INDUCTIVE LOOP DETECTOR ANALYSIS

Milton K. Mills
U.S. Department of Transportation
Federal Highway Administration
Office of Research
Traffic Systems Division, HRS-32
Washington, D.C. 20590

Introduction

The inductive loop detector (ILD) system is used nationwide to provide vehicle passage and/or presence information to traffic surveillance and control systems. The ILD system is composed of a typically 1.83 m (6 ft) by 1.83 m (6 ft) loop of 3 turns of 14 AWG wire embedded in the pavement, a transmission line to roadside, and roadside vehicle detection electronics. The ILD system design has become standardized as a result of considerable experimentation by users and manufacturers over a number of years. Field problems reported for ILD systems are short presence holding time because of low sensitivity, unstable operation, and lack of interchangeability of detector electronics from different manufacturers (i.e., detector electronics must be selectively fitted to some locations in order to obtain satisfactory operation).

The purpose of this paper is to present an approximate model of an ILD system which will provide assistance in understanding and solving those problems with ILD systems.

Background

Most prior analyses of ILD systems have concentrated on theoretically calculating the self inductance of different loop sizes and shapes and on the effect of transmission line inductance on sensitivity. The Link Inductive Loop Analysis Project final report² contains the development of an equation to calculate the self inductance of a multi-turn rectangular or square loop. Anderson³ describes the detector electronic design for self-tuning, bridge balance, and phase shift loop detector electronics. He provides design equations and graphs to illustrate the effect of different length transmission lines on the performance of inductive loop detectors. The effect of the loop quality factor on ILD performance is described in a City of Los Angeles, Department of Traffic, staff report. Hulscher⁵ describes a modern digital type loop detector.

Inductive Loop Detector System Model

Figure 1 is a block diagram of an ILD system. When a vehicle passes over the loop in the pavement, currents are induced in the vehicle's undercarriage. The magnetic field generated from the induced currents changes the driving point impedance of the loop. The loop driving point impedance change is principally a reduction in loop self inductance with a negligible increase in loop resistance. The loop is connected to a transmission line at the splice box. The transmission line connects the loop to roadside detector electronics which is typically housed in a traffic controller cabinet. When the loop is installed over a reinforcing steel mesh in the pavement, the magnitude of loop driving point impedance change decreases. The magnitude of the loop driving point impedance change is further reduced by the transmission line. The turning network which is part of the electronics resonates the equivalent inductance at the detector

terminals.

Moisture and temperature changes due to the environment cause changes in ILD system electrical parameters which can result in unstable operation.

Vehicle Model

The vehicle is modeled as a flat, perfectly conducting plate at a height above the loop approximately equal to the average undercarriage height of the vehicle. The width and length of the plate are equal to the width and length of the vehicle. The continuous, perfectly conducting plate can be simulated by a wire grid provided the mesh elements are sufficiently small. A FORTRAN IV computer program was developed which modeled the vehicle by a wire grid. The important effect on detector performance for vehicles with lateral and longitudinal displacement can be simulated with this model. The size of the loop relative to the size of the vehicle is not limited in this model.

In order to simplify the calculation of loop system sensitivity for calculation on a small programmable calculator, the rectangular loop simulating the vehicle was set equal to the size of the inductive loop in the roadway. When the wire grid is symmetrically located over a symmetrical loop, the currents induced in the grid by the loop cancel in the grid interior leaving a current flowing around the perimeter of the grid provided the grid width and length are equal to those of the loop. Thus, the grid can be replaced with a wire loop equal to the grid perimeter. This case represents a vehicle centered over the loop (i.e., maximum percentage inductance change of the loop). Magnetic effects on the loop due to the permeable material of the vehicle are assumed to be negligible.

Reinforcing Steel Model

The mesh elements of the reinforcing steel are considered sufficiently small and the size of the mesh sufficiently large compared to the loop so that the steel mesh can be replaced with an perfectly conducting plane of infinite extent. Based on image theory, the infinitely conducting plane is replaced with an image loop at twice the distance from the loop to the reinforcing steel. Figure 2 illustrates the geometry of the vehicle and reinforcing steel shorted turn model. The direction of induced currents determines the sign of mutual inductance terms used in the following circuit model.

Inductive Loop Circuit Model for Calculating Loop Sensitivity

Figure 3 shows the inductive loop, loop simulating vehicle, and loop simulating reinforcing steel mesh, modeled as an air core transformer. The inductive loop is the primary winding of the transformer with the shorted turn secondary winding modeling the vehicle. This shorted turn winding is movable to simulate changes in vehicle undercarriage height. The shorted turn secondary winding modeling

the reinforcing steel mesh is located at a distance of twice the steel mesh to inductive loop spacing.

Inductive Loop Circuit Equations

The circuit equations for Figure 3 are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 \quad (1)$$

$$0 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 \quad (2)$$

$$0 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3 \quad (3)$$

where

$$Z_{11} = R_{11} + j \omega L_{11} \quad (4)$$

$$Z_{22} = R_{22} + j \omega L_{22} \quad (5)$$

$$Z_{33} = R_{33} + j \omega L_{33} \quad (6)$$

$$Z_{12} = j \omega M_{12} \quad (7)$$

$$Z_{13} = j \omega M_{13} \quad (8)$$

All mutual impedances are symmetrical (i.e.,

$$Z_{12} = Z_{21}, Z_{13} = Z_{31}, \text{ etc.}).$$

Experimental measurements show that the quality factor, $\omega L_{11}/R_{11}$, of the loop driving point impedance, Z_{11} , is ≥ 10 .

