DETERMINATION OF THE IONIC CONDUCTIVITY
OF IONIZED AIR WITH A
RESONANT CAVITY

L. Ambrose, M. Bushell, E. Hirschorn, C. Kenyon,
G. Merkel, and W. D. Scharf

US Army Electronics Research and
Development Command
HARRY DIAMOND LABORATORIES
2800 Powder Mill Road
Adelphi, MD 20783

ABSTRACT

A resonant cavity technique has been employed to measure the ratio of the average ionic mobility, $\bar{\mu}_i$, to the ionic recombination constant, k , for ionized air (water content $1.1 \pm 0.1\%$ by volume). The measured value is $\bar{\mu}_i/k = 0.8 \times 10^8 (\text{m}^{-1} \text{V}^{-1})$. The result is compared with measurements made with other techniques. A possible basis for the variation of published values is the variation of humidity in the air sample measured.

I. INTRODUCTION AND THEORETICAL BACKGROUND

Chervenak and van Lint¹ have employed a "pie-pan" technique to obtain the ionic mobility, $\bar{\mu}_i$, and the ionic recombination parameter, k , of dry ionized air.^{2,3} These authors obtained a value of $\bar{\mu}_i/k = 2.4 \times 10^8 (\text{m}^{-1} \text{V}^{-1})$. This paper presents experimental results consistent with a value of $\bar{\mu}_i/k = 0.8 \times 10^8 (\text{m}^{-1} \text{V}^{-1})$ for humid air with a water content of $1.1 \pm 0.1\%$ by volume.

When a nuclear burst occurs or a flash X-ray machine (Aurora) discharges, the high energy photons (gammas, X-ray or bremsstrahlung) knock out photoelectrons or Compton electrons from the air molecules.⁴ These electrons eventually thermalize, and in the process of thermalization produce conduction electrons. Eventually, the conduction electrons are absorbed by the oxygen molecules in the air. At atmospheric pressures this absorption by oxygen molecules takes about 10^{-8} seconds to occur. In a nuclear burst, the electrons are always replenished because of the presence of late-time gammas produced by neutron-induced nuclear reactions. In a flash X-ray machine, the replenishment of electrons ceases when the flash X-ray machine pulse terminates and the conducting air finally consists of a mixture of neutral molecules, positive ions, and negative oxygen ions.

The post electron-attachment era can be divided into two time regimes in which two different sets of ionic conductivity parameters can be seen to dominate. The dynamics of the physical situation that occurs in these finalized ionic-air-conductivity regimes can be described by the following equations. The ionic conductivity is given by

$$\begin{aligned}\sigma &= Ne(\mu_+ + \mu_-) \\ &\approx 2Ne\bar{\mu}_i,\end{aligned}$$

where

N is the ion pair density ($\#/ \text{m}^3$),

μ_+ is the ionic mobility of the positive ions,

μ_- is the ionic mobility of the negative ions,

$\bar{\mu}_i$ is the average ionic mobility of the positive and negative ions, and

e is the charge of the electron

The ion density, N , obeys the relationship

$$\frac{dN}{dt} = -kN^2.$$

where

k is the ionic recombination constant ($\text{m}^3 \text{s}^{-1}$).

The above differential equation can be solved to yield

$$N = \frac{N_0}{1 + N_0 kt},$$

where

N_0 is the initial ion pair density.

Note that for

$$t \gg \frac{1}{N_0 k},$$

we can write

$$\begin{aligned}\sigma &\approx \frac{2e(\bar{\mu}_i/k)}{(1/N_0 k) + t}, \\ &\approx \frac{2e\bar{\mu}_i/k}{t}.\end{aligned}\quad (1)$$

In this final result, N_0 , the initial value, disappears. Figure 1 gives an example of how two different initial values of N_0 eventually yield the same value of ionic density, N .

Note that the flat portions of the curves in figure 1 correspond to the time,

$$t \ll \frac{1}{N_0 k}.\quad (2)$$

Of course, this equation does not necessarily

apply at early times if electron conductivity is significant. For a sufficiently sharp radiation pulse in the early post-electron-era, the conductivity reduces to

$$\sigma = N_0 e(\mu_+ + \mu_-) . \quad (3)$$

Equation (3) holds as long as the recombination of positive and negative ions has not yet achieved an appreciable reduction of the number of ions. At late time the conductivity is given by equation (1). One could conclude that early-time measurements of ionic conductivity can be interpreted to yield information about the ionic mobility $\bar{\mu}_i$ and the late-time measurements can yield information about the ratio $\bar{\mu}_i/k$. Of course, the above generalization is based on the assumption that the time given by relation (2) occurs after all the conduction electrons have attached to oxygen molecules.

