Group-Orthogonal OFDMA in Fast Time-Varying Frequency-Selective Fading Environments

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Abstract: This paper presents a group-orthogonal OFDMA (GO-OFDMA) suitable for broadband access systems in a fast time-varying frequency-selective fading environment when channel knowledge is not available at the transmitter. Sub-carrier grouping to achieve the diversity gain of a time-domain Rake receiver is discussed and the closed-form expression of its bit-error-rate (BER) performance over a frequency-selective Rayleigh fading channel is derived. The proposed GO-OFDMA scheme uses a split-and-group structure and a maximum-likelihood (ML) multi-user detection (MUD) to increase the number of supportable active users and to reduce the peak-to-average ratio (PAR). Analytical and simulation results are in an excellent agreement. Performance evaluation indicates that the proposed GO-OFDMA provides a lower PAR and similar BER as compared with the group-orthogonal multi-carrier CDMA (GO-MC-CDMA), and outperforms the random-hopping (RH)-OFDMA and matched-filter based MC-CDMA.

I. INTRODUCTION

Orthogonal-frequency-division multiple-access (OFDMA) scheme has recently attracted considerable attention in broadband wireless access for multimedia services [1], [2]. In OFDMA, the system bandwidth is divided into a large number of orthogonal overlapping narrow-band sub-carriers that can be shared among multiple users. With a sufficiently narrow band, each sub-carrier “sees” only *flat* fading even when frequency-selective fading occurs over the entire system band. Accordingly, bandwidth and power-efficient modulation and coding schemes adapted to available signal-to-noise ratios (SNR) in different sub-carriers can be applied. A cyclic prefix guard time longer than the channel delay spread is inserted in the transmitted signals to maintain the orthogonality among the sub-carriers [3]. Nevertheless, deep fades can occur and create extremely low SNR in one or more sub-carriers, causing data losses and performance degradation. In this situation, diversity techniques can be applied to enhance the system performance. One effective diversity approach is to transmit the same signal over multiple sub-carriers. The required number of sub-carriers for full diversity gain has to be not less than the channel order [4], [5]. Those multiple sub-carriers can be shared by many users for high bandwidth utility and the resulting multiple-access-interference (MAI) can be alleviated in a generalized MC-CDMA (GMC-CDMA) structure [4] or a mutually-orthogonal user code-receiver (AMOUR)

structure [6]. Both structures relied on block spreading under the assumption of time-invariant channel condition over the transmitted block period. The block spreading can be made channel independent [7]. In a time-invariant or slow time-varying channel, with channel knowledge available to the transmitter, group-optimal adaptive-tone-diversity (GO-ATD) technique [8] can be used to enhance the utility of available communications resources. For fast time-varying frequency-selective channels, it is difficult to obtain accurate and up-to-date channel knowledge at the transmitter. A group-orthogonal (GO) MC-CDMA was proposed [9], where all the sub-carriers were subdivided into orthogonal groups and the multi-user detection (MUD) was used to mitigate the MAI that existed in a group of users. A large number of sub-carriers needed to obtain the diversity gain increases the peak-to-average power ratio (PAR), implying low transmitter power efficiency due to large required amplifier back-off to maintain good linearity.

In this paper, we consider techniques to provide sub-carrier diversity for OFDMA in fast time-varying frequency-selective fading channels with emphasis on improving number of active users and PAR. Transmitters are assumed to have no prior channel knowledge. We will first examine the sub-carrier grouping to achieve the same diversity gain as the time-domain Rake receiver and derive the closed-form expression of the BER performance of an OFDMA system using M-ary QAM and sub-carrier grouping diversity receiver in a frequency-selective Rayleigh fading environment. The analytical results are shown in an excellent agreement with the simulation ones. To increase the number of supportable users and reduce the PAR, we propose a group-orthogonal (GO)-OFDMA scheme, in which an user data stream is first split into multiple sub-streams, spread by distinct Walsh codes, and finally re-grouped for transmission over the selected sub-carriers. Computer simulation results indicate that the proposed GO-OFDMA scheme has a similar performance to the GO-MC-CDMA and better PAR. The proposed GO-OFDMA scheme outperforms the RH-OFDMA and matched-filter based MC-CDMA.