Assuming small circuit loss to simplify calculations

$$Z_{11} = j \omega L_{11} \quad (9)$$

$$Z_{22} = j \omega L_{22} \quad (10)$$

$$Z_{33} = j \omega L_{33} \quad (11)$$

Inductive Loop Driving Point Impedance

The loop driving point impedance, Z_1 , from equations 1, 2, and 3 is:

$$Z_1 = \frac{V_1}{I_1} = \frac{Z_{11}Z_{22}Z_{33} - Z_{31}^2Z_{22} - Z_{32}^2Z_{11} - Z_{21}^2Z_{33} + 2Z_{21}Z_{31}Z_{32}}{Z_{22}Z_{33} - Z_{23}^2} \quad (12)$$

If the reinforcing mesh is spaced a great distance from the inductive loop

$$Z_{13} = Z_{23} = 0$$

then

$$Z_1 = \frac{Z_{11}Z_{22} - Z_{21}^2}{Z_{22}} \quad (13)$$

The loop driving point impedance, Z_1 , depends only on the effect of the vehicle.

Inductive Loop Sensitivity

The sensitivity, S_L , of an inductive loop is defined as:

$$S_L = \frac{\Delta Z_1}{Z_1^{NV}} \quad (14)$$

The change in loop driving point impedance, ΔZ_1 , is given by:

$$\Delta Z_1 = Z_1^{NV} - Z_1^V \quad (15)$$

where Z_1^{NV} , is the loop driving point impedance with no vehicle present and Z_1^V is the loop driving point impedance when sensing a vehicle.

The loop driving point impedance, Z_1 , can be put in the following form:

$$Z_1 = Z_{11} - \frac{Z_{13}^2Z_{22} + Z_{12}^2Z_{33} - 2Z_{12}Z_{13}Z_{23}}{Z_{22}Z_{33} - Z_{23}^2} \quad (16)$$

Then if no vehicle is present

$$Z_{12} = Z_{23} = 0$$

$$Z_1^{NV} = Z_{11} - \frac{Z_{13}^2}{Z_{33}} = \frac{Z_{11}Z_{33} - Z_{13}^2}{Z_{33}} \quad (17)$$

$$Z_1^V = Z_{11} - \frac{Z_{13}^2Z_{22} + Z_{12}^2Z_{33} - 2Z_{12}Z_{13}Z_{23}}{Z_{22}Z_{33} - Z_{23}^2} \quad (18)$$

$$\Delta Z_1 = \frac{Z_{13}^2Z_{22} + Z_{12}^2Z_{33} - 2Z_{12}Z_{13}Z_{23}}{Z_{33}(Z_{22}Z_{33} - Z_{23}^2)} \quad (19)$$

The loop sensitivity, S_L , is given by:

$$S_L = \frac{\Delta Z_1}{Z_1^{NV}} = \frac{Z_{13}^2Z_{22} + Z_{12}^2Z_{33} - 2Z_{12}Z_{13}Z_{23}}{(Z_{11}Z_{33} - Z_{13}^2)(Z_{22}Z_{33} - Z_{23}^2)} \quad (20)$$

$$\text{if } Z_{13} = Z_{23} = 0$$

$$S_L = \frac{Z_{12}^2}{Z_{11}Z_{22}} \quad (21)$$

Applying equations 9, 10, and 11 to equation 20 the loop sensitivity, S_L , in percent is:

$$S_L = 100 \frac{M_{12}^2L_{33}^2 + M_{13}^2M_{23}^2 - 2M_{12}M_{13}M_{23}L_{33}}{(L_{11}L_{33} - M_{13}^2)(L_{22}L_{33} - M_{23}^2)} \quad (22)$$

where

S_L = inductive loop sensitivity (%)

M_{12} = mutual inductance between inductive loop and loop simulating vehicle (Hy)

M_{13} = mutual inductance between inductive loop and loop simulating reinforcing mesh (Hy)

M_{23} = mutual inductance between loop simulating vehicle and loop simulating reinforcing mesh (Hy)

- L_{11} = self inductance of inductive loop (Hy)
 L_{22} = self inductance of loop simulating vehicle (Hy)
 L_{33} = self inductive of loop simulating reinforcing mesh (Hy)

Figure 4 shows the effect of reinforcing steel on a 1.83m (6ft) by 1.83m (6ft), 3 turn loop. This graph is calculated from equation 22.

Loop Self Inductance

The loop self inductance is comprised of a low frequency component, which is independent of operating frequency and a high frequency component, which is a function of frequency. The high frequency, inductive component L_{O}^{HF} , is proportional to loop turns, N , and the low frequency, inductive component L_{O}^{LF} , is proportional to loop turns squared. Because of leakage flux which does not link loop turns, the contribution of loop turns, N , must be modified by an experimentally determined, coupling coefficient, K .

