$\theta = \pi/2$ in both (10) and (13), and here (11) and (12) agree too.

In summary, the "best-fitting" step edge to

AB

CD

is found as follows.

If $|B - C| \leq |A - D|$, then $\theta = \pi/4 [1 - (B - C)/(A - D)]$, and a, b are given by (11).

If $|B - C| \geq |A - D|$, then $\theta = \pi/4 [3 + (A - D)/(B - C)]$, and a, b are given by (12).

The magnitude $|a - b|$ of the edge is $|A - D|$ in the first case, and $|B - C|$ in the second case. In other words, the magnitude is $\max(|A - D|, |B - C|)$. Note that this is just the magnitude of the Roberts operator, using the max of the absolute differences rather than the square root of the sum of the squares [7]. (The slope θ , on the other hand, is not the arc tangent of the ratio of these differences; but its value is reasonable, e.g., if

$$\frac{AB}{CD} = \frac{12}{34}$$

we get $\theta = \pi/6$.)

III. CONCLUSION

We have presented an elementary derivation of step edge fitting in a simple, but nontrivial case: a 2×2 neighborhood and three basis functions. It turns out that the magnitude of the best-fitting edge to

AB

CD

is $\max(|A - D|, |B - C|)$, which is a commonly used version of the Roberts edge detector. Thus, our derivation provides a new motivation for that detector.

We have assumed here, in common with some of the other "simplified Hueckel" schemes, that the edge passes through the center of the neighborhood, whereas Hueckel's original derivation does not require this. It would be of interest to extend our approach to the general case of a step edge that crosses a circular neighborhood along an arbitrary chord. For some recent work on edge orientation estimation which does not assume that the edge passes through the center of the neighborhood, see [8], [9].

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Finding Edges in Noisy Scenes

RAUL MACHUCA AND ALTON L. GILBERT

Abstract—Edge detection in the presence of noise is a well-known problem. This paper examines an applications-motivated approach for solving the problem using novel techniques and presents a method developed by the authors that performs well on a large class of targets. ROC curves are used to compare this method with other well-known edge detection operators, with favorable results. A theoretical argument is presented that favors LMMSE filtering over median filtering in extremely noisy scenes. Simulated results of the research are presented.

Index Terms—Average operator, edge detection, edge direction, index of detectability, moments, ramp edge, roof edges, ROC curves, step edge, vector fields.

INTRODUCTION

Research into methods of identifying edges in a noisy scene has been an active field of investigation for many years. Treatment of the subject may be found in many books written over the past decade [1]-[3] and many different approaches are proposed. Recently a survey and comparative analysis of the methods was made [7].

In this paper we motivate the edge detection problem from an actual application to be made. Constraints are placed upon the algorithm that arise from the physics of the problem and bounds on the resources used in its solution. The resulting algorithm achieves the objectives and compares favorably with other methods previously proposed. Comparison of methods was done by receiver operating characteristics (ROC) curve analysis, confirming some results of analytical evaluations of alternative approaches.

The body of this paper is segmented into five parts. In the first, we motivate the problem and provide some simple arguments based upon noise models that gradient methods should not be used. In the second we derive and define a "moment operator" which we show to work well for step and ramp edges. Third, we define and characterize second-order edges using the concept of the rotation of a point in a vector field and develop the detector analytically. In Section IV we develop the algorithms for implementing the previously defined operators. Finally, in Section V these algorithms are evaluated using ROC curves and compared with previously known techniques.

The detection of edges to isolate objects in a scene is motivated by many distinct problems. One such problem arises in a tracking system where the input video image is analyzed and the object to be tracked identified. Subsequent input and feedback to the drive controls causes the sensor to reorient to

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The authors are with the Department of the Army, U.S. Army White Sands Missile Range, White Sands Missile Range, NM 88002.

a new position in an attempt to maintain the same x - y coordinate position for the object in the field of view. While this problem motivated the research that led to this paper, the results herein discussed are much broader in scope and application. The constraints imposed by this problem led to a method that is useful in high data throughput systems.

