 3.5 GHz and a peak power of –10 dBm. The time-domain signal is then captured at 50 MS/s on a Tektronix RSA 6106A real-time spectrum analyzer.

Both spectral nulling methodologies are examined for the case of an in-band null with a relative offset of 1.5 MHz with respect to the start frequency and the case of an out-of-band null 11.5 MHz away from the start frequency. The in-band null lies directly between the two SF transmitted frequencies of the second and third pulses at 3.501 and 3.502 GHz, respectively. As explained in [10], if the null is within the spectral mainlobe of a stepped frequency pulse, it can be omitted to avoid the distortion associated with placing nulls in the mainlobe regions. Experiments were performed both for inclusion and omission of the second and third pulses. Also, note that the out-of-band null lies at a peak in the spectral sidelobes (*i.e.*, not in the location of a natural null).

Plots presented in the paper are compared to a measured baseline case where no nulling is present. In the following, all baseline traces are presented in black, SFPC cases in red, and PC-LFM cases in blue.

A. SFPC Results

The SFPC waveform is created from M (= 10) sub-pulses, N (= 40) chips per pulse, and a step frequency of Δf = 10 MHz. For this stepped-frequency waveform, the length of each sub-pulse is 1 µs, resulting in a burst time of 10 µs. To facilitate measurement of the signal on the RSA, the bursts are transmitted in a 40-µs pulse repetition interval (PRI).

Results from the first example, with a null located between the second and third step frequencies at 3.5015 GHz, are shown with and without the second and third pulses in Figures 1 and 2, respectively. In Figure 1, the power of the baseline signal at the null frequency is approximately –44 dBm compared to the nulled waveform's power level of –66 dBm, a relative decrease of 22 dB. Also note that the frequency content at the surrounding frequencies has increased as a byproduct of the phase offsets.

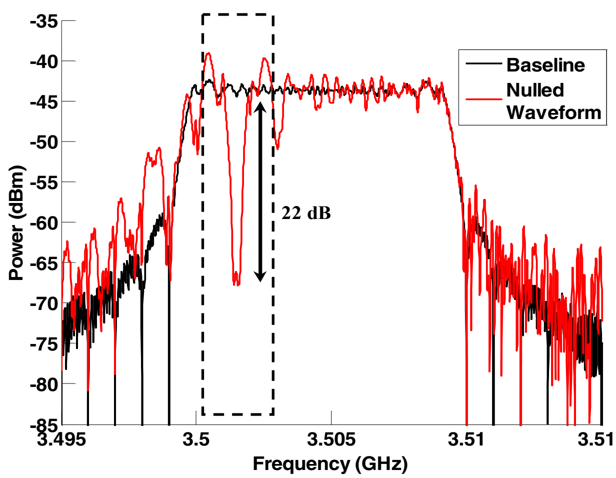

Figure 1. SFPC waveform with null at 3.5015 GHz

Figure 2 shows that omission of the two pulses near the null results in an increase of the null depth to 48.5 dB relative to the case of no phase perturbations. The power level of the SF waveform with pulses 2 and 3 skipped is –52.4 dBm at 3.5015 GHz, and the SFPC waveform has a power level of –100.9 dBm. When compared to the full 10-pulse SF waveform (baseline case in Figure 1), the depth of the null is 57 dB.

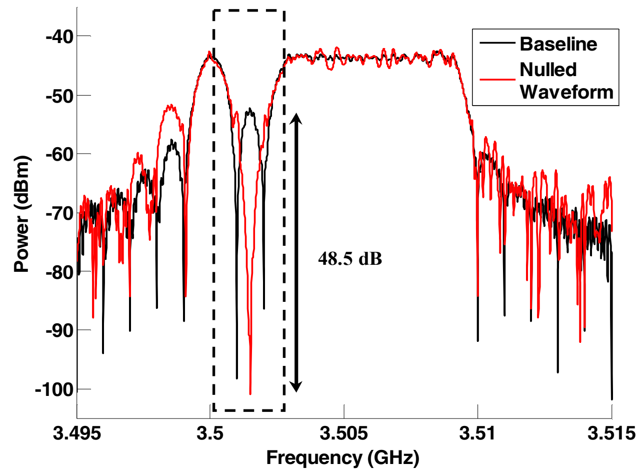

Figure 2. SFPC waveform with null at 3.5015 GHz and pulses 2 and 3 omitted

Figure 3 displays the second case with an out-of-band null at 3.5115 GHz. The null depth is 25 dB with the baseline power at this frequency measured to be –64.5 dBm; whereas the nulled waveform's power is –89.5 dBm. Note the increase in spectral power at natural nulls that occur in the baseline waveform at 3.5110 and 3.5120 GHz. There is an increase of approximately 10 dB in the nulled waveform with respect to the baseline waveform at those frequencies.