The self inductance, L , of the loop is calculated from the formula:

$$L = N L_{O}^{HF} + (KN)^2 L_{O}^{LF} \quad (23)$$

where

$$L_{O}^{HF} = \frac{\mu_0 \mu_w}{\pi} (A+B) \quad (24)$$

$$L_{O}^{LF} = \frac{\mu_0 \mu_m}{\pi} \left\{ 2B_1 - (A+B) - B_2 - B_3 + 2r + A \ln \left[\frac{(B_2+A)B}{(B_1+A)r} \right] + B \ln \left[\frac{(B_3+B)A}{(B_1+B)r} \right] \right\} \quad (25)$$

where

$$B_1 = \sqrt{A^2 + B^2} \quad (26)$$

$$B_2 = \sqrt{A^2 + r^2} \quad (27)$$

$$B_3 = \sqrt{B^2 + r^2} \quad (28)$$

$$\delta = \frac{1}{\sqrt{\mu_0 \mu_w \pi f \sigma}} \quad (29)$$

L = loop self inductance (Hy)

δ = skin depth (m)

μ_0 = permeability of free space = 4×10^{-7} H_y/m

μ_w = relative permeability of copper wire = 1

μ_m = relative permeability of medium = 1

N = number of loop turns

K = experimentally measured coupling coefficient ($0 < K < 1$) for 1.83m (6ft) by 1.83m (6ft), 3 turn loop, $K = 0.86$

r = radius of loop wire (m)

A = inductive loop width (m)

B = inductive loop length (m)

Loop Mutual Inductance

The mutual inductance, M , between the inductive loop and the single turn loop simulating the vehicle is calculated using the formula for the mutual inductance between two, equal, parallel current elements of length R and spacing D . A plus sign is used when the direction of current in the current elements is the same. A minus sign is used when the direction of current in the current elements is opposite. The mutual inductance formula for the two elements is:

$$M(R,D) = \pm \frac{\mu_0 R}{2\pi} \ln \left[\frac{R}{D} + \sqrt{1 + \left(\frac{R}{D} \right)^2} \right] \pm \frac{D}{R} \quad (30)$$

where

μ_0 = permeability of free space = 4×10^{-7} H_y/m

R = current element length (m)

D = current element spacing (m) (H_y in Figure 2)

The total mutual inductance of the two, parallel loops in figure 2 is given by the sum of the mutual inductances between the parallel sides and is:

$$M_T = 2N_1 N_2 \left[M_{13}(A, \sqrt{H_V^2 + B^2}) - M_{11}(A, H_V) + M_{24}(B, \sqrt{H_V^2 + A^2}) - M_{22}(B, H_V) \right] \quad (31)$$

where M_{13} is the mutual inductance between side 1 of the inductive loop and side 3 of the loop representing the vehicle in Figure 2. Note that $M_{13} = M_{31}$

N_1 = turns of inductive loop

N_2 = turns of loop simulating vehicle = 1

Using appropriate subscripts formula 31 is also used to calculate the mutual inductance between the inductive loop and the loop representing the reinforcing steel mesh, the loop representing the vehicle, and the loop representing the reinforcing steel mesh.

Loop Resistance

The series loop resistance, R_L , is comprised of the direct current wire resistance, R_{dc} , the high frequency or skin effect resistance, R_{ac} , and the ground resistance, R_g . The most important resistance is the ground resistance which is caused by currents induced in the conductive pavement and subgrade material. The ground resistance may limit loop sensitivity in locations with a large moisture content. The ground resistance is calculated by assuming the pavement and subgrade material causes a magnetic loss similar to that of a ferrite or iron core in an inductor. The permeability, μ_g , of the pavement and subgrade material is assumed to be one. Appendix I presents a derivation of the ground resistance.

The series loop resistance is:

$$R_L = R_{dc} + R_{ac} + R_g \quad (32)$$

$$R_{dc} = \left(\frac{1}{\pi r^2 \sigma} \right) (P)(N) \quad (33)$$

$$R_{ac} = R_{dc} \left(\frac{r}{48\delta} \right) \quad (34)$$

$$R_g = \tan \delta_g \omega L_L \quad (35)$$

where

R_L = series loop resistance (Ω)
 R_{dc} = direct current loop resistance (Ω)
 R_{ac} = skin effect resistance (Ω)
 R_g = ground resistance (Ω)
 r = loop wire radius (m)
 σ = conductivity of copper wire (mhos/m)
 $\quad = 0.58 \times 10^8$ mhos/m
 P = loop perimeter (m)
 N = number of loop turns
 δ = skin depth (m)
 $\tan \delta_g$ = loss tangent of pavement material
 L_L = loop self inductance (Hy)
 ω = operating frequency (radians)

The calculated loss tangent is 0.033 at 100 KHz. The loss tangent was calculated by subtracting the calculated loop wire resistance from the measured loop resistance in Table I and dividing the result by the loop inductive reactance. A value of 0.028 was measured by the MIT Laboratory for Insulation Research.

Loop Capacitance

Capacitance exists between loop wire turns and causes the inductance at the loop terminals to vary with frequency. At the self resonant frequency of the loop, the loop capacitance resonates with the loop inductance resulting in a real impedance across the loop terminals. The loop capacitance is:

$$C_L = \frac{1}{4\pi^2 L_L f_o^2} \quad (36)$$

where

C_L = loop capacitance (Fd)
 L_L = loop self inductance (Hy)
 f_o = loop self resonant frequency (Hz)

If the loop resistance, R_L , and dielectric loss are negligible, the apparent loop inductance, L_L^a , is:

$$L_L^a = \frac{L_L}{1 - 4\pi^2 f^2 C_L L_L} \quad (37)$$

or

$$L_L^a = \frac{L_L}{1 - \left(\frac{f}{f_o}\right)^2} \quad (38)$$

where f is any frequency (Hz) below the self resonant frequency, f_o .

The measured self resonant frequency, f_o , of a 3 turn, 1.83m (6ft) by 1.83m (6ft) loop buried in asphalt pavement at the Fairbank Highway Research Station is 697.06 KHz. The measured loop inductance at the lowest frequency, 20KHz, is 73.9 μ H. The calculated loop capacitance is 707 pF using equation 36.

