Modern Piezoceramic Equivalent Networks

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Abstract

Future applications for ferroelectric ceramics include thin-film resonant microstructures for timing, sensing, and actuation. The frequency range of interest extends from nearly DC through the UHF band. At high frequencies especially, it is difficult to obtain precise material parameters from broadband electrical measurements. We have developed highly accurate lumped equivalent networks for these frequencies, for the two canonical cases of thickness- and lateral-field excitation. Measurements on these circuits permit the extraction of the complex dielectric, piezoelectric, and elastic parameters of the ceramic material. A numerical example is given.

Introduction¹

Modern ferroelectric materials are being developed for a variety of mixed-effect microdevices that utilize the elastoelectric coupling stemming from the piezoeffect. These devices will be required to be uniform in their behavior to an extent unimagined in the past; this, in turn, necessitates the ability to characterize and measure precisely the material properties of the substances from which they are fashioned. The properties in question are the dielectric, piezoelectric, and elastic constants; the measurements are electrical in nature, and the characterization requires highly accurate equivalent electrical networks, valid over a wide frequency range, to describe the physical processes involved. Traditional equivalent circuits suffer from a variety of shortcomings that, e.g., prevent adequate resonator characterization for the purpose of extracting material coefficients, as well as use of electronic frequency-temperature compensation in oscillator (TCXO) applications.

The Butterworth-Van Dyke Circuit²

The traditional Butterworth-van Dyke (BVD) equivalent circuit of a resonant system driven by exposure to a capacitive electric field has been exploited for many years to represent piezoelectric resonators. The traditional BVD circuit is given in Fig. 1a. A distinction must be made according to the direction of the exciting field; the distinction is represented by the differences in Figs. 1a and 1b.

Simple Thickness Modes³⁻⁵

In order to treat our subject without undue complication, we assume that the structure under consideration is a piezoelectric plate resonator having an unbounded lateral extent. For these plate modes, it is found that the capacitance ratio, $r = C_0/C_1$, of the BVD circuit is related to the piezoelectric coupling factor, k , by the relation: $r = \pi^2/8k^2$.

Thickness and Lateral Excitation^{5,6}

The usual case of transduction is via a pair of electrodes placed on the major surfaces of the plate, to produce a thickness-directed driving field; this is known as thickness excitation (TE). Driving fields impressed from the side produce lateral excitation (LE). In the LE case the driving field is orthogonal to the wave-produced field in the thickness direction; this situation is appropriate to the BVD circuit of Fig. 1a. The TE situation, where the driving and wave-produced fields are parallel, is shown by the theory to be characterized by the alternative BVD circuit of Fig. 1b; the presence of the negative capacitor is a manifestation of the interaction of the driving and wave-produced fields. Thickness excitation is the overwhelmingly predominant form of excitation for piezoelectric plates.

In Fig. 1 the BVD circuits for LE and TE are contrasted. It is seen from the figure that the capacitance

ratios r and r' differ by unity. A number of important electrical attributes, e.g., the frequency difference ($f_A - f_R$), of the networks, are related to the capacitance ratio. Depending upon the size of the piezocoupling factor, k , the use of the traditional BVD network of Fig. 1a (pertinent to the LE case), when the circuit of Fig. 1b is called for (in the usual case of TE), can result in a significant error. Consider the case of a poled ceramic where $k = 51\%$. From the relation given above, $r = \pi^2/8k^2$, we find that $r = 4.74$, and $r' = 3.74$; the difference is greater than 20%, and is therefore quite significant. For a low coupling substance such as AT-cut quartz, where $k = 8.80\%$, r and r' differ by only 0.6%; this is frequently of no consequence.

Inclusion of Loss⁷

We may include the phenomenological effects of loss by two simple expedients. Internal friction within the piezo material is accounted for by making the elastic stiffness complex: $c^E \rightarrow \dot{c}^E = c^E + j\omega\eta$; the imaginary part is the acoustic viscosity term. This has the effect of making the velocity and wavenumber complex, and leads, in the lumped BVD description, to the presence of the R_1 term. The second provision is for dielectric and ohmic losses. These are accommodated by making the dielectric permittivity complex: $\epsilon^S \rightarrow \dot{\epsilon}^S = \epsilon' - j\epsilon''$, and combining ohmic and dielectric loss terms; the total effect is additive, so that the total effective σ is the ohmic conductivity σ plus the dielectric losses, $\omega\epsilon''$. In the traditional BVD circuit these losses appear as a shunt resistor across the static capacitor C_0 . When the LE version of the BVD circuit is taken, the further addition of the shunt resistor constitutes the ceramic resonator equivalent circuit almost universally used at low frequencies. Viscous and dielectric mechanisms are incorporated using two time constants, each of which is a real number. The motional time constant is $\tau_1 = R_1 C_1 = \eta/c$, where c is the lossless piezoelectrically stiffened elastic stiffness for the mode in question. The "static" time constant is $\tau_0 = R_0 C_0 = \epsilon'/(\sigma + \omega\epsilon'')$, which reduces to $\tau_0 = \epsilon'/\sigma$ in the DC limit.