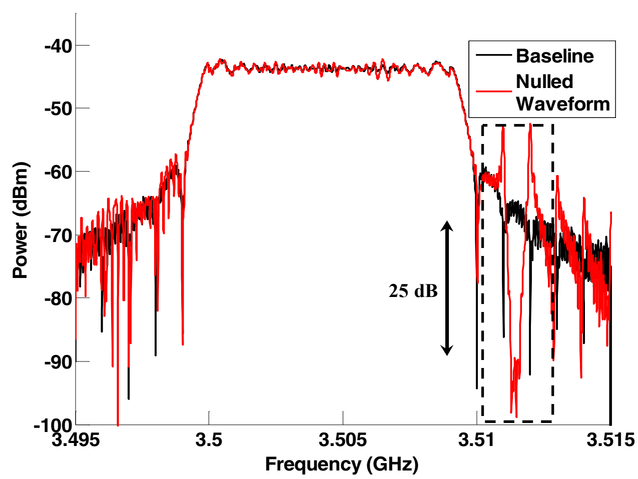

Figure 3. SFPC waveform with null at 3.5115 GHz

B. Polyphase-Coded LFM Results

The baseline PC-LFM waveform has a bandwidth of 10 MHz and a pulse length of 10 µs. The start frequency is 3.5 GHz and the stop frequency is 3.510 GHz. The PRI for this waveform is 40 µs (same as SFPC waveform experiment). Figure 4 shows the PC-LFM with a null at a 3.5015 GHz and the baseline LFM waveform with no phase coding. A narrow null with a depth of 51 dB is achieved with the bottom of the null having a power of –94.6 dBm compared to the baseline case at –43.6 dBm. A second, unintended null that is generated at 3.51205 GHz is simply a byproduct of the polyphase coding. Also note that the spectral sidelobes are disturbed at frequencies greater than the LFM band, but not at those less than the LFM band.

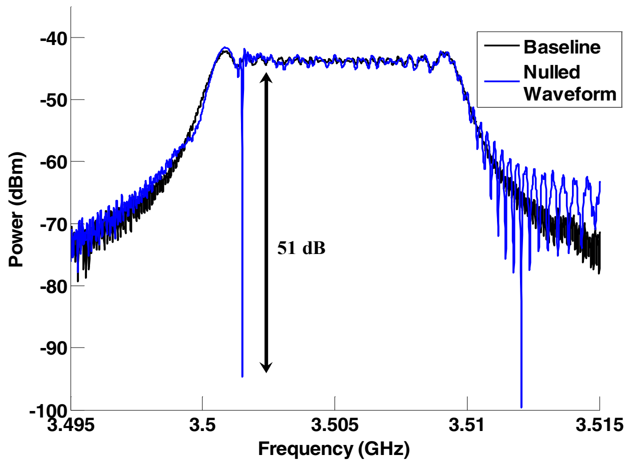

Figure 4. PC-LFM waveform with null at 3.5015 GHz

Results from the second experiment of the PC-LFM waveform, where a null is generated at 3.5115 GHz, are displayed in Figure 5. Power at this frequency is –64.5 dBm for the baseline waveform and –96.5 dBm for the PC-LFM waveform, which results in a null depth of 32 dB. The mainlobe frequencies of the baseline and nulled waveforms are nearly identical, and the spectral sidelobes have a high degree of similarity. The width of the null is still narrow as is seen in Figure 4.

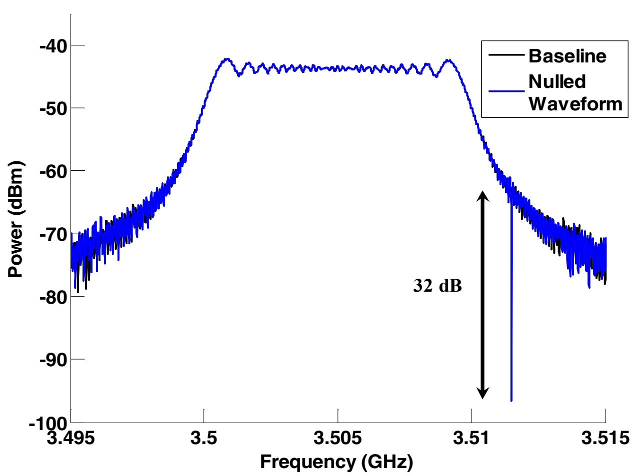

Figure 5. PC-LFM waveform with null at 3.5115 GHz

C. Comparisons

Some interesting differences are seen between the SFPC case and the PC-LFM results. First, note that the spectral levels in the bandwidth of the LFM waveforms are mostly undisturbed with the exception of the null; whereas the SFPC waveform with the in-band null distorts the mainlobe, especially at frequencies near the null. Another point of interest is the effect that the SFPC waveforms impart on the spectral sidelobes. All measurements displayed in Figures 1-3 show an increase in spectral power outside the null region. However, the PC-LFM waveforms only show a significant increase for the in-band null at frequencies greater than mainlobe region.