The loop capacitance is estimated using the formula for the capacitance of a parallel wire transmission line:

$$C(F/m) = \frac{K}{36 \ln\left(\frac{D}{r}\right)} \times 10^{-9} \quad (39)$$

where

K = dielectric content of material around conductor
 D = line center to center spacing (m)
 r = conductor radius (m)

Applying equation 39 to the loop geometry,

$$C_L(F) = \frac{(N-1) PK}{36 \ln\left(\frac{D}{r}\right)} \times 10^{-9} \quad (40)$$

where

N = number of loop turns
 P = loop perimeter (m)

Based on equation 40, the calculated loop capacitance for the above loop of number 14 gauge wire with a wire diameter of 1.628mm, polyethylene insulation, with dielectric constant of 2.5, and center to center spacing of 3.42mm is 708 pF. When a loop is installed in a slot in the pavement surface, voids in the sealing material could fill with water causing an increase in loop capacitance. This effect could result in unstable loop operation.

Loop Quality Factor

The quality factor, Q_L of an inductive loop is defined by

$$Q_L = \frac{\omega W_s}{P_L} \quad (41)$$

where

W_s = peak value of stored energy in magnetic field
 P_L = average power in the loop wire resistance and pavement material

The peak value of energy stored in the loop is:

$$W_s = \frac{1}{2} L_L I_{\text{peak}}^2 \quad (42)$$

The average power dissipated is:

$$P_L = \frac{1}{2} I_{\text{peak}}^2 R_L \quad (43)$$

where R_L is total loop resistance in ohms.

The quality factor, Q_L , for the loop is:

$$Q_L = \frac{\omega L_L}{R_L} \quad (44)$$

Tables I and II present measured loop quality factors. Note increased ground resistance at higher frequencies causes the quality factor to decrease in value.

Table I. Measured inductive loop parameters with vehicle not present.

f_o (KHz)	f_2 3db(KHz)	f_1 3db(KHz)	$Q_1 = \frac{f_o}{f_2 - f_1}$	C_1 (μF)	$L_1 = \frac{1}{(2\pi f_o)^2 C_1}$ (μH)	$R^1 = \frac{2\pi f_o L_1}{Q_1}$ (Ω)
20	20.334	19.702	31.7	0.856490	73.9	0.29
25	25.372	24.688	35.5	0.548109	73.9	0.33
30	30.388	29.644	40.3	0.380000	74.1	0.35
35	35.427	34.607	42.7	0.278693	74.2	0.38
40	40.470	39.573	44.6	0.212056	74.3	0.42
45	45.514	44.530	45.7	0.167887	74.5	0.46
50	50.571	49.472	45.5	0.135704	74.7	0.52
55	55.633	54.407	44.9	0.111845	74.9	0.58
60	60.704	59.342	44.1	0.093450	75.3	0.64
65	65.783	64.264	42.8	0.079329	75.6	0.72
70	70.886	69.154	40.4	0.061825	75.9	0.83
75	76.050	74.024	37.0	0.059101	76.2	0.97
80	81.194	78.844	34.0	0.051682	76.6	1.13
85	86.324	83.711	32.5	0.045634	76.8	1.26
90	91.532	88.534	30.0	0.040486	77.2	1.46
95	96.690	93.355	28.5	0.036172	77.6	1.63
100	101.882	98.174	27.0	0.032464	78.0	1.82
105	107.172	102.885	24.5	0.029300	78.4	2.11
110	112.388	107.680	23.4	0.026538	78.9	2.33

Comments:

Loop Size: 1.83m (6ft) by 1.83m (6ft).

Loop Number of Turns: 3.

Loop Wire Size: 14 Gauge.

Loop Lead in Length: 1.52m (60in).

Loop Self-Resonant Frequency: 697.06 kHz

Note: 50 pf residual capacitance in decade box neglected in C_1 value.

Table II. Measured inductive loop parameters with vehicle centered over loop.

f_o^v (KHz)	f_2^v 3db(KHz)	f_1^v 3db(KHz)	$Q_v = \frac{f_o^v}{f_2^v - f_1^v}$	C_v (μF)	$L_v = \frac{1}{(2\pi f_o^v)^2 C_v}$ (μH)	$R_v = \frac{2\pi f_o^v L_v}{Q_v}$ (Ω)	$\Delta L = L_1 - L_v$ (μH)	$S_L = 100 \frac{\Delta L}{L_2}$ (%)	θ_v (deg)
20	20.392	19.651	27.0	0.940236	67.4	0.31	6.5	9.0	-65.5
25	25.435	24.608	30.2	0.602169	67.3	0.35	6.6	9.0	-67.7
30	30.480	29.558	32.5	0.417769	67.4	0.39	6.7	9.0	-69.8
35	35.512	34.522	35.4	0.306880	67.4	0.42	6.8	9.0	-70.8
40	40.560	39.480	37.0	0.234615	67.5	0.46	6.8	9.0	-71.6
45	45.603	44.440	38.7	0.184988	67.6	0.49	6.9	9.0	-72.0
50	50.666	49.373	38.9	0.149662	65.4	0.55	7.0	9.0	-72.3
55	55.729	54.309	38.7	0.128095	68.0	0.58	9.5	13.0	-72.4
60	60.810	59.237	38.1	0.103450	68.4	0.67	7.3	10.0	-72.2
65	65.900	64.144	37.0	0.087676	68.6	0.76	7.2	10.0	-72.0
70	71.040	69.045	35.1	0.075329	68.9	0.86	7.3	10.0	-71.5
75	76.180	73.874	32.5	0.065392	69.1	1.00	7.3	10.0	-70.8
80	81.315	78.728	30.9	0.057248	69.4	1.12	7.5	10.0	-70.0
85	86.493	83.550	27.9	0.050590	69.6	1.33	7.4	10.0	-69.2
90	91.614	88.441	28.4	0.044934	69.9	1.39	7.6	10.0	-68.2
95	96.773	93.264	27.1	0.040154	70.2	1.54	7.7	10.0	-67.4
100	102.000	98.090	25.6	0.036097	70.2	1.72	7.8	10.0	-66.4
105	107.260	102.810	23.6	0.032581	70.5	1.97	7.9	10.0	-65.4
110	112.447	107.622	22.8	0.029558	70.8	2.15	8.1	10.0	-64.3

Comments:

Loop Size: 1.83m (6ft) by 1.83m (6ft).