II. SUB-CARRIER GROUPING & PERFORMANCE ANALYSIS

We consider an uplink quasi-synchronous OFDMA with N available tones shared by K active users [8]. The transmitted signal for user k and symbol interval n is

$$s_{k,n}(t) = \sum_{i=0}^{N+L_{CPI}-1} s_{k,n,i} P_T(t - iT) \quad (1)$$

where T is the sampling interval, P_T is the unit rectangular pulse of duration T , L_{CPI} is the length of the cyclic prefix insertion (CPI), $\{s_{k,n,l} \doteq s_{k,n,N+l}, l = 0, 1, \dots, L_{CPI}-1\}$ are the inserted cyclic prefix, and $\{s_{k,n,i+L_{CPI}}\}$ are the output samples from the inverse discrete Fourier transform (IDFT),

$$s_{k,n,i+L_{CPI}} = \mathcal{A}_k \sum_{m=0}^{N-1} S_{k,n,m} C_{k,n,m} e^{j 2 \pi m i / N} \quad (2)$$

$i = 0, 1, \dots, N-1$

where $\mathcal{A}_k \doteq \sqrt{E_k / (N_o N_p)}$ is the signal amplitude, E_k is the transmitted symbol energy for user k , N_o is the white Gaussian noise power spectral density, and $N_p = N + L_{CPI}$. $S_{k,n,m}$ is the n -th symbol from user k at tone m , and with unit energy. $C_{k,n,m}$ is an index variable for tone allocation among the K active users. It is set to 1 if the m -th tone is allocated for user k and symbol n , and set to 0 otherwise. For any given n and m , only one $C_{k,n,m}$ is set to 1 to avoid collisions.

At the receiver, signals from K active users arrive with different time delays $\tau_{k,l}$ and experience different multipath fading $\alpha_{k,l}$. The received samples can be written as

$$V_{n,i} = \sum_{k=1}^K \sum_{l=0}^{L_t-1} \beta_{k,l} s_{k,n,i-l} + \eta_{n,i} \quad (3)$$

$i = L_{CPI}, L_{CPI}+1, \dots, L_{CPI}+N-1$

where $\eta_{n,i}$ is zero-mean, unit-variance Gaussian distributed random variable; $\{\beta_{k,l}\}$, the equivalent fading parameters with sample-spacing delay profile, are transformed from $\{\alpha_{k,l}\}$ which have arbitrary time-spacing delay profile. The channel order L_t is the maximum value of $\lfloor v_{k,l} + 2 \rfloor$ over $l = 1, 2, \dots, L_{k,t}$ for user k with $v_{k,l} \doteq \lfloor \tau_{k,l} / T \rfloor$.

The DFT can be conducted on $\{V_{n,i}\}$ in order to extract the transmitted information-bearing symbol over each tone to obtain

$$Y_{n,m} = \sum_{k=1}^K \mathcal{A}_k N [H_{k,m} \cdot S_{k,n,m} \cdot C_{k,n,m}] + W_{n,m} \quad (4)$$

where $Y_{n,m} \doteq \sum_{i=0}^{N-1} V_{n,i+L_{CPI}} e^{-j 2 \pi i m / N}$, $H_{k,m} \doteq \sum_{l=0}^{L_t-1} \beta_{k,l} e^{-j 2 \pi l m / N}$, and $W_{n,m} \doteq \sum_{i=0}^{N-1} \eta_{n,i+L_{CPI}} e^{-j 2 \pi i m / N}$, denote the m -th DFT of $\{V_{n,i}\}$, $\{\beta_{k,l}\}$, and $\{\eta_{n,i}\}$, respectively. For statistically independent and identically distributed (i.i.d.) Gaussian noise samples $\{\eta_{n,i}\}$ with zero mean and unit variance, the DFT transformed noise components $\{W_{n,m}\}$ are also i.i.d. but with variance N . As shown in [5], by forming a sub-carrier group of L_f sub-carriers evenly distributed in the system spectrum, a diversity gain equivalent to that of a time-domain Rake receiver can be achieved with $L_t \leq L_f$. For convenience, L_f is selected to be divisible by N so that $K = N / L_f$. The sub-carriers occupied by user k ($0 \leq k \leq K-1$) have indices $\{m_i = k + iK, i = 0, 1, \dots, L_f-1\}$.