One final consideration is the width of the nulls. The SFPC displays significantly wider nulls than PC-LFM. In Figure 1, the width of the SFPC null is approximately 1 MHz at the power level of the baseline waveform. A 10 percent decrease is seen in the null width of the PC-LFM waveform of Figure 4. Similar results are seen when comparing Figures 3 and 5, with the SFPC null width at 3.5115 GHz being approximately 10 percent greater than in the PC-LFM waveform. The reason for this dissimilarity is that the temporal extent of the phase perturbation imparted by the SFPC waveform is only over the 1-µs duration of a single sub-pulse; whereas the extent for the phase perturbation of the PC-LFM waveform is the full 10-µs pulse width. The greater time support results in a narrower null width for the PC-LFM waveform.

IV. Range Processing

The SF processing done in this paper is performed by the IFFT method discussed in [9]. On the other hand, PC-LFM processing is performed with the standard matched filtering operation; however, the uncoded LFM waveform is used as the reference. In the following section, the measured baseline and in-band nulled waveforms examined in Section III are processed as described above. Recall that the waveform is measured on the RSA at a sample rate of 50 MS/s. The SFPC waveform has a pulse bandwidth of 1 MHz and spans a 10 MHz interval. The LFM waveform has a 10 MHz bandwidth.

A. SFPC Range Processing

Processing the SF waveform is accomplished through "range gating," with the width of a single range interval being the same as the width of a single pulse. A sample is taken from each pulse of the measured waveform at time intervals chosen from (2). An IFFT is then taken over the samples to form the range profile for the range cell.

The first experiment considered is the SFPC waveform with an in-band null at 3.5015 GHz and all pulses being used. In Figure 6, a single burst with all 10 pulses is processed, simulating the response of a single point target within a range cell. A sample is taken from the middle of each pulse and an oversampled IFFT is performed. The result from the nulled waveform closely resembles that of the baseline, with a slight offset near the edges. The width of the mainlobe is 2 µs wide, and the peak sidelobe level (PSL) is 13 dB down from the peak. Range sidelobe nulls occur at 1 µs intervals. This range cell profile demonstrates the ability of an SF or SFPC waveform to increase the resolution achieved by the instantaneous bandwidth of a single pulse by a factor of the number of pulses used.

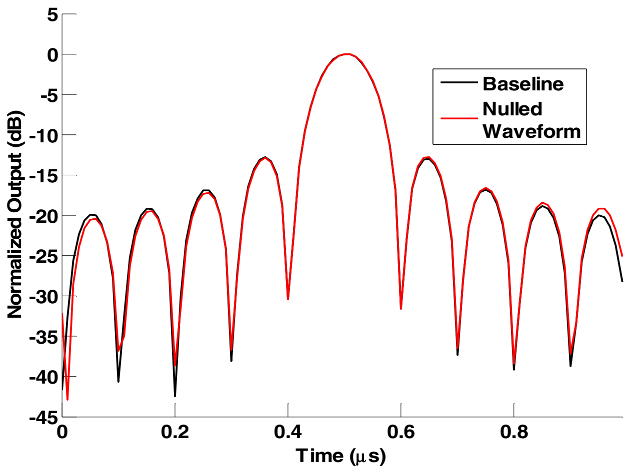

Figure 6. Range processing of SFPC waveform with null at 3.5015 GHz

Figure 7 shows the results for an SFPC waveform with an in-band null at 3.5015 GHz when the 2nd and 3rd pulses are skipped. In this instance, only 8 samples are collected for the IFFT. As before, samples are taken from the middle of the pulses. Notice that the omission of 2 pulses results in the width of the mainlobe decreasing to 0.25 µs and in the PSL decreasing to 12.3 dB relative to the peak. These observations illuminate a trade-off that can be made between increasing the null depth and decreasing the range resolution and PSL. Differences between the baseline SF waveform and the nulled SFPC waveform are minimal, as seen in Figures 6 and 7.