Loop Number of Turns: 3.

Loop Wire Size: 14 Gauge.

Loop Lead in Length: 1.52m (60in).

Loop Self-Resonant Frequency: 697.06 kHz

Note: 50 pf residual capacitance in decade box neglected in C_1 value.

Inductive Loop Transmission Lines

Most states use shielded, twisted pair type transmission lines to connect the loop to roadside detector electronics although Dorsey found some States which use nonshielded underground feeder, UF, type cable. Table III presents a comparison of shielded and nonshielded cable.

The transmission line is assumed to be less than one eight wave length long so that the transmission line can be modeled as a series inductance and resistance with shunt capacitance. Figure 5 is the inductive loop system circuit model. The inductance of the transmission line, which is proportional to cable length, reduces the sensitivity of frequency and period shift type vehicle detector on electronics.

Nonshielded Single Pair Transmission Line Inductance

The inductance per unit length of an unshielded, two conductor transmission line of the UF type is:

$$L_c^{NS} = \frac{\mu_0}{\pi} \ln \left(\frac{2D}{d} \right) \quad (45)$$

where

L_c^{NS} = nonshielded transmission line inductance (Hy/m)

$\mu_0 = 4\pi \times 10^{-7}$ (Hy/m)

D = conductor center to center spacing (mm)

d = conductor diameter (mm)

In terms of micro Henries per meter

$$L_c^{NS} = 0.4 \ln \left(\frac{2D}{d} \right) (\mu \text{ Hy/m}) \quad (46)$$

Shielded Single Pair Transmission Line Inductance

The inductance per unit length of a shielded single pair transmission line is derived from the formula for the velocity of propagation of a low loss transmission line and is:

$$L_c^S = \frac{100 \epsilon_r}{9C_c} \quad (47)$$

where

L_c^S = shielded transmission line inductance ($\mu \text{ Hy/m}$)

ϵ_r = dielectric constant

C_c = cable capacitance per unit length (pF/m)

For example, Belden type 8720 shielded cable uses polyethylene as an insulator with an estimated dielectric constant of 2.3. The measured capacitance¹¹ in the balanced mode is 79 pF/m. The calculated inductance based on equation 47 is 0.32 $\mu \text{ H/m}$, (0.1 $\mu \text{ H/ft.}$)

When one conductor is connected to the shield, an unbalanced mode results and the shielded cable approximates a coaxial cable. The measured capacitance¹² in the unbalanced mode is 154 pF/m. The calculated inductance using equation 47 is 0.17 $\mu \text{ H/m}$, (0.05 $\mu \text{ H/ft.}$)

The inductance per unit length of a shielded single pair transmission line, when the cable capacitance per unit length is unknown, is calculated from the cable geometry and is:

$$L_c^S = 0.4 \ln \left(1.2 + \frac{2.4 t}{d} \right) \quad (48)$$

where

t = conductor insulation thickness (mm)

d = conductor diameter (mm)

Table III. Comparison* of Transmission Line Types

Cable Type	Advantages	Disadvantages
Shielded, Twisted Pair	a) The shield reduces interference from external electric fields b) The shield reduces the inductance of transmission line which improves sensitivity of period shift type digital/detectors. c) The cable twist reduces interference from external magnetic fields which reduces crosstalk from nearby cables. d) The stranded conductor makes the cable more flexible.	a) The shield increases cable capacitance b) The capacitance can change due to water penetrating the jacket. c) The cable jacket can tear in pulling through conduit.
UF Cable	a) The cable can be run in conduit or direct burial. b) The cable has a tough molded jacket that reduces water penetration. c) The cable has less cable capacitance.	a) The cable is subject to interference from electric and magnetic fields since the cable conductors are neither twisted nor shielded.

*See table 9 of Dorsey Report

Equation 48 is derived in Appendix II. For Belden type 8720 cable, the conductor insulation thickness is 0.81mm and conductor diameter is 1.8mm. The calculated inductance using equation 48 is 0.32 $\mu\text{H/m}$, (0.1 $\mu\text{H/ft.}$)

Selection of Transmission Line for Minimum Inductance per Unit Length

To maximize detection sensitivity of frequency and period shift type vehicle detector electronics, a transmission line with minimum inductance per unit length should be selected. The inductance per unit length of a shielded single pair cable is proportional to the t/d ratio in equation 48. Hence, a minimum t/d ratio should be selected. For temperature stability, a cable with polyethylene dielectric and outer jacket is desirable with minimum t/d ratio.