More generally, after a sample of air is subjected to an ionizing radiation burst, the electron contribution dominates the air conductivity because of the high mobility of electrons. The electrons then rapidly attach to oxygen ions. The radiation source (Aurora) does not cut off sharply (i.e., down by $\sim 10^3$ in < 200 ns) so the analysis must account for the residual electronic conductivity by employing the general differential rate equations of charge carrier density. The electron density, N_e , positive ion density, N_+ , and negative ion density, N_- , are given by⁴

$$\frac{dN_e}{dt} = S - k_1 N_e - k_2 N_e N_+ , \quad (4)$$

$$\frac{dN_+}{dt} = S - k_2 N_e N_+ - k_3 N_+ N_- , \quad (5)$$

$$\frac{dN_-}{dt} = k_1 N_e - k_3 N_+ N_- , \quad (6)$$

where S is the rate of production of electron-ion pairs, and the rate constants k_1 , k_2 , and k_3 are for the following three respective reaction groups:

where M is a third body. On a time-scale long compared with the radiation pulse width (~ 100 ns for Aurora) and the attachment time (~ 10 ns), the ionic contribution to the conductivity dominates.

The shape of the source term, S , in equations (4) and (5), when the Aurora flash x-ray is the source of the ionization, is shown in figure 2. The application of the above air-

chemistry equations to a source term with a time history of the form shown in figure 2 and with a total radiation dose of 90 rads(Si) yields an air conductivity time history of the form shown in figure 3. The results in figure 3 indicate that the late-time conductivity is sensitive to the parameter $\bar{\mu}_i/k$.

II. EXPERIMENTAL TECHNIQUE

To attempt to measure the ionic conductivity of air using the conventional one-pass microwave absorption techniques,⁵ one would be forced to install at least 100 feet of waveguide into the Aurora test cell. This is not very practical because: (1) waveguide is expensive, and (2) the waveguide would take up a lot of space.

Another approach is to effectively elongate the waveguide by changing it into a two-port cavity in which an electromagnetic wave is reflected many times before it leaks out. The higher the Q of the cavity, the more reflections the wave undergoes and the longer the effective length of the cavity. Of course, a high- Q cavity takes a long time to respond, but since the ions of air have a long recombination time, lack of time resolution is not a problem.⁶

A number of such measurements have been made throughout the Aurora test cell. The heart of the apparatus was the large cavity (10' x 23" x 11- $\frac{1}{2}$ ") which was exposed to the Aurora pulse. The cavity was, in fact, a ten-foot section of 2300 waveguide partially blocked off from the adjoining sections by aluminum-foil obstacles. As shown in figure 4, one port of the cavity is connected to an rf source, the other to an rf detector.

III. ANALYSIS

The simple equivalent circuit shown in figure 5 is the starting point for the analysis of the data listed in Table I. The Q of the two-port cavity was determined to be approximately 10,000 by measuring the width of the cavity resonance.

When the cavity air is ionized by exposure to an Aurora discharge, a new loss mechanism is introduced. If the ionized air is a collision-dominated plasma, the added loss mechanism can be incorporated into the equivalent circuit by inserting a time-varying resistor, $R_I(t)$, across the capacitance in the circuit shown in figure 5. Its value is determined by

$$R_I(t) = \frac{\epsilon}{\sigma C} . \quad (10)$$

As will be discussed more fully, if the ionic plasma is not completely collision-dominated, and ion inertia or ion oscillations play a role, the time-varying "resistor," $R(t)$, shown in figure 5 becomes complex because the value of σ employed to determine R has a reactive term. As will be shown, a complex σ tends to detune the cavity from its resonance frequency.

The equation of motion for the interaction of the ions with the electric field in the cavity is

$$Ee = m \frac{dv}{dt} + mvv \quad (11)$$

where mv is an effective frictional resistance resulting from collisions between ionized and neutral air molecules, E is the electric field strength and m the ionic mass.

