Wavelet Transform Signal Processing for Dispersion Analysis of Ultrasonic Signals

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ABSTRACT

The Wavelet Transform provides a new tool for analyzing the time-frequency evolution of transient signals as an alternative to the classical Short-Time Fourier Transform. The purpose of the present paper is to provide an overview of the applicability of the wavelet transform technique to the analysis of the propagation of dispersive ultrasonic waves. The wavelet transform is briefly introduced, with special emphasis on the relationship between the wavelet transform and the group velocity of dispersed signals. A complex mother wavelet is utilized to obtain the time evolution of the various spectral components of the ultrasonic signal, and the magnitude of the wavelet transform is used to represent the envelope of the ultrasonic pulse and to determine the time of arrival of the acoustic energy. This approach results in a time-scale representation of the ultrasonic signal which is extremely useful in the characterization of thin coatings using the dispersion behavior of the surface wave velocity. Numerical simulations and experimental results are presented to discuss the usefulness of the wavelet transform.

INTRODUCTION

Processing and analysis of a signal involves the division of a signal into different components which are used for extracting the information of interest. The most common technique used for analyzing transient nonstationary signals is the Short Time Fourier Transform, in which the signal is decomposed into its harmonic components. As an alternative, the Wavelet Transform (WT) can also be used to analyze and identify the various components of an ultrasonic signal.

The application of the WT in ultrasonics is discussed, with emphasis on the time-frequency analysis of dispersed signals. Since the type and amount of information extracted from the analysis is directly related to the choice of the mother wavelet, it is shown that an analytic mother wavelet is particularly suited to represent the envelope of the acoustic energy in time and frequency domain. The magnitude of the analytic wavelet transform is directly related to the rate of energy arrival, thus it is an optimal estimator for the measurement of group delay or velocity in ultrasonic analysis of dispersed signals.

In ultrasonics, the wavelet transform has been used to detect pulses buried in noise and also to measure dispersion of surface acoustic waves. The WT has been successfully utilized to enhance ultrasonic signal detection in presence of background noise, and this technique was successfully applied in ultrasonic flaw detection. In particular the change in measured time delay between two pulses has been calculated as a function of the SNR of the input signal. Numerical results show that the system is capable of performing accurate time measurements even with SNR of -15 dB[1]. Improvements in detection were also experimentally quantified by tests performed on steel samples of different thickness with flaws of variable sizes[2].

The technique here presented was applied for the measurement of elastic constants of chromium coatings on steel substrates using laser-generated surface acoustic waves. The time of arrival of the acoustic energy was utilized to determine the group velocity as a function of frequency of the wave and thus to characterize the variations in depth of the elastic properties of the coating/substrate system.

ANALYTIC WAVELET TRANSFORM IN ULTRASONICS

In the dispersion analysis of acoustic signals it is important to analyze the time-frequency evolution of the received ultrasonic transient signals, with as little distortion as possible. This is commonly achieved using signal processing techniques such as the Short-Time Fourier Transform (STFT) which provides accurate information about the signal simultaneously in the time domain and the frequency domain. In the STFT, the time variation of the spectral components of the signal is analyzed using a sliding window in time. The choice of the window determines the resolution in time domain and frequency domain of this analysis. The uncertainty principle states that the precise measurement of time and frequency are fundamentally incompatible, since sharp localization in time and frequency are mutually exclusive. Assuming that the window is localized in a time interval Δ and in a frequency band Ω, the product $\Delta \cdot \Omega$ is larger than C, where C is a constant depending on the units used. Thus, the choice of Δ and Ω affects very much the results obtained from the analysis.

The continuous wavelet transform (WT) is a new method of processing transient nonstationary signals simultaneously in time and frequency domain. Similar to the STFT, which utilizes modulation in time domain to translate the window in frequency, the WT uses scaling in time domain to scale a single function in frequency. This function, commonly referred to as the mother wavelet, is used to extract details and information in time and frequency domain, from the transient signal under analysis. This approach results in a more natural description of the signal, since the size of the window in the time domain is now function of the scaling.

By definition, the WT of a signal $s(t)$ is the correlation between the signal and a set of basic wavelets $h_{a,b}(t)$, and is expressed by the following relationship[3]:

$$W_s(a,b) = \int_{-\infty}^{\infty} s(t) \cdot h^*_{a,b}(t)\,dt = s(t) \otimes \frac{1}{\sqrt{a}}\, h^*\left(\frac{t}{a}\right) \quad (1)$$

The function $h(t)$, referred to as the mother wavelet, must satisfy the admissibility condition. The set of daughter wavelets $h_{a,b}(t)$ is generated by dilation/compression in time of $h(t)$[4].