Effect of Transmission Line on Detector Tuning

Most ILD electronics use a parallel resonant "tank" circuit where the inductance of the "tank" circuit is provided by the apparent loop inductance and the capacitance of the "tank" circuit is provided by the tuning capacitance. Manufacturers typically specify the input inductance range of vehicle detector electronics. The apparent detector inductance is calculated by applying equation 37 to the sum of the apparent loop inductance and series cable inductance and is:

$$L_D^a = \frac{L_L^a + L_C l}{1 - 4\pi^2 f^2 C_c l [L_L^a + L_C l]} \quad (49)$$

L_D^a = apparent inductance at detector electronic (Hy)

L_L^a = apparent inductance of loop (Hy)

L_C = cable inductance (Hy/m)

C_c = cable capacitance (Fd/m)

f = operating frequency (Hz)

l = cable length (m)

Vehicle Detector Electronics

Loop detector electronics sense the change in electrical parameters at the electronics or detector input terminals. The sensitivity of all types of detector electronics is proportional to the normalized inductance change at the detector terminals. The quality factor also affects detector sensitivity. Types of detector electronics are:

- . Analog phase shift
- . Analog frequency shift
- . Digital frequency shift
- . Digital period shift

The analog phase shift detector which is described by Anderson¹³ produces a dc level proportional to phase shift. The dc level is threshold detected to actuate a output relay driver.

The analog frequency shift detector produces a phase shift proportional to the difference between a loop oscillator and reference oscillator. A dc level proportional to phase shift is threshold detected to actuate an output relay driver. The threshold reference voltage may track slowly varying changes in the loop oscillator to prevent false alarm. This type of detector is similar to a phase locked loop, PLL.

The digital frequency shift detector counts the frequency of the loop oscillator and compares the frequency count with a reference count in memory which is periodically updated. When the loop count exceeds the reference count, a detector output is generated.

The digital period shift detector is described by Hulscher¹⁴ and produces counts proportional to the period of the loop oscillator. When the loop period count exceeds the reference period count by a selected number, a detector output is generated. The reference count is periodically updated.

Phase Shift Detector Sensitivity

The phase shift, $\Delta\theta_D$, of a parallel resonant "tank" circuit due to a change in "tank" circuit inductance is derived in Appendix III and is:

$$\Delta\theta_D(\text{rad}) = \tan^{-1} \left[\frac{S_D Q_D \left(1 + \frac{S_D}{Q_D^2} - \frac{1}{Q_D^2} \right)}{1 + \frac{1}{Q_D^2}} \right] \quad (50)$$

where

$$Q_D = \frac{\omega L_D}{R_D} \quad (51)$$

and

$$S_D = \frac{\Delta L_D}{L_D} \quad (52)$$

For $Q_D > 5$

$$\Delta\theta_D(\text{rad}) = \tan^{-1}(S_D Q_D) \quad (53)$$

For $\Delta\theta_D(\text{rad}) \leq 0.2 \text{ rad}$

$$\Delta\theta_D(\text{rad}) = S_D Q_D \quad (54)$$

$$\Delta\theta_D(\text{deg}) = 57.3 S_D Q_D \quad (55)$$

This equation shows that the phase shift for a phase shift type detector is proportional to both detector sensitivity, S_D , and detector quality factor.

Phase Shift Detector System Sensitivity

The phase shift, $\Delta\theta_D$, at the detector terminals is required as a function of loop and cable parameters.

$$\text{Since } R_D = R_L + R_C \quad (56)$$

$$\text{and } L_D = L_L + L_C \quad (57)$$

$$\Delta L_D = \Delta L_L \quad (58)$$

$$\text{then } \Delta\theta_D(\text{deg}) = 57.3 \left(\frac{S_L Q_L}{1 + \frac{R_C}{R_L}} \right) \quad (59)$$

where $\Delta\theta_D \leq 10 \text{ deg}$ and $Q_L > 5$

This equation shows that cable resistance, R_C , is important relative to phase detection sensitivity. If the cable resistance is one tenth or less the loop resistance, R_L , the cable has negligible effect ILD system sensitivity. This implies that a

cable should be selected with large diameter conductors. The loop resistance, R_L , contains both loop wire resistance and ground resistance due to currents in pavement and subgrade material which will change with moisture content of pavement and subgrade materials.

Frequency Shift Detector Sensitivity

The frequency shift, Δf_D , of a parallel resonant "tank" circuit due to a change in "tank" circuit inductance is derived in Appendix IV and is:

$$S_D = \frac{1}{2} \left(\frac{f_V^2 - f_{NV}^2}{f_V^2} \right) \left(1 + \frac{1}{Q_D^2} \right) \left(1 + \sqrt{1 - \frac{4}{Q_D^2(1 + \frac{1}{Q_D^2})}} \right) \quad (60)$$

where

- f_V = resonant frequency with vehicle present (Hz)
- f_{NV} = resonant frequency with no vehicle present (Hz)
- Q_D = quality factor at detector
- S_D = detector sensitivity

For $Q_D > 5$

$$S_D = \frac{f_V^2 - f_{NV}^2}{f_V^2} = \left(\frac{f_V - f_{NV}}{f_V} \right) \left(\frac{f_V + f_{NV}}{f_V} \right) \quad (61)$$

For small frequency shifts

$$f_V = f_{NV} \text{ and } Q_D > 5$$

$$S_L = 2 \left(\frac{f_V - f_{NV}}{f_{NV}} \right) \quad (62)$$

Frequency Shift Detector System Sensitivity

The frequency shift, Δf_D , at the detector terminals is required as a function of loop and cable parameters.

$$\text{Let } f_V - f_{NV} = -\Delta f_D \quad (63)$$

$$\text{and } f_{NV} = f_D \quad (64)$$

$$\text{Then } \frac{\Delta f_D}{f_D} = -\frac{1}{2} S_D \quad (65)$$

$$\text{Since } L_D = L_L + L_C \quad (66)$$

$$\text{and } \Delta L_D = \Delta L_L \quad (67)$$

$$\frac{\Delta f_D}{f_D} = -\frac{1}{2} S_L \left(\frac{1}{1 + \frac{L_C}{L_L}} \right) \quad (68)$$

This equation shows that the cable inductance, L_C , is important relative to frequency shift detection sensitivity. If the cable inductance is one tenth or less of the loop inductance, the transmission line has a negligible effect on ILD sensitivity provided the quality factor, Q_D , is five

or greater. The frequency shift detector system sensitivity results also applies to period shift detector systems.