Assuming that $E = E_0 e^{j\omega t}$, $v = v_0 e^{j\omega t}$, a solution of equation (11) is

$$v = \frac{Ee}{mv + j\omega m} \quad (12)$$

The ion current density is

$$J = Nev = \frac{Ne^2 E}{mv + j\omega m} \quad (13)$$

where N is the ion density. In the ionized region, Maxwell's curl of H equation can be written

$$\begin{aligned} \text{curl } H &= j\epsilon_0 \omega E + J \\ &= j\epsilon_0 \omega E + \sigma^* E \end{aligned} \quad (14)$$

where

$$\sigma^* = \frac{Ne^2}{m} \frac{1}{v + j\omega} \quad (15)$$

Let the impedance of the capacitance, C , in figure 5 be given by

$$Z = \frac{1}{j\omega C} \quad (16)$$

If the volume of the capacitance, C , is filled with a conductive medium with conductivity, σ , the altered impedance (capacitor with a lossy dielectric) can be obtained by substituting $C\{1 - (j\sigma/\omega\epsilon_0)\}$ for C in equation (16),

$$Z = \frac{1}{j\omega C(1 - \frac{j\sigma}{\omega\epsilon_0})} \quad (17)$$

(This corresponds to the well-known technique of replacing ϵ_0 with $\epsilon_0 - (j\sigma/\omega)$ in $\nabla \times H = j\omega\epsilon_0 E$ so that the $\nabla \times H$ Maxwell's equation becomes $\nabla \times H = (j\omega\epsilon_0 + \sigma)E$ when the medium becomes conductive.)

Substitution of the explicit expression of complex conductivity, (15), into expression (17) yields

$$\begin{aligned} \frac{1}{Z} &= j\omega C(1 - \frac{\omega^2 p}{v^2 + \omega^2}) + \frac{1}{R_c} \\ &= j\omega C_I + \frac{1}{R_c} \end{aligned} \quad (18)$$

where

$$R_c = \frac{1}{C} \frac{v^2 + \omega^2}{\omega^2 p} \quad (19)$$

$$\omega^2 p = \frac{ne^2}{\epsilon_0 m}$$

and

$$C_I(t) = C(1 - \frac{\omega^2 p}{v^2 + \omega^2}) \quad (20)$$

The factor $1 - \frac{\omega^2 p}{v^2 + \omega^2}$ is the basis for the statement that a plasma has a dielectric constant less than 1. The ionic conductivity has been incorporated into the equivalent circuit shown in figure 5; the effect of the ionic conductivity is to change the oscillation frequency and to increase the loss mechanism in

the circuit.

Our approach to analysis of our data is to employ the equivalent circuit shown in figure 5. In order to employ this circuit it is necessary to know the values of C and L . One obvious relation between C and L is given by the resonance relation,

$$\omega_0^2 = \frac{1}{LC} \quad (22)$$

But, the choice of a specific L or C is somewhat arbitrary. As is well known, the concept of capacitance, when applied to a cavity, is open to interpretation. We have derived the capacitance C in terms of stored electrical energy. Then⁸

$$C = \frac{1}{4} \epsilon_0 ad/b,$$

where a and d are the waveguide or cavity cross section dimensions and b the cavity length. A solution of the equivalent circuit to figure 9 yields

$$I = \frac{F_0}{K} \left[e^{-pt/2} \frac{\cos(\omega_0 t - \phi) \sin \epsilon}{\cos \phi} + \sin(\omega_0 t - \epsilon) \right] \quad (23)$$

where

$$F_0 = \frac{V_0}{L} \left[\left(\frac{\sigma_r(t)}{\epsilon_0} \right)^2 + \omega^2 \right]^{\frac{1}{2}},$$

$$p = \frac{R}{L} + \frac{\sigma_r(t)}{\epsilon_0},$$

$$r = \frac{R\sigma_r(t)}{L\epsilon_0} + \frac{1}{LC},$$

$$K = [(r - \omega^2)^2 + (pw)^2]^{\frac{1}{2}},$$

$$\epsilon = \tan^{-1} \left(\frac{pw}{r - \omega^2} \right),$$

$$\phi = \tan^{-1} \left(\frac{p}{2\omega_0} - \frac{\omega}{\omega_0} \cot \epsilon \right),$$

$$\omega_0 = \frac{1}{\sqrt{LC(t)}},$$

and

$$\omega = \text{rf source frequency.}$$

Note that the value of the capacitance, C , always appears in combination with the inductance, L , in the above equations. The conductivity, $\sigma(t)$, was extracted from the measured cavity power output by making equation (23) a best fit to that cavity output.

IV. RESULTS

Figure 6 shows a comparison of the calculation of the envelope response of an empty chamber to that of a chamber filled with ionized air for two different scope sweep times. Table I lists a small part of the data obtained during a large number of Aurora shots. Figures 7 to 9 show comparisons of theoretical predictions and experimental data for different air-chemistry parameters.

Table II lists a number of other experimental determinations of the parameters Π_i , k , and/or Π_i/k . The results in Table II indicate the necessity of more measurements.