I. THE PROBLEM

Consider a video process $v(t)$ with a frame period τ_L . Let $s(t)$ be a sampling function

$$s(t) = \sum_n \sum_m \delta \left(t - \frac{k_n}{\tau_L}, t - \frac{k_m}{\tau_L} \right)$$

where $\delta(\alpha, \beta)$ is the two-dimensional Kronecker delta function. If n is a modulo k function and m is modulo 525, then $s(t)$ will form a sampling matrix of k equally spaced samples per line and of 525 lines per frame. Letting $s(t)$ serve as the sampling function for $v(t)$, let

$$\bar{V}(n, m) = v(t) s(t).$$

If k is chosen to be 512, then $V(n, m)$ is a 512×525 matrix of sampled video values for each frame. For simplicity in notation we will let

$$\bar{V}_{nm}^i = \bar{V}^i(n, m)$$

where the superscript denotes the i th frame. The superscript will be omitted where no loss of clarity will result.

In most edge-detecting algorithms the objective is to form some function

$$F(n, m, v)$$

such that

$$\Delta_{n,m,n',m'} = |F(n, m, v) - F(n', m', v')|$$

is maximized if (n, m) and (n', m') fall on opposite sides of an edge. If $\Delta > T$, where T is an arbitrarily chosen threshold, we conclude that an edge lies between (n, m) and (n', m') . A common such function is the gradient operator where $k_x = n' - n$, $k_y = m' - m$, and

$$F(n, m, v) = \frac{1}{k_x k_y} \bar{V}(n, m)$$

so that

$$\Delta_{n,m,n',m'} = \frac{1}{k_x k_y} \left| \bar{V}(n, m) - \bar{V}(n', m') \right|.$$

If we let

$$\frac{n + n'}{2} = n_0, \quad \frac{m + m'}{2} = m_0,$$

then as k_x and k_y become small

$$\Delta(n_0, m_0) \simeq \left| \frac{\partial}{\partial n} \frac{\partial}{\partial m} \bar{V}(n_0, m_0) \right|$$

where the concept of the partial derivative is broadened to accommodate a closely sampled function.

From Fourier analysis we can see that

$$\Delta(w_n, w_m) = (-jw_n) (-jw_m) \bar{V}(w_n, w_m)$$

and the power density spectrum

$$\Delta(w_n, w_m) \Delta^*(w_n, w_m) = w_n^2 w_m^2 |\bar{V}(w_n, w_m)|^2$$

so that by applying the gradient operator a parabolic power spectrum is introduced that emphasizes the high spatial

Fig. 1. Example center of mass vectors for (1) an edge and (2) a region of uniform intensity.

frequency components that correspond to edges. When additive noise is present, however, we have

$$Z(t) = V(t) + N(t)$$

so that the power density spectrum is

$$\Delta \Delta^* = w_n^2 w_m^2 [VV^* + NV^* + N^*V + NN^*]$$

where the arguments are eliminated for simplicity. If $V(t)$ and $N(t)$ are uncorrelated and $N(t)$ is zero-mean, then

$$\Delta \Delta^* = w_n^2 w_m^2 [VV^* + NN^*]$$

and the noise now has a parabolic power spectrum, making the detection of edges more difficult. If the noise is not zero-mean and/or not uncorrelated with the signal (a more common occurrence in imaging systems), then the gradient method experiences even greater difficulties in edge detection.

Similar analysis can be made of the many edge detector algorithms. For a recent excellent treatment of the subject see [7]. Many such algorithms depend upon a preprocessing algorithm that either averages or finds the median [3] of a local area about (a, b) and assigns the determined average or median to $Z(a, b)$. It will be shown later that such a preprocessing step can appreciably improve performance, and that under high noise conditions the averaging filter outperforms the median, while the opposite is true for low noise conditions (where in each case the noise is considered wide band and zero-mean).

II. EDGES FROM MOMENTS

First-order edge detection methods work in the following way. A picture function $f(x, y)$ is transformed to another picture function $F(x, y) = Tf(x, y)$ in such a way that the edges of objects in the scene will be in the set $\{(x, y): F(x, y) \geq W\}$ for some W . The usual method is to transform the picture using T equal to the gradient operator. Different edge detection methods correspond to different numerical approximations to the gradient.

The method used in our edge detection program is not based on derivatives. To reduce the effect of noise, this edge detection method uses integrals.

Edges can be found by using moments [10] as follows. A digitized picture can be thought of as a lamina whose density at each point is $f(x, y)$, so points of high intensity correspond to points of high density. A point (a, b) on an edge in the original function (see Fig. 3) would correspond to a point in this lamina (digitized picture) with high densities on one side and lower densities on the other side. Thus, if we look at a small lamina centered at point (a, b) and compute the center of mass of this small lamina, we can expect the center of mass to lie within an area of high densities (Fig. 1).