A frequency-domain Rake receiver in [10] is adopted to combine the L_f received samples $\{Y_{n,m_i}\}$. By assuming perfect channel knowledge at the receiver, the resulting decision

variable can be represented by

$$D_k = \sum_{i=0}^{L_f-1} H_{k,m_i}^* Y_{n,m_i} = \sum_{i=0}^{L_f-1} H_{k,m_i}^* W_{n,m_i} + \frac{1}{\sqrt{L_f}} \mathcal{A}_k N S_{k,n,m} \sum_{i=0}^{L_f-1} |H_{k,m_i}|^2 \quad (5)$$

where $1/\sqrt{L_f}$ is the normalized factor applied to keep the transmitted signal power constant. W_{n,m_i} is the noise component, C_{k,n,m_i} is 1, and the transmitted symbol $S_{k,n,m}$ is taken out from the summation since it is the same for all m_i . Then the signal-to-noise ratio (SNR) of user k with equal-power distribution over L_f tones, denoted by γ_k , is

$$\begin{aligned} \gamma_k &= \frac{1}{L_f} \mathcal{A}_k^2 N \sum_{i=0}^{L_f-1} |H_{k,m_i}|^2 \\ &= \frac{N}{N_p} \frac{E_k}{N_o} \sum_{l=0}^{L_t-1} |\beta_{k,l}|^2 \end{aligned} \quad (6)$$

where $\{\beta_{k,l}\}$ are the equivalent fading parameters with sample-spacing delay profile. The above equation indicates that the received signal combines energies from all paths. The resulting SNR is equivalent to that achieved by the single-user CDMA Rake receiver in a system except a small energy loss due to the cyclic prefix insertion. This result confirms the idea of frequency spreading over the system band to mitigate the frequency-selective fading [1], and indicates that, for $L_t \leq L_f$, all $K = N / L_f$ users can accomplish the same error-rate performance of a single-user time-domain Rake receiver.

From Eq. (6), the average received SNR per bit can be calculated as

$$\begin{aligned} \bar{\gamma}_{b,k} &= \frac{E_{b,k}}{N_o} \frac{N}{N_p} \sum_{l=0}^{L_t-1} \overline{|\beta_{k,l}|^2} \\ &= \sum_{l=0}^{L_t-1} \bar{\gamma}_{b,k,l}. \end{aligned} \quad (7)$$

The instantaneous SNR of an individual path is a function of $|\beta_{k,l}|^2$. If $\beta_{k,l}$ are i.i.d. complex Gaussian variables with Rayleigh distributed amplitude, $|\beta_{k,l}|^2$ has a chi-square probability density function, and so does the instantaneous SNR of an individual path. Eq. (14.5-26) in [11] shows that the probability density function of the instantaneous SNR, $\gamma_{b,k}$, for $[L_t-1]$ independent paths with unequal powers is $p(\gamma_{b,k}) = \sum_{l=0}^{L_t-1} \frac{\pi_{k,l}}{\bar{\gamma}_{b,k,l}} \exp\left(-\frac{\gamma_{b,k}}{\bar{\gamma}_{b,k,l}}\right)$, where $\pi_{k,l} = \prod_{i=0, i \neq l}^{L_t-1} \frac{\bar{\gamma}_{b,k,i}}{\bar{\gamma}_{b,k,i} - \bar{\gamma}_{b,k,l}}$.

The conditional bit error probability for M-ary QAM signaling is in the form of [12]

$$P_b(e|\gamma_{b,k}) = \frac{1}{\log_2(I \cdot J)} \left(\sum_{a=1}^{\log_2 I} P_I(a) + \sum_{b=1}^{\log_2 J} P_J(b) \right) \quad (8)$$

where I and J are the dimensions of the corresponding PAM signals, and

$$P_I(a) = \frac{1}{I} \sum_{c=0}^{(1-2^{-a})I-1} \left\{ (-1)^{\lfloor \frac{c \cdot 2^{a-1}}{I} \rfloor} \left[2^{a-1} - \left\lfloor \frac{c \cdot 2^{a-1}}{I} + \frac{1}{2} \right\rfloor \right] \cdot \operatorname{erfc} \left[(2c+1) \sqrt{\frac{3 \log_2(I \cdot J) \gamma_{b,k}}{I^2 + J^2 - 2}} \right] \right\}$$

$$P_J(b) = \frac{1}{J} \sum_{c=0}^{(1-2^{-b})J-1} \left\{ (-1)^{\lfloor \frac{c \cdot 2^{b-1}}{J} \rfloor} \left[2^{b-1} - \left\lfloor \frac{c \cdot 2^{b-1}}{J} + \frac{1}{2} \right\rfloor \right] \cdot \operatorname{erfc} \left[(2c+1) \sqrt{\frac{3 \log_2(I \cdot J) \gamma_{b,k}}{I^2 + J^2 - 2}} \right] \right\}$$