The significance of air water-content needs investigation.

The value of late-time ionic conductivity during a nuclear burst can be approximated by

$$\sigma_i = \sqrt{\frac{KS}{k}} e\bar{\mu}_i.$$

The value of $\frac{\bar{\mu}_i}{k}$ in most EMP codes is:

$$\frac{\bar{\mu}_i}{k} = 1.25 \times 10^8 \text{ (mks)},$$

the value of $\frac{\bar{\mu}_i}{k}$ that yields the best fit to our data is

$$\frac{\bar{\mu}_i}{k} = 0.8 \times 10^8 \text{ (mks)},$$

the value of $\frac{\bar{\mu}_i}{k}$ obtained by Chervenak and van Lint¹ is

$$\frac{\bar{\mu}_i}{k} = 2.4 \times 10^8 \text{ (mks)}.$$

Figure 7 shows the result of a number of calculations corresponding to a series of cavity total average dose assumptions. If the dose is small the knee in the curves shown in figure 1 occurs at a late time. At early time the ionic conductivity is given by

$$\sigma_i = \frac{e2\bar{\mu}_i N_0}{1 + kN_0 t} \approx e2\bar{\mu}_i N_0,$$

and the ionizing cavity response is determined by $\bar{\mu}_i$ not $\bar{\mu}_i/k$. By employing a radio frequency bridge or nulling technique it should be possible to expose a cavity to a low average dose such as 5 rads and determine the value of $\bar{\mu}_i$ by the techniques presented in this paper. The less accurate steady state analysis, employed by the authors of this paper in reference 6, yields values of $\bar{\mu}_i/k$ that are 1.7 times larger than the more accurate transient analysis presented in this paper. The various curves shown in figure 10 of this paper would converge much faster if the method of analysis in reference 6 were correct.

V. CONCLUSIONS

The values of the ionic mobility, $\bar{\mu}_i$, and the ionic recombination parameter, k , have a significant effect on late-time source region EMP calculations and on the prediction of nuclear EMP induced lightning. The experimental measurements tabulated in Table II indicate that some unanswered questions still should be addressed; for example, the dependence or lack of dependence of the ionic mobility, $\bar{\mu}_i$, and the ionic recombination constant, k , on the water content of the ionized air.

The authors would like to express their thanks to the members of the Aurora crew and staff for their kind cooperation and helpfulness during the experiments reported in this paper.

REFERENCES

1. J. G. Chervenak and V. A. J. van Lint, "Experiments on Air Conductivity Relevant to Source Region EMP," IEEE Trans. Nucl. Sci., NS-29 (December 1982).
2. W. F. Crevier, C. L. Longmire, G. Merkel, and W. J. Spohn, "Air Chemistry and Boundary Layer Studies with AURORA," IEEE Trans. Nucl. Sci., NS-24 (December 1977).
3. J. G. Chervenak and V. A. J. van Lint, "Conductivity of Highly Dosed Air," IEEE Trans. Nucl. Sci., NS-27 (December 1980).
4. Conrad L. Longmire, "On the Electromagnetic Pulse Produced by Nuclear Explosions," IEEE Trans. on Antennas Propagat., AP-26 (January 1978).
5. M. A. Heald and C. B. Wharton, "Plasma Diagnostics With Microwaves," John Wiley & Sons, New York, 1965, p. 155.
6. L. M. Ambrose, M. Bushell, C. Kenyon, G. Merkel, and W. D. Scharf, "Microwave Measurements of Conductivity in Air and Other Gases," IEEE Trans. Nucl. Sci., NS-28 (December 1981).
7. Heald and Wharton (reference 5), p. 6.
8. Simon Ramo and John R. Whinnery, "Fields and Waves in Modern Radio, John Wiley & Sons, New York, First Edition, 1944, p. 382.
9. H. J. Longley and C. L. Longmire, "Electron Mobility and Attachment Rate in Moist Air," Mission Research Corporation, Santa Barbara, CA., MRC-N-222, December 1975.
10. Edward L. Ginzton, "Microwave Measurements," McGraw-Hill Book Company, New York, 1957, p. 391.
11. Klaus Kerris, "The AURORA Dosimetry System," HDL-TR-1754, March 1976.