Frequency Shift Detector System Sensitivity Example

The ILD electronics are operated at a detector sensitivity of 0.16 percent to minimize detector time response and are connected to a 1.83m (6ft) by 1.83m (6ft), three turn inductive loop of number 14 AWG wire by 99m (324ft) of Belden 8720 transmission line cable. The loop is installed in asphalt pavement. What is the maximum vehicle undercarriage height that can be detected? If the loop is installed in concrete pavement and spaced 1.3cm (0.5in) from reinforcing steel mesh, what is the maximum vehicle undercarriage height that can be detected?

Neglecting the effect of loop capacitance, the loop inductance from equation 23 is 78.9 μ H. From equation 47 the cable inductance is 31.7 μ H. From equations 65 and 68 the loop sensitivity is 0.22 percent. From Figure 4, a vehicle with an undercarriage height of 1.16m (5.8ft) or less will be detected with the loop in asphalt pavement. From Figure 4, a vehicle with an undercarriage height of 0.74m (2.43ft) or less will be detected with the loop in concrete pavement and spaced 1.3cm (0.5in) from a steel reinforcing mesh.

Results and Conclusions

The material above presents an approximate model of an ILD system. The model can be extended to include arbitrary vehicle dimensions and positions relative to the loop, various loop sizes and shapes, exact geometry of reinforcing steel, and long transmission lines. Reinforcing steel 5cm (2in) from a buried loop in concrete can halve the sensitivity of the same loop buried in asphalt. Moisture in pavement and subgrade materials can increase the loss tangent of the material resulting in an increase in the loop ground resistance. This increased loop resistance reduces the ILD system quality factor which, if less than five can reduce sensitivity of frequency shift or period shift ILD electronics.

No measurements were conducted to validate the accuracy of the effect of reinforcing steel on loop sensitivity in Figure 4. A 1974 Mercury station wagon with an average undercarriage height of 0.15m (0.5ft) was centered over the loop in asphalt to collect the loop sensitivity data in Table II. The measured loop sensitivity of 9 to 10 percent compares favorably with the 9 percent loop sensitivity from Figure 4 for the no steel case.

Incomplete sealing of the loop wires in the pavement can allow water to change the loop capacitance resulting in variation in loop inductance. Unstable detector operation may result.

For frequency shift and period shift ILD electronics, transmission lines should be selected with low inductance. For phase shift ILD electronics, transmission lines should be selected with low resistance.

A transformer with low leakage inductance (i.e., total series leakage inductance less than transmission line inductance) can be placed between the loop and transmission line to transform the loop inductance to a value larger than the transmission line inductance. The transformer will remove the reduction in sensitivity caused by the transmission line.

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APPENDIX I

LOOP GROUND RESISTANCE DERIVATION

The complex impedance, $\hat{Z}_L$, of the loop results from a complex permeability, $\hat{\mu}_g$, and is:

$$\hat{Z}_L = j \omega \hat{\mu}_g L_L \quad (I-1)$$

$$= j \omega (\mu_g' - j \mu_g'') L_L \quad (I-2)$$

The material loss tangent, $\tan \delta_g$, is:

$$\tan \delta_g = \frac{\mu_g''}{\mu_g'} \quad (I-3)$$

Letting $\mu_g' = 1$

The loss tangent is

$$\tan \delta_g = \mu_g'' \quad (I-4)$$

Then

$$\hat{Z}_L = j \omega (1 - j \tan \delta_g) L_L \quad (I-5)$$

$$\hat{Z}_L = \omega \tan \delta_g L_L + j \omega L_L \quad (I-6)$$

$$\hat{Z}_L = R_g + j X_L \quad (I-7)$$

$$R_g = \tan \delta_g \omega L_L \quad (I-8)$$

APPENDIX II

SHIELDED SINGLE PAIR TRANSMISSION LINE INDUCTANCE DERIVATION

From reference 15, the characteristic impedance for a shielded, two conductor transmission line operated in a balanced mode is:

$$Z_0 = \frac{120}{\sqrt{\epsilon}} \ln \left\{ 2 \sqrt{\frac{(1 - \sigma^2)}{(1 + \sigma^2)}} \right\} \quad (II-1)$$

$$\text{where } \sigma = h/d \quad (II-2)$$

$$\sigma = \frac{h}{D} \quad (II-3)$$

and

h = center conductor center to center spacing (m)

D = shield diameter (m)

d = center conductor diameter (m)

The velocity of propagation, v , is given by

$$v = \frac{1}{\sqrt{L' C'}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} \quad (II-4)$$

and the characteristic impedance is:

$$Z_0 = \sqrt{\frac{L'}{C'}} \quad (II-5)$$

Then using II-4 and II-5

$$L' = 120 \pi Z_0 \epsilon_0 \sqrt{\epsilon_r} \quad (II-6)$$

where $L' = \text{inductance per unit length (H/m)}$
 $Z_0 = \text{characteristic impedance (ohms)}$
 $\epsilon_0 = \text{permittivity of free space}$
 $(\frac{1}{36\pi} \times 10^{-9} \text{ F/m})$
 $\epsilon_r = \text{dielectric constant}$