Suppose we now look at a point (c, d) such that the densities around it are fairly constant. Then the center of mass of a small lamina about it would be close to (c, d) . In this case, a vector from (c, d) to the center of mass would be very small compared to a vector from (a, b) to the center of mass in the previous case.

We conclude that one way to transform $f(x, y)$ to $F(x, y)$ such that edges of the original picture lie in the set $F(x, y) \geq W$ is to replace every $f(x, y)$ by the length of the vector from (x, y) to the center of mass of a small lamina centered about (x, y) . If the density at any point (r, s) is $f(r, s)$, we replace the picture function $f(x, y)$ by

$$F(x, y) = \bar{Y}^2 + \bar{X}^2$$

where

$$MX = \int_{-k}^k \int_{-h}^h tf(x+v, y+t) dt dv$$

$$MY = \int_{-k}^k \int_{-h}^h vf(x+v, y+t) dv dt$$

$$M = \int_{-k}^k \int_{-h}^h f(x+v, y+t) dv dt$$

$$\bar{X} = MY/M$$

$$\bar{Y} = MX/M.$$

That is, $F(x, y)$ is the magnitude of the vector from (x, y) to the center of gravity of a square lamina centered at (x, y) whose density is given by the picture function $f(x, y)$. In the one-dimensional case, these formulas reduce to

$$MX = \int_{-h}^h f(x+t) t dt$$

$$M = \int_{-h}^h f(x+t) dt.$$

If we make the change of variables $T = -u$ and use additive properties of the integral, these integrals become

$$MX = \int_0^h [f(x+t) - f(x-t)] t dt$$

$$M = \int_0^h [f(x+t) + f(x-t)] dt$$

and

$$F(x) = \frac{\frac{1}{h} \int_0^h (f(x+t) - f(x-t)) t dt}{\frac{1}{h} \int_0^h (f(x+t) + f(x-t)) dt}$$

so that replacing a function $f(x)$ by $F(x)$ amounts to replacing a function with a value calculated by the following process.

- 1) Take a small neighborhood about X .
- 2) Calculate the average of symmetric differences of the intensities multiplied by the distance from X .
- 3) Calculate the average of intensities.
- 4) Divide the value obtained in step 2) by the value obtained in step 3).

Fig. 6(b) is an example of how this method works on a scene [Fig. 6(a)] typical of those we study at White Sands Missile Range.

Once the coordinates $(\bar{X}, \bar{Y})$ of the center of mass of a lamina about (x, y) are calculated, the direction of the edge (if any) can easily be found. Since $(\bar{X}, \bar{Y})$ points to where the inten-

Fig. 2. (a) Image of rocket and plume. The plume is the large region of highest intensity. (b) Ramp and step edges found by using the moment operator. (c) The vector field generated by the moment operator. (d) Second-order edges detected by using the vector field.

sity of the picture is the highest, the direction of the edge is perpendicular to the direction of the vector from (x, y) to $\bar{X}, \bar{Y}$. If we take $(x, y) = (0, 0)$, then the direction of the edge is $\theta = \text{Arctan}(\bar{Y}/\bar{X}) + \pi/2$.

Thus, this model gives for each point in the scene a quantity that measures the probability that a point is an edge point and a direction which is the direction of a possible edge through that point.

The model introduced in Section I will not work for roof edges, since at the very peak of the roof (exactly where the edge is situated) both $\bar{X}$ and $\bar{Y}$ are equal to zero. In order to detect roof edges we need to take advantage of the direction information, and as Fig. 2(a)–(c) shows, we need to detect the shearing cause by the change in direction of the vector field at the edge points. One way of doing this is by using a tool from the theory of vector fields, namely, the rotation of a vector field about a point.

III. SECOND-ORDER EDGES

After a scene is processed by the moment edge detector, each point is assigned a direction and a magnitude. In effect, this specifies a vector at each point of the plane in question, i.e., these vectors define a vector field over the scene. To define the rotation of a vector field (see [4] and [5]), suppose a vector of the vector field Φ at the point (x, y) is given by

$$\Phi(x, y) = \{\phi(x, y), \psi(x, y)\}$$

$$\phi(x, y) = \bar{X}(x, y)$$

$$\psi(x, y) = \bar{Y}(x, y).$$

Fig. 3. A curve Γ and its corresponding vector field $\Phi(t)$.