Lumped Circuits^{2,4,8}

Exact expressions for the input admittance, Y_{in} , of a single excited mode in the one-dimensional approximation, when driven in TE and LE, are:

$$Y_{in}(TE) = j \omega C_0 / [1 - k^2 T(X)];$$

$$Y_{in}(LE) = j \omega C_0 [1 + k^2 T(X)]$$

where $T(X) = \tan(X)/(X)$, and $X = \kappa h$. The parameter $\kappa = \omega/v$ is the acoustic wavenumber and v is the piezoelectrically stiffened acoustic velocity. Piezoelectric transformer turns ratio, n , appearing in Fig. 2, is found from the relation $n^2 = (eA/2h) = 2 C_0 k^2 f_1/Y_0$, where Y_0 is the acoustic admittance.

The bisected, exact, transmission line circuit model characterizing a lossy plate resonator is shown in Fig. 2. Included here is the extension to the case where the electrode is of a thickness where its wave transmission time cannot be neglected, and the electrode must itself be represented by a transmission line. The figure shows the presence of ohmic and dielectric conductivities modeled by the shunt conductance $G_0 = 1/R_0$, as in the BVD case. Of particular note is the presence of a negative G_0 shunting the negative C_0 of the TE circuit; the negative G_0 and C_0 arise in the lossy case for the same reason as the negative C_0 in the lossless case: the interaction of the driving and wave-produced fields.

Passage from the distributed transmission line description to lumped parameters is accomplished by using the partial fractions expansion of the tangent function describing the transmission line. Loss is included by making the argument of the tangent function complex. The individual partial fractions then each have lumped network equivalents as series RLC circuits, and represent the harmonic plate resonances.

When one simplifies Fig. 2 appropriately, one obtains the network of Fig. 3, the new high frequency TE resonator equivalent circuit pertinent to high coupling ceramic, and other ferroelectric materials. For LE, one simply short-circuits the negative elements to arrive at the appropriate network topology. As regards the numerical circuit values pertinent to the two excitation types, we note that values of k for TE and LE are mutually independent, stemming from different piezoelectric coefficients; generally, the electrode geometry is also different, and therefore the two types of piezoelectric driving are represented not only by different circuit topologies, but also by different numerical values. The simplifications necessary to yield Fig. 3 from the distributed analog network are the following:

1) a single resonant harmonic is retained, and modeled by R_M, L_M, C_M ; M is an odd number representing the harmonic.

2) the effect of all other harmonics is subsumed into another capacitor, C_R , shunting the R_M, L_M, C_M branch.

3) electrode inertia is modeled by an inductor L_e .

The more accurate circuit of Fig. 3 is to be contrasted with the network usually given for ceramic resonators; viz., an LE BVD with shunt G_0 . If necessary to expand the frequency measurement regime to more than one harmonic, the circuit of Fig. 3 may be augmented by the placement of additional series RLC arms shunting that shown, with a concomitant decrease in the value of C_R .

Circuit Element Values

We assume a thickness-excited (TE) piezo resonator of thickness $2h$, coated with coextensive electrodes of area A , mass density ρ_e , and thickness h_e on the major plate surfaces. The plate is of mass density ρ ; complex dielectric permittivity and total effective conductivity in the thickness direction are ϵ^* and σ , effective piezoelectric constant is e , and the stiffened elastic constant and viscosity governing the modal velocity and attenuation are denoted by c and η . The requisite values of ϵ^* , σ , e , η and c are obtained from the material tensor schemes by straightforward methods. The piezoelectric coupling coefficient is k .

In terms of these quantities, the circuit elements of Fig. 3 are, to our level of approximation:

$$\begin{aligned} C_0 &= \epsilon' A/2h; & R_0 &= 1/G_0 = \tau_0/C_0; \\ C_M &= 8k^2 C_0/M^2 n^2; & C_R &= C_1(\pi^2/8 - \Sigma 1/N^2); \\ & \Sigma = \text{sum on odd } N \leq M; \\ L_M &= h^2/2v^2 C_0 k^2; & L_e &= \mu h^2/v^2 C_0 k^2 = 2\mu L_M; \\ R_M &= \tau_1/C_M; & k^2 &= e^2/\epsilon' c. \end{aligned}$$

More generally, C_0 and k^2 are complex, which render the lumped circuit elements complex. The effects can virtually all be subsumed into the same circuit form with modified values. The quality of fit can be further enhanced by adding, in series with C_R , an R_R and L_R .