The WT transform can be seen as a bank of filters which are constructed by dilation/compression of the single function $h(t)$. The filter constructed by the dilated version of the mother wavelet processes the low frequency information of the signal $s(t)$, and the one related to the compressed version of $h(t)$, analyzes the high frequency information. The output of the filter bank is thus related not only to the input signal $s(t)$ but also to the type of filters $h_{a,b}(t)$ that are utilized. By choosing a different mother wavelet, different characteristics of the input signal $s(t)$ can be obtained as output. When performing a time-frequency decomposition of a signal using either the STFT or the WT, no theoretical gain is expected from one method as opposed to the other, in the case all the information is used. But, if only partial information is retained in the analysis, such as using only the magnitude and not the phase and looking only at the output of particular filters, the choice of the technique and of the mother wavelet will affect the information that can be extracted. The flexibility of choosing the proper mother wavelet is one of the strongest advantages of using the WT, since the choice of the mother wavelet for a particular problem improves the signal processing capability of the technique. Tailoring of the wavelet to the actual problem is possible and should be done.

Scaling of the mother wavelet also results in a more appropriate decomposition of the various spectral components of transient non-stationary signals. In the STFT the width Δ of the time window is a constant, while in the WT it varies depending on the scaling a. Assuming that $h(t)$ represents a band-limited signal with a central frequency f_o and bandwidth Ω, a daughter wavelet $h_{a,b}(t)$ is a similar signal with center frequency $f_a = f_o / a$ and bandwidth $\Omega_a = \Omega / a$. *The relative bandwidth is constant for* every daughter wavelet and equal to the one of the mother wavelet:

$$\frac{\Omega_a}{f_a} = \frac{\Omega / a}{f_o / a} = \frac{\Omega}{f_o} = Constant \quad (3)$$

It follows that $\Delta_a = a \cdot \Delta$, so also the time window will change as a function of the scaling a. This results in larger time windows for lower frequencies and the opposite for higher frequencies, which is a more natural procedure for decomposing a transient signal, since a longer observation time is necessary for analyzing slower varying components of the signal.

The utilization of analytic signals for analyzing transient signals was first proposed by Gabor[5] and later introduced in ultrasonics by Gammel[6]. The total energy of a bounded oscillator is given by the sum of the kinetic energy, which is proportional to the square of the particle velocity, and of the potential energy, proportional to the square of the particle position. The total energy of the system can be calculated using a complex function whose real and imaginary parts are the square roots of the kinetic energy and potential energy, respectively[7]. *The total energy of the oscillator can thus be expressed as a single* side band complex signal, whose real and imaginary part are Hilbert transforms of each other. This fact is extremely important for understanding the usefulness of the utilization of analytic wavelet transform in ultrasonics. An ultrasonic transducer is sensitive to the local pressure field, and therefore, generates a signal proportional to the local displacement, i.e. the square root *of the potential energy.*

An analytic mother wavelet $h^A(t)$ can be constructed by zeroing the negative frequency components of the real wavelet $h(t)$. The magnitude of the complex analytic WT output thus describes the envelope of the ultrasonic signal. Furthermore the square of this quantity is equal to the true rate-of-arrival of the sound energy traveling through the material[8]. Using the real mother wavelet $h(t)$, the square of the real WT is only proportional to the rate-of-arrival of the potential energy, while the square of the analytic WT, obtained using $h^A(t)$, is proportional to the total, kinetic plus potential, instantaneous energy.

DISPERSION ANALYSIS OF ULTRASONIC SIGNALS

From wave theory, it is known that the ultrasonic signal can be expressed as a liner combination of harmonic functions. The dispersion relation describes the relationship between the frequency and wavelength of each component, i.e. the relationship between the spatial and temporal rate-of-change of the waveform. In a non-dispersive medium, the dispersion relation is linear, thus all wave components travel with the same phase velocity, which in this case coincides with the group velocity[9]. The signal thus maintains its original shape in time and space and does not disperse. In a dispersive medium, an arbitrary waveform will evolve in time and space due to the dependence of the phase velocity on the frequency. Since each progressive wave component propagates with a different phase velocity, the initial shape of the transient waveform is distorted in time. A typical