Fig. 4. Vector field at a step or ramp edge point.

Fig. 5. Vector field at a roof edge point.

If a curve Γ on the plane (scene) is given in the form

$$\Gamma: x = x(t), y = y(t) \quad a \leq t \leq b,$$

then $\Phi(t) = \{\phi[x(t), y(t)], \psi[x(t), y(t)]\}$ is defined on the interval $[a, b]$ (see Fig. 3).

For each $t \in [a, b]$ there is determined an angle, the angle in radians between $\Phi(t)$ and $\Phi(a)$ measured from $\Phi(a)$ to $\Phi(t)$. This angle is a many valued function of t . The continuous branch of this function (vanishing for $t = a$) is designated by $\Theta(t)$ and called an angular function of the field Φ on a curve Γ . The rotation of the field Φ on the curve Γ is defined to be

$$\gamma(\Phi, \Gamma) = \frac{1}{2\pi} [\Theta(b) - \Theta(a)].$$

If Γ is a closed Jordan curve, then the rotation is found by subdividing Γ into two curves (not closed), computing the rotation of each, and adding. In the following, Γ is taken to be a small circle about a point.

We can write the rotation as

$$\gamma = \frac{1}{2\pi} [\Theta(b) - \Theta(a)] = \frac{1}{2\pi} \int_a^b \frac{d\Theta(t)}{dt} dt.$$

With $\Theta(t) = \text{Arctan } \bar{Y}/\bar{X} + \pi/2$, we make the following observations.

1) If $\Theta(t) = \text{constant}$, then $d\Theta(t)/dt = 0$ and $\gamma = 0$. So $\gamma = 0$ when x is a point on the edge of an object in a scene (see Fig. 4).

2) If Θ is symmetric about x and Γ is a small circle about $x = \text{edge point on a roof edge}$ (see Fig. 5), then with

$$\Gamma = \Gamma_1 + \Gamma_2$$

(where $\Gamma_1 = \text{one half of the circle}$ and $\Gamma_2 = \text{the other half}$)

$$\int_{\Gamma} \frac{d\Theta(t)}{dt} dt = \int_{\Gamma_1} d\Theta(t) + \int_{\Gamma_2} d\Theta(t) = \pi + \pi = 2\pi.$$

An example of how these observations can be used to detect second-order edges appears as Fig. 6(c) and (d).

IV. ALGORITHMS FOR IMPLEMENTATION

A. Calculation of Moment

Since we are interested in real-time applications of these methods, we simplify the calculation of $\bar{X}$ and $\bar{Y}$ by setting

$$M = \int_{-h}^h \int_{-k}^k f(x+t, y+u) dt du = 1.$$

This can be justified by observing that $M/4hk$ is the average of the intensities over a small neighborhood of (x, y) and so this value can be approximated by the average value of intensities over the entire picture. This would then be just a scale factor and so could be left out.

To calculate the integrals involved (see Fig. 7), we use an integral formula [6] of order $O(h^6)$. The formula for integration is

$$\iint F(x, y) = \sum_{i=1}^9 W_i * D_i$$

with

$$W_1 = W_3 = W_5 = W_7 = 25/324$$

$$W_2 = W_4 = W_6 = W_8 = W_9 = 10/81.$$

If we apply this to the integrals for $\bar{X}$ and $\bar{Y}$ and factor out all scale factors we get

$$\bar{Y} = 5 * (D1 - D5) + 4 * (D8 + D2 - D6 - D4)$$

$$\bar{X} = 5 * (D7 - D3) + 4 * (D8 + D6 - D2 - D4)$$

and we use $abs(\bar{X})^2 - abs(\bar{Y})^2$ for the associated magnitude. If we sweep a 3×3 window across the digitized scene, $D7$ can be taken as the upper left-hand corner while $D3$ is the lower right-hand corner. In this case the direction of a possible edge is equal to

$$\Theta = \text{Arctan} \left(\frac{\bar{Y} + \bar{X}}{\bar{Y} - \bar{X}} \right) + \pi/2.$$

B. Calculation of the Rotation

The vector field of a roof edge will look like the vector field of Fig. 5. To find roof boundary points we have to find points for which in a neighborhood of such a point

$$\int_c d\Theta = 2\pi.$$

The smallest region in the discrete case over which we can take an integral is a 2×2 window. Thus, our algorithm sweeps a 2×2 window across a scene and computes the integral

$$\int_c d\Theta$$

for each of these four windows. If it turns out that this integral is equal to 2π , then those four points which make up the window are classified as boundary points.