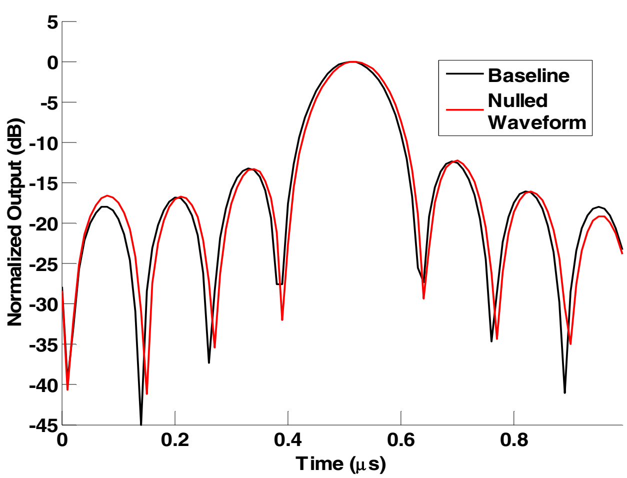

Figure 7. Range cell processing of SFPC waveform with null at 3.5015 GHz

B. *PC-LFM Range Processing*

The matched filter for pulse compressing the LFM is generated from a simulated waveform with no noise present. The baseline LFM waveform and the PC-LFM waveform with a null at 3.5015 GHz are convolved with the matched filter (Figures 8 and 9). The complete range profile is shown in Figure 8, and Figure 9 displays a close-up view that corresponds to the range interval processed by the SF waveforms in Figures 6 and 7.

In Figure 8, the range-time sidelobes of the nulled waveform have increased with respect to the baseline LFM waveform. A moderate increase is seen in the PC-LFM waveform between –4 and 4 µs. A large increase occurs between 4 and 8 µs. The levels within 1 µs of the peak closely resemble those of the baseline wavefrom. The close-up view in Figure 9 further emphasizes that both results are similar near the peak.

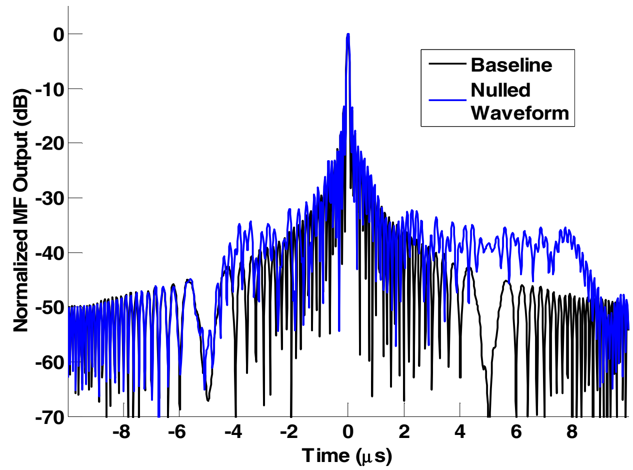

Figure 8. Matched filter response of PC-LFM waveform with null at 3.5015 GHz

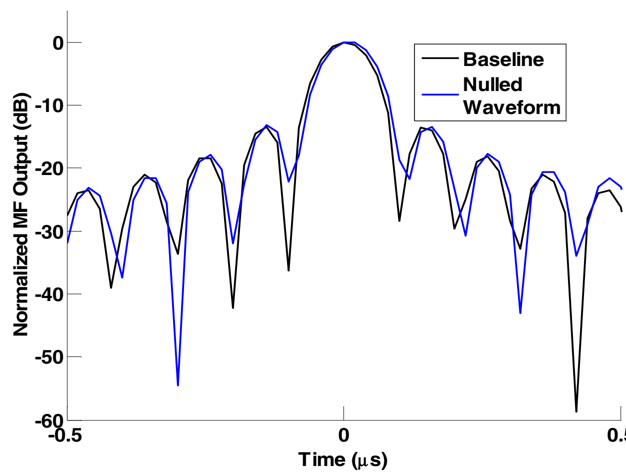

Figure 9. Close-up of matched filter response of PC-LFM waveform with null at 3.5015 GHz

V. CONCLUSIONS

In this paper, preliminary experimental results have been presented which demonstrate the application of phase-only perturbations to stepped-frequency and LFM waveforms to produce nulls in the transmitted spectrum. Nulls can be placed in-band or out-of-band for both waveforms. For the SFPC waveforms, wide nulls are observed at depths greater than 22 dB with distortion resulting in the surrounding frequencies. For cases with in-band nulls, step frequencies near the null can be skipped to increase the null depth at the cost of decreased range resolution and dynamic range. IFFT range cell processing was found to be similar for the unperturbed SF waveform compared to the SFPC waveform.

The methodology for spectral nulling with PC-LFM results in narrower nulls than those generated for the SFPC waveforms. However, the PC-LFM waveform yields significantly less distortion at out-of-band frequencies than the SFPC waveform. Spectral suppression of 51 dB for the in-band null and 32 dB for the out-of-band null have been measured. The pulse compression range profiles are similar for the LFM and PC-LFM waveforms near the match point; however, the range sidelobes increase at offsets greater than 1 μs for the PC-LFM waveform.

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