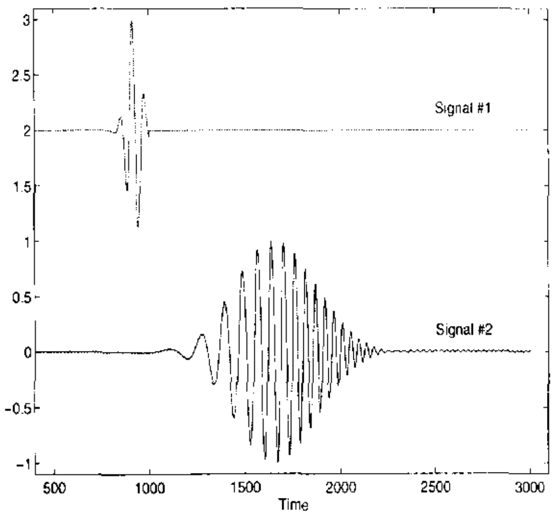

Figure 1. Typical ultrasonic signal (Signal #1). A linear phase delay was used to simulate dispersion in Signal #2.

non-dispersed signal is plotted as the top curve in figure 1 (labeled Signal #1). A linear phase delay was introduced for the dispersion relation, resulting in Signal #2 on the same figure.

The majority of absolute ultrasonic velocity data have been obtained using time-of-flight methods[10], such as pulse-echo-overlap or double-pulse superposition methods[11],[12]. In order

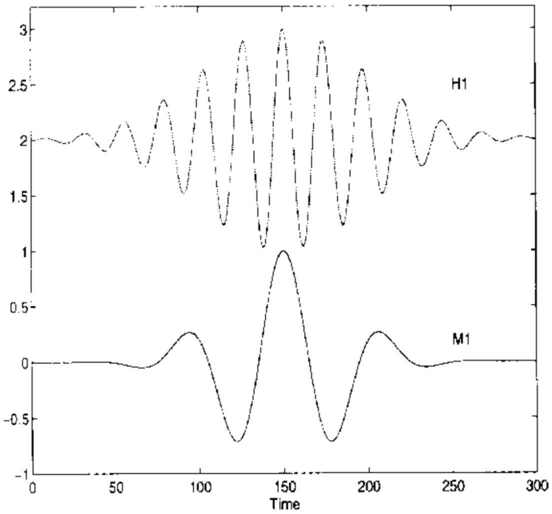

Figure 2. Plot of the real part of H1 and M1 mother wavelets.

to utilize in dispersion analysis the pulse-echo technique, the signal must be decomposed into a series of components which travel with a pseudo-constant phase and group velocity. When the components making up an original transient waveform are spread over a wide spectral range, the resulting evolution of the spectrum becomes a natural candidate for the wavelet transform analysis. Analogous to the wave-packet Fourier decomposition, the signal decomposition obtained using wavelet components can be utilized to analyze the time-frequency behavior of dispersive phenomena.

An optimal mother wavelet for dispersion analysis is the Morlet wavelet[13], defined as:

$$h_M(t) = e^{-(t/\sigma)^2} e^{j 2\pi f_o t} \qquad (3)$$

which represents a complex harmonic signal with frequency f_o, whose amplitude is modulated by a gaussian waveform of width σ. The choice of σ affects the temporal width of $h_M(t)$, and consequently its frequency bandwidth Ω. Two mother wavelets, H1 and M1, are plotted in figure 2. They were both obtained using eq. 2 with different choices of σ.

Using H1, which exhibits a larger time-window Δ, but a smaller frequency bandwidth Ω, the signal is decomposed into a series of wavelet components which have a gaussian bandwidth about a center frequency f_o / a, where a is the scale defined in eq. 1. Each wavelet component can be considered as a narrowband ultrasonic echo that travels through the material without any dispersion. Pulse-echo overlap can thus be used to estimate the acoustic delay and group velocity. This discussion is graphically summarized in figure 3, where dispersion is simulated using a signal composed by two echoes, the first of which is Signal #1 of figure 1, and Signal #2 is the other. Using H1 the top curve of figure 3 is decomposed into various wavelet components, three of which are also plotted in figure 3. For the first echo, all three components overlap in time, while this does not occurr for the second echo. Thus each wavelet component is characterized by an associated group of waves that have a particular propagation velocity. The center frequency of a wavelet component determines the group velocity and thus the arrival time. The detected point for the group delay time is the peak point of the