To calculate the integral of the 2×2 windows (see Fig. 8) we use the approximation

$$\int_c d\Theta = \sum_{i=1}^4 \Delta_i \Theta$$

where $\Delta_i \Theta$ is computed by the program in Appendix A.

Fig. 6. (a) Original image of disk with a roof edge. (b) Vector field generated by applying moment operator to image shown in Fig. 2(a). (c) Edge points of image in Fig. 2(a) found by identifying those points for which $\int_C de = 2$.

Fig. 7. Points used in calculation of the integral.

Fig. 8. A typical 2×2 window.

For the purposes of this experiment the procedure used to generate a file of detected second-order edges is the following.

1) From the original file (scene) two files are generated; one (ACI) contains $\sqrt{(\bar{X})^2 + (\bar{Y})^2}$; the other (ANG) contains the angle of $(\Theta, 0 \leq \Theta \leq 255)$, a possible edge.

2) From the ANG and ACI files a new file AAA is created by sweeping a 2×2 window across the ANG file. The rotation is calculated and, if a point is classified as boundary, then to the corresponding point of AAA (initialized at zero) is added the average of those elements of ACI that have the same subscripts as those of the 2×2 window being swept across ANG.

Examples of how this method works are illustrated in Figs. 6(d) and 2(d).

V. EVALUATION

The methods described above were tested on disks whose edges were step, ramp, and roof edges. The step and ramp edges had edge height equal to 16 while the roof edge was constructed by beginning at the center with gray value equal to 100, incrementing by one to gray value equal 132, and then decrementing by one to gray value equal 100. All files were $128 \times 128 \times 8$.

To test the effectiveness of the different operations considered here, we added Gaussian noise of different standard

Fig. 9. Originals: (a) step edge; (b) ramp edge; (c) roof edge. Edges: (d) moment operator; (e) moment operator; (f) rotation operator. Originals + Noise: (g) SNR = 6; (h) SNR = 6; (i) SNR = 10; (j) PF = 0.11, PD = 0.95; (k) PF = 0.11, PD = 0.60; (l) PF = 0.11, PD = 0.28.

deviation to achieve a given signal-to-noise ratio (SNR) and then tested the algorithms (Fig. 9).

The SNR ratio was measured in dB; that is, we used $\text{SNR} = 10 \log_{10} (16/\sigma_n)^2$ where σ_n = standard deviation of the noise. For the ramp and step edges we used SNR = 4, 5, 6, ..., 14 while for the roof edge the SNR ratios used were 10, 11, 12, ..., 20. To measure the effectiveness of the different algorithms we graphed PF = the probability of false alarms versus

Fig. 10. (a) ROC curve for roof edge at SNR = 13. Moment edge operator and Sobel edge operator applied after a 3×3 averaging. (b) This roof edge was found using moment operator. SNR = 13, $PF = 0.10$, $PD = 0.58$. (c) Roof edge which was found using Sobel operator. SNR = 13, $PF = 0.10$, $PD = 0.51$.

PD = the probability of detection. (See Fig. 11(a) and 11(b); for details, see [7].) Fig. 10(b) and (c) contain examples of processed roof edge disks with SNR = 13. The graphs of PF versus PD (ROC curves) for the corresponding operators appears in Fig. 10(a).

The information contained in these curves can be summarized by the following process.

1) For each SNR map each ROC curve into a straight line by sending

$$(PF, PD)^{T^{-1}} \rightarrow (x, y)$$

where

$$T(x) = \int_{-\infty}^x e^{-t^2} dt.$$

The resulting lines are called normalized ROC curves [Fig. 11(c) and (d)].

2) For each SNR compute the detectability index D_n (see [8], [9]) using $D_n \approx$ the average distance of a line from line $y = x$ over values of x where the data are concentrated.

3) Graph N versus D_n , where n is the signal-to-noise ratio.

For the step and ramp edges we tested the Sobel and moment operators alone and these operators when the scenes had been preprocessed by a 3×3 averaging [Fig. 11(e) and (f)] or 3×3 median operator. The roof edge was easily blurred by noise so we increased the signal-to-noise ratio and computed the rotation using the Sobel and moment operators only when this scene had been preprocessed either by a median or averaging operator.