Numerical Example

We give as an example the circuit parameters and electrical input behavior of a thin-film, thickness-mode ceramic resonator for operation in the VHF-UHF frequency bands. The material chosen is a modified lead titanate. Piezoelectrically stiffened, lossless acoustic velocity is $v = 4400$ m/s; piezocoupling is $k = 51\%$; and dielectric constant in the thickness direction is $\epsilon_{33}^T/\epsilon_0 = 263$. We choose specifically a fundamental mechanical response frequency of $f_1 = 250$ MHz; the resonator thickness required is $2h = v/2f_1 = 8.8 \mu\text{m}$. Mass density is taken as $7.52 \cdot 10^3$ kg/m³. To obtain the permittivity ϵ^S , necessary to calculate C_0 , we make the reasonable approximation that $\epsilon^S \approx \epsilon_{33}^T (1-k^2)$. This yields $\epsilon^S \approx 194.59 \epsilon_0 \approx 0.7399 \epsilon_{33}^T \approx 1.723$ nF/m. The piezo-transformer turns ratio squared is therefore $n^2 \approx 0.842$. We also assume a viscous (or motional) time constant τ_1 of value $\tau_1 = 10^{-12}$ s (loss one hundred times greater than quartz); this is presently unattainable at VHF/UHF, but is anticipated with future advances in material science. A value of total σ (conduction plus dielectric loss) of 10^{-7} S/m is used in the calculations, corresponding to a static time constant, $\tau_0 \approx \epsilon^S/\sigma$, of 17.23 ms.

We assume also a square electrode patch on each surface of the thin film with dimensions 1 mm by 1 mm by 250 nm. The electrode area is thus $A = 10^{-6}$ m²,

and the width-to-plate-thickness ratio is $1\text{mm}/8.8\mu\text{m} \approx 114$, so the one-dimensional approximation is valid. The acoustic (mechanical) impedance of the ceramic for this particular mode is $Z_0 = 1/Y_0 = A \rho v \approx 33.09$ mechanical ohms (kg/s).

Assuming further that the electrode is of aluminum, with mass density, ρ_e of $2.70 \cdot 10^3$ kg/m³, the normalized mass loading $\mu = (\text{electrode mass per unit area}/\text{resonator mass per unit area}) = \rho_e h_e/\rho h = (2.70 \times 250\text{nm})/(7.52 \times 4.4\mu\text{m}) \approx 2.04\%$. Because the acoustic velocity in the electrode is roughly of the same magnitude as that in the ceramic, this is also very nearly the ratio of the acoustic lengths of the electrode and plate in the thickness direction. Because μ is small, it is therefore permissible to substitute a lumped inductor L_e for the transmission line representing the electrode; see Fig. 2. Traditional treatments of mass loading show that the normalized mass loading approximates closely the ensuing frequency lowering due to the electrode mass; more accurate results are reported in Ref. [5]. In our case, $\mu = \Delta f/f_1 \approx 2.04\%$, or, since $f_1 = 250$ MHz, the mass loaded frequency of the purely mechanical resonance is reduced to 244.9 MHz; we shall see that this is indeed the effect of the inductor L_e , representing the mass loading.

The above data permit us to calculate, using the formulas given earlier, the parameters associated with the equivalent network of Fig. 3; these are given, to an accuracy consistent with our other approximations, as follows:

$$\begin{aligned} C_0 &\approx 195.8 \text{ pF}; & R_0 &= 1/G_0 = 88 \text{ M}\Omega; \\ L_e &\approx 0.401 \text{ nH}. \end{aligned}$$

$$\begin{aligned} \text{For } M = 1 \text{ (fundamental harmonic), } C_M &= C_1 \approx 41.3 \text{ pF}; \\ C_R &\approx 9.65 \text{ pF}; & R_M &= R_1 \approx 1/41.3 \Omega; \\ L_M &= L_1 \approx 9.82 \text{ nH}; & (2\pi f_1)^2 L_1 C_1 &= 1. \end{aligned}$$

Table I gives the resonance, antiresonance, and fractional difference frequencies for aluminum electrodes. The condition - $C_0 = -G_0 = 0$ is pertinent to LE, while the first entry is for TE; the entries where $L_e = 0$ correspond to the absence of mass loading ($\mu = 0$). The last column in Table I shows the fractional separations for various alterations in the circuit of Fig. 3. It is seen that there are weak dependencies upon the condition of excitation (TE versus LE), the presence or absence of mass loading, and the inclusion or exclusion of effects of the other (nonresonant) harmonics. Additional details are contained in Ref. [9].

Conclusion

Novel, lumped electrical equivalent circuits have been derived from transmission line analog networks; these yield accurate results for characterizing highly piezoelectric lossy materials, such as ceramics and other ferroelectrics, for high frequency operation.

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TABLE I. RESONANCE AND ANTIRESONANCE FREQUENCIES, AND FRACTIONAL SEPARATIONS; ALUMINUM ELECTRODES.

Circuit condition	f_R (MHz)	f_A (MHz)	$[(f_A - f_R)/f_1]$ (%)
Complete circuit	215.60	244.91	11.72
$-C_0 = -G_0 = 0$	244.91	268.88	9.58
$C_R = L_e = 0$	221.99	249.91	11.16
$L_e = 0$	220.45	249.91	11.78
$C_R = 0$	217.59	244.96	10.94

FIGURE CAPTIONS

Figure 1. Capacitance ratios for traditional BVD representations for TE and LE.

Figure 2. Exact network description of a bisected, one-dimensional, thickness-excited plate resonator.

Figure 3. High frequency ceramic resonator equivalent circuit for thickness excitation.

Figure 1.

Figure 2.

Figure 3.