TABLE 1

Synthesizer frequency 391, 322, 400 MHz

Q = 9783.1

Shot	Scope 1	Scope 2	Dosimeter Reading (rad)			Average Total Dose (rad)
3272	1 μ s/div	0.5 μ s/div	66.46 80.20	65.05 80.84	71.82 79.43	73.97
3273	5 μ s/div	2 μ s/div	64.53 83.75	80.82 86.36	70.84 82.42	76.45
3274	20 μ s/div	10 μ s/div	59.01 71.39	65.06 73.76	64.54 78.09	68.64
3275	100 μ s/div	50 μ s/div	118.00 131.00	97.24 104.00	95.58 106.00	108.64
3276	500 μ s/div	0.2 μ s/div	62.92 78.89	70.80 86.04	68.16 76.72	73.93
3277	1ms/div	0.1 μ s/div	65.07 81.71	75.69 75.57	68.27 85.29	75.27

TABLE II
EXPERIMENTALLY DETERMINED
VALUES OF k AND μ_i

	$k(m^3/s)$	$\mu_i (m^2/Vs)$	$\mu_i/k(m^{-1}V^{-1})$
SAYERS ^a	$2.2 \pm 0.1 \times 10^{-12}$ (DRY AIR)		
McGOWAN ^b	2.2×10^{-12} (DRY AIR) 1.45×10^{-12} (AIR + 1% H ₂ O)		
CHERVENAK ^c	$7.0 \pm 0.4 \times 10^{-13}$	$1.7 \pm 0.3 \times 10^{-4}$	2.4×10^0
THIS PAPER			0.8×10^0
BRADBURY ^e		1.9×10^{-4}	
QUOTED BY ^f C. LONGMIRE AS TYPICAL VALUES	2.0×10^{-12}	2.5×10^{-4}	1.25×10^0

- a) $t < 0.1s$, cold trap purified dry air
b) $0.2ms < t < 400ms$, cold trapped dry air
c) $0.1ms < t < 10ms$, 45 rad dose
d) $5\mu s < t < 100\mu s$, 70-110 rad dose, 1.1% H₂O, 20 deg C.

- a) J. Sayers, Proc. Roy. Soc., A169, 83 (1938).
b) S. McGowan, Can. J. Phys., 45, 439 (1967).
c) J. G. Chervenak and V. A. J. van Lint, IEEE Trans. Nucl. Sci., NS-29, 1880, (December 1982).
e) W. E. Bradbury, Phys. Rev., 40, 508, 524, 1932.
f) Conrad L. Longmire, IEEE Trans. on Antennas and Prop., AP-26, 3, (January 1978).

Figure 1. Qualitative behavior of ionic density after all of the conduction electrons have attached to oxygen molecules. Ionic density remains constant until $t \approx 1/N_0k$, where N_0 is the initial density. At late time, $N \approx 1/kt$ irrespective of initial density, N_0 .

Figure 2. AURORA Bremstrahlung pulse at early and late times measured on the beam center line.¹¹

Figure 3. The air conductivity resulting from the Bremstrahlung pulse shown in figure 2, calculated from complete air chemistry package (Longley-Longmire).

Figure 4. Block diagram of 2300 waveguide connected to RF electronics. The two scopes are set at different sweep rates.

Figure 5. Equivalent circuit for two-port cavity when the cavity is filled with ionized air. The resistor $R_I(t)$ is given by $R_I(t) = \epsilon / (C\sigma(t))$.

The empty cavity Q is given by

$$Q_e = \omega_0 L / R.$$

The ionized cavity Q is given by

$$\frac{1}{Q(t)} = \frac{1}{Q_e} + \frac{\sigma(t)}{\omega \epsilon}.$$

See reference 10 for further discussion of equivalent circuits for a two-port cavity. Experimental voltage response is proportional to current I given by equation (23).

Figure 6. Typical response envelopes of resonant chamber with and without ionized air. Figure (a) shows the early time cavity response with and without the Aurora radiation pulse (electron conductivity is taken into consideration). Aurora pulse reaches maximum at about 100 ns. Figure (b) corresponds to a much longer time scale. The cavity output voltage trace appearing on the oscilloscope is proportional to the current given by equation (23). (See figure 5.)

At resonance, a standing wave is set up in the cavity. On the average the electromagnetic wave bounces back and forth in the cavity thousands of times before it leaks out. Each traversal of the cavity samples the ionic conductivity.

Figure 7.

Figure 8.

Figure 9.

Figures 7, 8, and 9. Comparison between unirradiated cavity response (solid line), theoretical calculation of irradiated cavity response (dotted line), and experimental measurement of irradiated cavity response (dashed line). Parameter values used in the calculations are shown on the figures. The results are sensitive to μ_i/k , and very insensitive to μ_i .

Figure 10. Comparison of a number of calculated cavity output voltages corresponding to a series of different initial radiation doses.