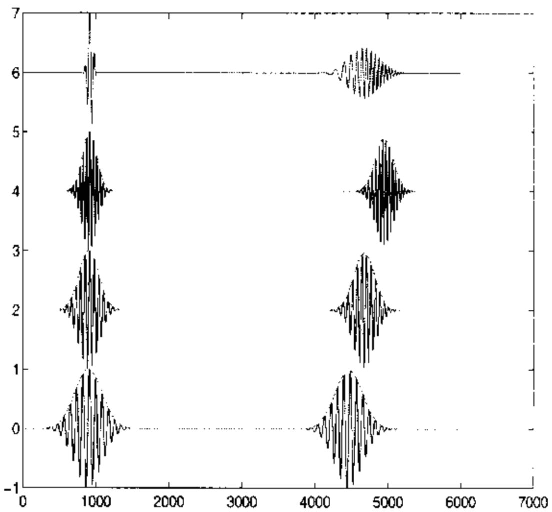

Figure 3. The top curve represents a composite of the signals in figure 1. Three wavelet components obtained for different values of the dilation a are also plotted. Dispersion on the second echo is clearly seen as a variation of the time of arrival of the different wavelet components.

envelope curve of the received signal waveform. A plot of the estimated group delay as a function of the normalized frequency is shown in figure 4. For comparison, also the theoretical expected group and phase delay are plotted as lines.

The mother wavelet M1, which has been extensively used to detect ultrasonic broad-band echoes buried in high level noise, was also utilized in the dispersion analysis. Results are shown in figure 4 as open circles. Unfortunately, M1 being a broad-band signal itself, it cannot be used in dispersion analysis since its components would disperse. This is clearly seen in figure 4, where the estimation for the group delay clearly exhibit a different behavior from the expected values.

From the above it seems that the optimal choice for the analysis of the dispersion is to have a mother wavelet as narrow as possible in frequency, since the peak point of the envelope curve is used for the delay measurement. Unfortunately, the decomposition obtained using H1 is very sensitive to the noise, compared to the one obtained using M1, resulting in less accurate time delay measurements. Clearly, as in the STFT case, the uncertainty principle does hold, but it can be adapted to the problem at hand. Techniques such as time-averaging can usually be utilized to increase the signal-to-noise ration (SNR), and thus

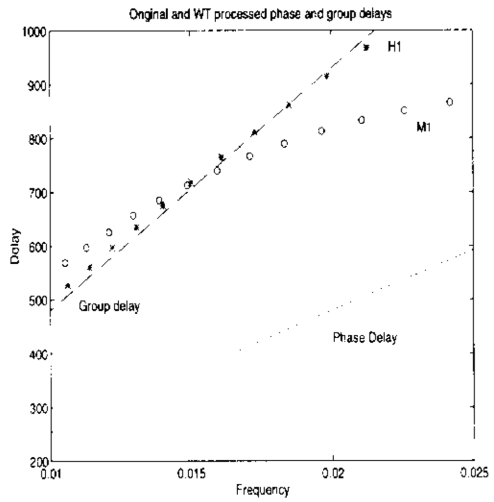

Figure 4. Calculated values of group delay using H1 and M1 mother wavelets are shown as open circles and stars, respectively. For comparison, the theoretical group and phase delay are also plotted as lines.

improve the time accuracy.

ULTRASONIC CHARACTERIZATION OF COATINGS

Surface waves are utilized to determine elastic properties near the surface of materials. Because of the energy concentration near the free boundary, the phase and group velocities are sensitive to the elastic conditions at the surface. Measurements made at different frequencies can be used to probe different depths below the surface because the depth to which particle motion occurs is comparable to the wavelength. The variation in depth of elastic properties can thus be inferred from the measurement of the surface acoustic velocity propagating along the surface of a steel substrate coated with chromium film.

By measuring the ultrasonic signals in two different positions, r_1 and r_2, and comparing the two wavelet transforms, the group delay $\tau = (r_2 - r_1) / v$, is easily obtained as a function of a and thus of the center frequency $f = f_o / a$. The surface velocity of the coating can be estimated from the dispersion curve obtained from the wavelet transform decomposition, as discussed previously.

The top signal in figure 5 represents typical surface wave signal obtained on a steel sample coated with a chromium film. The surface acoustic wave, generated using a Nd:YAG laser, Q-switched to produce pulses of approximate duration of 5 nsec, and energy of 5 mJ, is dispersive due to the higher velocity of sound in the coating[14]. The higher frequency component arrives at approximately 14 μsec., while lower frequency components arrive at later times, as shown in figure 5. For high frequencies the acoustic wavelength is proportional to the coating thickness, thus the group delay at these frequencies yields the group surface velocity for the coating. The measurement for much smaller values of frequencies is used to calculate the substrate surface velocity. The time-scale decomposition of the signal is represented in figure 6 as a pseudo-color density plot. The magnitude of the WT is plotted in the bottom as a function of the time delay b and the dilation coefficient m, such that $a = 2^m$. For comparison the magnitude of the WT for a signal obtained on a steel sample without chromium, is given in the top plot of fig. 6. As expected, the arrival time for the wavelet components with lower dilation coefficient m is less than the ones with higher values of m, which correspond to lower frequencies. The measured dispersion curve is given in figure 7. The acoustic velocity calculated using the WT is plotted as a function of the center frequency as open circles. For comparison also results obtained using the STFT are given as crosses. Values obtained using the WT agree better with indipendent measurements.