Using II-1

$$L_C^s = 0.4 \ln \left\{ 2 \ln \left[\frac{1-\sigma^2}{1+\sigma^2} \right] \right\} \quad (\text{II-7})$$

where $L_C^s = \text{inductance per unit length } (\mu\text{H/m})$

For the cable geometry in Figure II-1

$$D_S = 2d + 4t \quad (\text{II-8})$$

$$h = d + 2t \quad (\text{II-9})$$

Then

$$\nu = 1 + \frac{2t}{d} \quad (\text{II-10})$$

$$\sigma = 1/2$$

$$L_C^s = 0.4 \ln (1.2 + \frac{2.4t}{d}) \quad (\text{II-11})$$

APPENDIX III

Phase Shift Detector Sensitivity

The input admittance of a parallel tuned circuit is:

$$Y(j\omega) = G + jB = \frac{1}{R_p} + j\omega(C_p - \frac{1}{\omega^2 L_p}) \quad (\text{III-1})$$

Using series to parallel transformation

$$Y(j\omega) = \frac{R_s}{R_s^2 + \omega^2 L_s^2} + j\omega(C_p - \frac{L_s}{R_s + \omega^2 L_s^2}) \quad (\text{III-2})$$

$$\Theta = \tan^{-1} \frac{B}{G} = \tan^{-1} \left\{ \frac{\omega(C_p(R_s^2 + \omega^2 L_s^2) - L_s)}{R_s} \right\} \quad (\text{III-3})$$

$$\text{Let } \alpha = \frac{\Delta L_s}{L_s}, \text{ then } \alpha L_s = \Delta L_s \quad (\text{III-4})$$

$$\text{and } L_s + \Delta L_s = (1 + \alpha)L_s \quad (\text{III-5})$$

$$\tan(\Theta + \Delta\Theta) = \frac{\omega \{ C_p [R_s^2 + \omega^2 (1 + \alpha)^2 L_s^2] - (1 - \alpha)L_s \}}{R_s} \quad (\text{III-6})$$

At resonance

$$X_c = X_{Lp} \quad (\text{III-7})$$

$$\frac{1}{\omega C} = \omega L_s (1 + \frac{1}{Q^2}) \quad (\text{III-8})$$

$$C_p = \frac{1}{\omega^2 L_s (1 + \frac{1}{Q^2})} \quad (\text{III-9})$$

$$Q = \frac{\omega L_s}{R_s} \quad (\text{III-10})$$

Then

$$\tan(\Theta + \Delta\Theta) = \frac{\alpha Q (1 + \frac{\alpha}{Q^2} - \frac{1}{Q^2})}{1 + \frac{1}{Q^2}} \quad (\text{III-11})$$

At resonance $\Theta = 0^\circ$

$$\tan \Delta\Theta = \frac{Q (1 + \frac{\alpha}{Q^2} - \frac{1}{Q^2})}{1 + \frac{1}{Q^2}} \quad (\text{III-12})$$

$$\text{where } \alpha = \frac{\Delta L_D}{L_D} = S_D \quad (\text{III-13})$$

APPENDIX IV

Frequency Shift Detector Sensitivity

The input admittance of a parallel resonant circuit is:

$$Y_{in}(j\omega) = \frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p}) \quad (\text{IV-1})$$

At resonance the susceptance term is zero

$$\omega_o^2 = \frac{1}{C_p L_p} \quad (\text{IV-2})$$

The parallel inductance, L_p , in terms of series circuit inductance, L_s , and resistance, R_s , is:

$$L_p = \frac{R_s^2 + (\omega_o L_s)^2}{\omega_o^2 L_s} \quad (\text{IV-3})$$

Then

$$\omega_o^2 = \frac{1}{L_s C_p} - \frac{R_s^2}{L_s^2} \quad (\text{IV-4})$$

when no vehicle is present over the loop, let $L_s = L_o$, $R_s = R_o$

$$\omega_o^2 = \frac{1}{L_o C_p} - \frac{R_o^2}{L_o^2} \quad (\text{IV-5})$$

when a vehicle is present assuming no change in R_o

$$\omega_v^2 = \frac{1}{L_v C_p} - \frac{R_o^2}{L_v^2} \quad (\text{IV-6})$$

$$L_o = \frac{1}{2 \omega_o^2 C} (1 + \sqrt{1 - 4 R_o^2 C^2 \omega_o^2}) \quad (\text{IV-7})$$

$$L_v = \frac{1}{2 \omega_v^2 C} (1 + \sqrt{1 - 4 R_o^2 C^2 \omega_v^2}) \quad (\text{IV-8})$$

and

$$C = \frac{1}{L_o \omega_o^2 (1 + \frac{1}{Q^2})} \quad (\text{IV-9})$$

since

$$\Delta L_o = L_o - L_v \quad (\text{IV-10})$$

and

$$S_D = \frac{\Delta L_o}{L_o} \quad (\text{IV-11})$$

$$S_D = \frac{1}{2} \left(\frac{\omega_v^2 - \omega_o^2}{\omega_v^2} \right) \left(1 + \frac{1}{Q^2} \right) \left(1 + \sqrt{1 - \frac{4}{Q^2 (1 + \frac{1}{Q^2})^2}} \right) \quad (\text{IV-12})$$