The results for different operators and step, ramp, and roof edges appear, respectively, in Fig. 12(a)–(c). These graphs show that the performance of the moment operator is in all cases better than that of the Sobel operator. A significant improvement is obtained by first applying the average and then the moment operator. When the signal-to-noise ratio is high the median gives better results than the average, but there is a crossover point at which the average filter gives better results than the median. The reason for this behavior is given in Appendix B.

APPENDIX A

Program to Compute Integral of 2×2 Window

{The possible values of Θ are from ϕ to 255. Before this}
{function is called the first time sign is initialized to +1.}

Function $\Delta_I \Theta$ (sign, $z\phi$, $z2$: integer)

var del, jdel, sign, isign, ki, kj: integer;

begin

del: = $z\phi - z2$

{first compute the complementary angle to del and call it jdel}

if del $\geq \phi$, then

begin

isign: = + 1

jdel: = del - 256

end

else

begin

isign: = - 1

jdel: = del + 256

end;

ki: = abs (del)

kj: = abs (jdel)

if $kj \leq 74$ then del: = jdel;

{if either the angle or its complement are less than 74 then}
{use that one which is less than 74 as $\Delta_I \Theta$ }

if $ki \leq 74$ or $kj \leq 74$ then $\Delta_I \Theta$ = del else

begin

{let the direction of rotation agree with the last significant}
{rotation}

if isign $\neq$ sign then

begin

del: = jdel;

isign: = - 1;

end

{if the amount of rotation is not a noise strobe store the}

Fig. 11. (a) ROC curves for Sobel operator on ramp edge disk. SNR = 4-14. (b) ROC curves for ramp edge disk first processed by taking average (3×3) and then by moment operator. SNR = 4-14. (c) ROC curves of (a) after being normalized. (d) Normalized ROC curves for average-moment operator (b). (e) Edge detected using Sobel operator alone. SNR = 6, $PF = 0.11$, $PD = 0.23$. (f) Edge detected by first doing 3×3 averaging and then using moment operator. SNR = 6, $PF = 0.11$, $PD = 0.60$.

Fig. 12. (a) Signal-to-noise ratio versus index of detectability for disk with ramp edge. (b) Signal-to-noise ratio versus index of detectability for step edge. (c) Signal-to-noise ratio versus index of detectability for roof edge.

{direction of the change}

if abs (del) ≥ 40 then sign: = isign;

$\Delta_I \Theta$: = del

end

APPENDIX B

Median versus Mean Estimators in Noise

1) Mean: Let

$$x_{ij} = f(i, j) + n_{ij}$$

where n_{ij} is $N(0, \sigma^2) \forall i, j$ and

$$E[\eta_{ij}, \eta_{kl}] = 0 \quad \forall i \neq k, j \neq l.$$

Let $\hat{x}_{ij}$ = estimate of $f(i, j)$ where

$$\begin{aligned} \hat{x}_{ij} &= \frac{1}{(K+1)(L+1)} \sum_{i=I-(K/2)}^{I+(K/2)} \sum_{j=J-(L/2)}^{J+(L/2)} x_{ij} \\ &= \frac{1}{(K+1)(L+1)} \sum_{i=I-(K/2)}^{I+(K/2)} \sum_{j=J-(L/2)}^{J+(L/2)} [f(i, j) + \eta_{ij}] \\ &= \frac{1}{(K+1)(L+1)} \left[\sum \sum f(i, j) + \sum \sum \eta_{i,j} \right] \\ &= \hat{f}_{IJ} + \frac{1}{(K+1)(L+1)} \sum_{i=I-(K/2)}^{I+(K/2)} \sum_{j=J-(L/2)}^{J+(L/2)} \eta_{ij} \\ &= \hat{f}_{IJ} + \hat{n}_{IJ}. \end{aligned}$$

Now

$$E[\hat{n}_{ij}] = 0$$

$$\begin{aligned} E[\hat{n}_{IJ}^2] &= \left[\frac{1}{(K+1)(L+1)} \right]^2 (K+1)(L+1) \sigma^2 \\ &= \frac{\sigma^2}{(K+1)(L+1)}. \end{aligned}$$