Using the WT, the two largest wavelet components were extracted and plotted in the middle section of figure 5. Also the sum of only these two components is shown in the botttom plot of the same figure. As expected the group delay of the higher frequency wavelet component corresponds to the velocity of the surface wave in the chromium coating, while the other lower frequency component, to the valocity in the steel substrate. Furthermore the signal resulting from sum of these two components closely matches the original signal, thus showing the

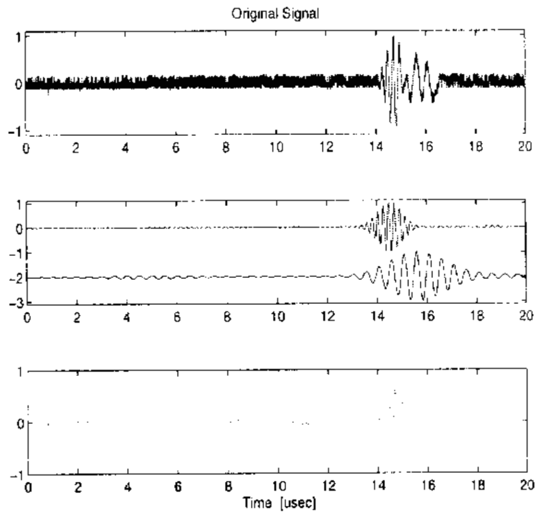

Figure 5. (Top) Surface wave signal on a steel substrate coated with chromium film. (Middle) Two largest wavelet components of the top signal, and (Bottom) their sum.

value of the proposed wavelet decomposition.

CONCLUSIONS

The preceding results and discussion demonstrated the usefulness and effectiveness of wavelet transform as a signal processing technique for the analysis of ultrasonic waveforms. The WT seen as a bank of matched filters can be used for the analysis of transient waves propagating in a dispersive medium The time-scale representations resulting from the WT demonstrate a very efficient means to obtain the velocity dispersion in an ultrasonic medium. The technique was successful in estimating the dispersion relation of surface acoustic wave group velocity induced by laser generation on a steel substrate with a chromium coating.

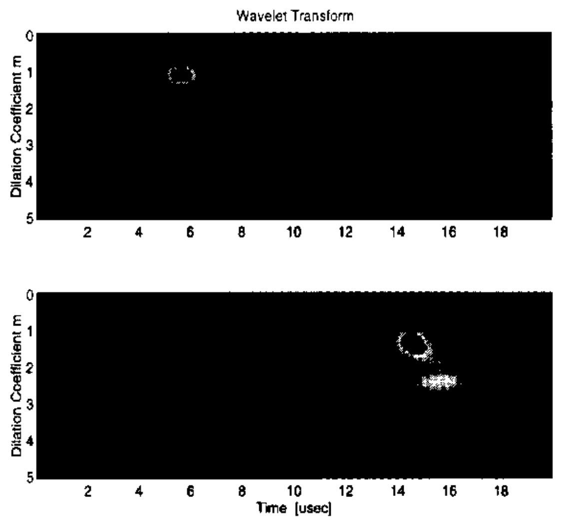

Figure 6. Pseudo color plot of the magnitude of the WT for a signal obtained on the steel substrate (Top plot), and for the top signal in figure 5 (Bottom plot)

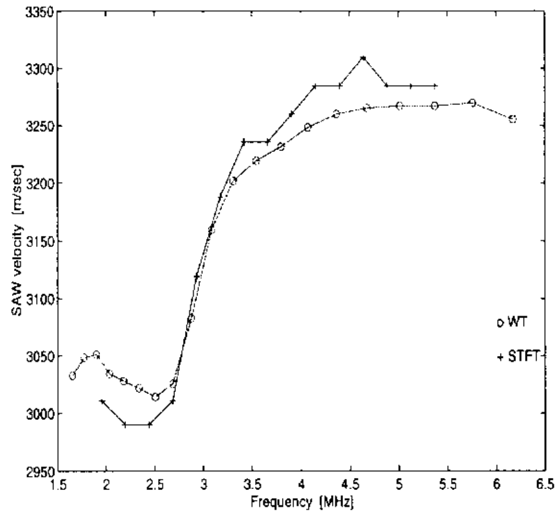

Figure 7. Dispersion curve of the top signal in fig. 5, calculated using the WT (o) and the STFT (+).

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