The average bit error rate is then obtained by $P_b = \int_0^\infty P_b(e|\gamma_{b,k}) p(\gamma_{b,k}) d\gamma_{b,k}$. By evaluating the integral in the same way as Eq. (14.5-28) in [11], the following closed-form BER expression is obtained for M-ary QAM signaling over multipath frequency-selective Rayleigh fading channels:

$$P_b = \frac{1}{\log_2(I \cdot J)} \left\{ \sum_{a=1}^{\log_2 I} \frac{1}{I} \sum_{c=0}^{(1-2^{-a})I-1} [\Theta(c, a, I) \cdot \Omega(c, I, J)] + \sum_{b=1}^{\log_2 J} \frac{1}{J} \sum_{c=0}^{(1-2^{-b})J-1} [\Theta(c, b, J) \cdot \Omega(c, I, J)] \right\} \quad (9)$$

where

$$\Omega(a, b, c) = \sum_{l=0}^{L_t-1} \pi_{k,l} \left[1 - \sqrt{\frac{\bar{\gamma}_{b,k,l} \cdot 2\aleph(a, b, c)}{2 + \bar{\gamma}_{b,k,l} \cdot 2\aleph(a, b, c)}} \right] \quad (10)$$

$$\aleph(a, b, c) = \frac{3(2a+1)^2 \log_2(bc)}{b^2 + c^2 - 2} \quad (11)$$

$$\Theta(a, b, c) = (-1)^{\lfloor \frac{a \cdot 2^{b-1}}{c} \rfloor} \left[2^{b-1} - \left\lfloor \frac{a \cdot 2^{b-1}}{c} + \frac{1}{2} \right\rfloor \right] \quad (12)$$

It can be checked that, for $I=2$ and $J=1$ (BPSK signaling), Eq. (9) is reduced to Eq. (14.5-28) in [11].

III. GO-OFDMA SCHEME

OFDMA with sub-carrier grouping provides the diversity gain to mitigate the effect of frequency-selective fading, but reduces the supportable number of active users to $\lfloor N/L_f \rfloor$ and increases the peak-to-average power ratio (PAR) corresponding to L_f . To improve the system capacity and PAR, we develop the Group-Orthogonal OFDMA (GO-OFDMA) scheme to be presented in this section.

To increase the supportable number of active users, we consider the use of orthogonal Walsh sequences to allow several active users to share a sub-carrier group as follows. Data from an active user, U_i , is first split into $N_s \leq L_f$ sub-streams, $U_i S_j$'s. Each data sub-stream, $U_i S_j$, is spread by a distinct Walsh sequence of length L_f in frequency

domain, and transmitted over L_f sub-carriers in the same group. Within a sub-carrier group, the Walsh codes provide separation between sub-streams. Since the sub-carrier groups are orthogonal, the same Walsh code family can be re-used in every sub-carrier group. Fig. 1 illustrates the sub-carrier and sub-stream assignment of a GO-OFDMA system with $L_f = 4$ and $N_s = 2$. As the data sub-streams of the

Fig. 1. Sub-carrier and sub-stream assignment of GO-OFDMA

same user are transmitted over the same sub-carrier group, they experience the same channel condition. At the receiver, maximum-likelihood (ML) multi-user detection (MUD) can be used to recover user data in one sub-carrier group. Since all sub-carrier groups have the same instantaneous SNR as shown in Eq.(6), and hence the same performance, newly admitted users can be randomly allocated to the sub-carrier groups. It is desired to achieve an equal distribution of user data sub-streams in all sub-carrier groups to keep the same level of interference.

In general, the PAR is increased with the number of sub-carriers occupied by a user in multi-carrier transmission. The split-and-group structure of GO-OFDMA reduces the PAR by allowing user to send the sum of several spread data sub-streams. For example, Fig. 2 illustrates the difference between using one spreading sequence (case A) and using two spreading sequences (case B). For the same reference

Fig. 2. Difference in input signals by sending two sequences

transmitted power, in case B, the normalizing amplitude factor $1/\sqrt{2}$ is applied to the two spread data streams that are summed to form the inputs to the IFFT. Due to the properties of the two Walsh codes used in case B, half of the input signals to the IFFT is zero. This effectively reduces the number of sub-carriers in use by 50%, and hence its corresponding PAR, as compared to the user in case A with only one data stream.

IV. ILLUSTRATIVE RESULTS

We first examine the PAR reduction for various N_s by simulation. Table I compares the different PAR values of the

GO-OFDMA and the GO-MC-CDMA for various N_s . The results clearly show the proposed GO-OFDMA has reduced PAR as compared with the GO-MC-CDMA. The PAR reduction achieved by the GO-OFDMA is 2, 4, and 4.686 for $N_s=2, 4$, and 8, respectively.