2) Median: Let

$$x_{ij} = f(i, j) + n_{ij}$$

and let

$$\bar{X}_{IJ} = \left\{ x_{ij} \mid I - \frac{K}{2} \leq i \leq I + \frac{K}{2}, J - \frac{L}{2} \leq j \leq J + \frac{L}{2} \right\}$$

Let the elements of $\bar{X}_{IJ}$ be arranged in ascending order such that

$$\bar{X}_{IJ} = \{X_m \mid 1 \leq m \leq (K+1)(L+1), X_p \leq X_q \forall p \leq q\}.$$

Let

$$M = \frac{(K+1)(L+1) + 1}{2}.$$

Then choose

$$\hat{X}_{ij} = X_M;$$

then

$$\hat{X}_{ij} = f(M) + n_M$$

and

$$E[n_M] = 0$$

$$E[n_M^2] = \sigma^2.$$

We conclude, therefore, that in low signal-to-noise environments, the noise removal properties of the mean filter exceed those of the median filter for $K, L \gg 1$.

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Book Reviews

Computer Image Processing and Recognition—Ernest L. Hall (New York: Academic, 1979, 584 pp.). *Reviewed by R. W. Ehrlich.*

After working on narrow research topics for a period of time one tends to forget about the scope and depth of one's general research area. For those of us involved in digital picture processing, there is a new book available that really drives home that point. Its title is *Computer Image Processing and Recognition*, written by E. L. Hall of the University of Tennessee.

The author's intent is to provide a coverage of five areas of picture processing—enhancement, communications, reconstruction, segmentation, and recognition. This is not a text on scene analysis, but those looking for a solid text on the foundations of the field will be well rewarded. The author believes the text suited to a one-year course in picture processing and pattern recognition for seniors and graduate students in electrical engineering, computer science, or one of the related disciplines. An instructor using the text for an audience of students without background in system theory or Fourier analysis may have to do some fancy footwork in a few places, but then, some topics such as sampling theory just cannot be explained clearly without that perspective. The techniques discussed in the book are well illustrated by examples which give the reader a good means of assessing their effectiveness. Another characteristic of the book is that the chapters are only minimally interdependent, and this gives an instructor using the text a great deal of freedom to select the topics he wishes to include in his course.

The book begins quite appropriately with some radiometry, photometry, and a discussion of image formation. This chapter is a little bit surprising in its heavy orientation toward human perception and nonlinear models of the visual system. This is probably the only section that seems peripheral to the principal subject of the book.

The next two chapters are rather classical in their presentations of 3-D imaging, sampling theory, transformations, enhancement, and restoration. Here the book reveals its system-theoretic orientation; mention image representation to a scene

analysis person and he will expect piecewise polynomial decompositions or, still more likely, hierarchical data structures. This, of course, is not the intent or orientation of the book.

Chapter 5 was particularly pleasurable to this reviewer, since general treatment of three-dimensional transmission mode reconstruction (tomography) is not usually part of image processing texts. The chapter begins by describing 3-D reconstruction conceptually using a well-known Fourier method. Next come descriptions of more recent algebraic reconstruction algorithms followed by a discussion of the problems involved in displaying and visualizing such reconstructions. The last part of Chapter 5 is devoted specifically to medical tomography using X-ray and gamma ray sources, and several devices are described. It is a nice idea to include in a text that is essentially theoretical, a discussion of real world implementations of theory, since this gives the reader some perspective on what the theory has actually achieved.

Chapter 6 is dedicated to image communications and more specifically to image coding and compression. The chapter begins with PCM, since digital communications is the central subject. Next comes a dose of classical information theory including noiseless coding and rate distortion theory. The remainder of the chapter is devoted to a description of numerous transform coding schemes including run length encoding, minimum distortion coding, predictive and interpolative encoding, as well as a number of other coding schemes.

The remaining third of the book is spent on image analysis problems such as segmentation and image matching. Segmentation includes clustering, region growing, and edge detection techniques. Since segmented images are relatively useless without good descriptions of the basic regions, there is also a section on description including old standbys such as boundary fitting, Fourier descriptors, and moments. This is followed quite naturally by a section on relational representation of scenes and finally by a discussion of picture grammars.

The last chapter is rather specialized and deals with image matching. It begins by a brief discussion of sequential decision theory, template matching, and correlation. The matching section begins with Barnea and Silverman's technique [1] of sequential matching that has been used so successfully for registering large images, and it continues with hierarchical matching at various resolution levels, some of which is the author's own work. The chapter ends with additional discussion of the per-