TABLE I
REDUCED PAR FOR DIFFERENT N_s

L_f		4	8	16	32
GO-MC-CDMA		4	8	16	32
GO-OFDMA	$N_s = 2$	2	4	8	16
	$N_s = 4$	1	2	4	8
	$N_s = 8$	—	1.7071	3.4142	6.8284

Next, to verify Eq. (9) for higher order QAM signaling, we simulate the BER performance for 8, 16, 32 and 64 QAM over a two-tap Rayleigh channel with power profile [0,-6]dB. The curves and markers shown in Fig. 3 indicate a very good agreement between the analytical and simulation results for various M-ary QAM.

Fig. 3. BER performance with various M-ary QAM

As discussed in the previous section, the MUD can be used in the GO-OFDMA to achieve the maximum likelihood estimate. The resulting system performance is expected to approach that of a single-user receiver. However, as the number of sub-streams per user, N_s , affects the cross-correlation matrix R , the selection of N_s has an impact on the performance of the MUD. As some 'users' in GO-OFDMA have identical channel condition and their corresponding elements in R are highly correlated, the performance degradation is expected. In Fig. 4, the simulation results on the symbol error rate (SER) of GO-OFDMA for $L_f=8$, and $N_s=2, 4$ and 8, are shown and compared with the theoretical ones of a single-user Rake receiver. A 6-tap Rayleigh fading channel model with exponential decaying power profile (i.e., 0, -2.6, -5.2, -7.8, -10.4, -13.0dB) is used. As N_s increases, the system performance gets worse. On the other hand, larger N_s implies

lower PAR. Thus, when designing the value of N_s , the trade-off between PAR reduction and error rate performance should be considered.

Fig. 4. The effect of N_s on the error rate performance

To study further our user-loading algorithm, Fig. 5 compares the system symbol error rate (SER) to the theoretical SER of a single-user Rake receiver at different loading values. The simulation is performed with $E_s/N_0=12$ dB, $L_f=8$, $N_s=2$, and QPSK in an eight-tap Rayleigh fading channel model with exponential decaying power profile (i.e., 0, -1.7, -3.5, -5.2, -6.9, -8.7, -10.4, -12.2dB). The system SER shows little degradation as compared to the optimal case at all loading levels thanks to the use of the MUD for maximum likelihood estimation.

Fig. 5. SER performance of GO-OFDMA at different system load

Finally, we compare the performance of GO-OFDMA with other multi-carrier schemes, namely GO-MC-CDMA, Random-Hopping (RH)-OFDMA, and a matched filter based MC-CDMA, in a typical fixed-access broadband wireless

channel represented by the modified Stanford University Interim (SUI)-3 [13] with power profile of [0, -5, -10]dB, delay profile of [0, 0.4, 0.9] μ s and Rician fading K-factors of [1, 0, 0]. The sampling frequency is set to 5MHz with $L_f=8$ and $N_s=2$. QPSK modulation and 512-point FFT are considered. Without any data split, 256-point FFT are used in other multi-carrier schemes. Please note that even though the the GO-OFDMA is using twice of the FFT size than that used in other schemes, L_f is kept unchanged. The resulting SERs are shown in Fig. 6. The SER performance of GO-OFDMA is

Fig. 6. SER comparison of different multi-carrier systems

slightly worse than that of GO-MC-CDMA scheme but better than the RH-OFDMA and MC-CDMA. As SNR improves, the performance of GO-OFDMA approaches that of a single-user Rake receiver.

V. CONCLUSIONS

A group-orthogonal (GO) OFDMA scheme is proposed to provide diversity gain in a fast time-varying, frequency-selective fading environment when channel knowledge may not be available at the transmitter. Sub-carrier grouping to achieve the diversity gain of a time-domain Rake receiver was discussed and the closed-form expression of its bit-error-rate (BER) performance over a frequency-selective Rayleigh fading channel derived. The proposed GO-OFDMA scheme uses a split-and-group structure and a maximum-likelihood (ML) multi-user detection (MUD) to increase the number of supportable active users and to reduce the peak-to-average ratio (PAR). Analytical and simulation results are in an excellent agreement. Performance evaluation indicates that the proposed GO-OFDMA provides a lower PAR and similar BER as compared with the group-orthogonal multi-carrier CDMA (GO-MC-CDMA), and outperforms the random-hopping (RH)-OFDMA and matched-filter based MC-CDMA.

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