Analysis of Fiber Interferometer Utilizing 3 × 3 Fiber Coupler

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Abstract—**An analysis of the properties of the 3 X 3 fiber coupler is made. The analysis reveals that four parameters are required to characterize the power transfer properties of this type of coupler. The performance of a fiber interferometer is analyzed in terms of these four parameters.**

A RECENTLY proposed single mode fiber optic interferometric sensor employs a 3 X 3 fused fiber coupler device as the arm combiner [1]. The advantage of this configuration is that three optical output signals are available. The three output intensities are sinusoidal functions of the optical path length difference between the two arms of the interferometer. With proper design of the 3 X 3 coupler, there is a 120° phase difference between any two of the three output sinusoids. This phase difference can be exploited in electronic processing techniques for avoiding thermal drift induced fading [2]. The performance of an interferometer of this type depends sensitively on the characteristics of the 3 X 3 coupler. Up to the present, no general analytic method for relating interferometer performance to relevant coupler properties has appeared. In principle, the operation of the 3 X 3 coupler can be predicted from coupled mode equations. In practice, however, the coupling coefficients and required geometric parameters are not known nor easily determined. It is the purpose of this paper to present an alternative analysis based on conservation principles. The analysis shows how interferometer performance is related to four easily measured coupler characteristic parameters.

The form of the interferometer utilizing the 3 X 3 coupler is shown in Fig. 1. The 2 X 2 coupler is used as the power divider while the 3 X 3 coupler is the arm combiner. The signal and reference arms are typically long lengths of single mode optical fiber. An important aspect of this type of device is that fiber lengths are not controlled on the order of an optical wavelength.

The starting point for the analysis is the construction of a transfer matrix description for the 2 X 2 and 3 X 3 couplers. In the case where the couplers are polarization independent, this is a straightforward task because a single polarization analysis can be used. For purposes of illustration, the 2 X 2 coupler will be discussed first and the formalism then applied to the 3 X 3 coupler. Finally, an analysis of the full interferometer will be given.

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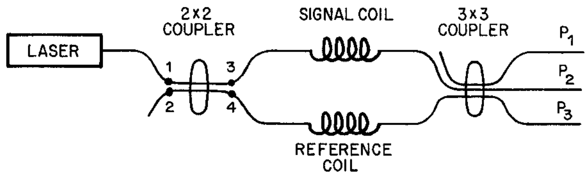

Fig. 1. Single mode fiber interferometer. Four reference points for the 2 X 2 coupler are indicated.

In the single polarization analysis the electric field at any point along a fiber can be regarded as a scalar quantity of the form

$$E(t) = E\, e^{i\omega t}. \tag{1}$$

The frequency of the laser source is ω. The complex number E is a function of position along the fiber guide. Rather than explicitly keeping track of the positional dependence of E, it is more convenient to denote a value at a specific reference location by a subscript. Four such reference points for the 2 X 2 coupler are indicated in Fig. 1. The action of the 2 X 2 coupler can be described by the operation of a 2 X 2 transfer matrix M_2 on a column vector specifying the electric field at the input points (1 and 2). The product of M_2 and the column vector is another column vector specifying the electric field at the output reference points (3 and 4)

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = M_2 \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}. \tag{2}$$

The general form of M_2 is

$$M_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{3}$$

The elements A, B, C, and D are complex. However, for purposes of this analysis, there is some freedom in how the phases of the elements are chosen. By relocating the points 1, 2, 3, and 4 by distances on the order of one wavelength, (3) can be transformed to

$$M_2' = \begin{bmatrix} Ae^{i\phi_A} & Be^{i\phi_B} \\ Ce^{i\phi_C} & De^{i(\phi_B + \phi_C - \phi_A)} \end{bmatrix}. \tag{4}$$

Since the length of fiber in an interferometer is never controlled to within a wavelength, any specific choice of ϕ_A, ϕ_B, and ϕ_C can be made without loss of essential generality. The most convenient choice is to select these parameters such that the (1, 1), (1, 2), and (2, 1) components of M_2' are positive real numbers. Consequently, the transfer matrix may be considered to be of the form (3) with A, B, and C real positive.

The conservation of optical energy condition can be expressed as [3]

$$(E_1^* E_2^*)\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = (E_3^* E_4^*)\begin{pmatrix} E_3 \\ E_4 \end{pmatrix}. \tag{5}$$

This equation uses the convention that the product of a row vector and a column vector is a scalar. Equation (5) implies the condition

$$M_2^+ M_2 = \text{unit matrix}. \tag{6}$$

The matrix M_2^+ is the Hermitian conjugate of M_2. By an elementary theorem of linear algebra [4], (6) implies

$$M_2 M_2^+ = \text{unit matrix}. \tag{7}$$

An expansion of (6) or (7) into element equations allows a straightforward proof that $B = C$, $D = -A$, and $A^2 + B^2 = 1$. It is interesting to note that with the choice of phase mentioned above, D, as well as A, B, and C is real. The form of M_2 is now

$$M_2 = \begin{pmatrix} A & B \\ B & -A \end{pmatrix}. \tag{8}$$

The power transfer properties of the 2×2 coupler can be characterized by a single parameter. This is the splitting ratio ($= B^2/A^2$).

The 3×3 coupler can be analyzed in the same manner as the 2×2 coupler. The general form of the 3×3 transfer matrix may be written as

$$M_3 = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}. \tag{9}$$

There is a reference point for each input fiber lead and for each output fiber lead. As in the previous case, the positions of these reference points can be adjusted such that a, b, c, d, and g are real positive. The conservation of optical energy principle results in the equivalent conditions

$$M_3^+ M_3 = \text{unit matrix}$$
$$M_3 M_3^+ = \text{unit matrix}. \tag{10}$$

On expanding (10) into element equations it is found that there are 9 conditions on the 13 parameters (one each for the real components, 2 each for the others) in (9). Therefore 4 parameters characterize the power transfer properties of the 3×3 coupler. The most convenient choice for these parameters are the moduli of the complex parameters e, f, h, and k. The element equations derived from (10) give the real parameters in terms of the moduli of e, f, h, and k.

$$\begin{aligned} b^2 &= 1 - e^2 - h^2 \\ c^2 &= 1 - k^2 - f^2 \\ d^2 &= 1 - e^2 - f^2 \\ g^2 &= 1 - k^2 - h^2 \\ a^2 &= e^2 + f^2 + h^2 + k^2 - 1. \end{aligned} \tag{11}$$

The phase of the complex elements may also be derived from the element equations

$$\begin{aligned} \cos(\phi_e) &= \frac{c^2 f^2 - a^2 d^2 - b^2 e^2}{2adbe}, \quad \phi_e > 0 \\ \cos(\phi_f) &= \frac{b^2 e^2 - a^2 d^2 - c^2 f^2}{2adcf}, \quad \phi_f < 0 \\ \cos(\phi_h) &= \frac{c^2 k^2 - a^2 g^2 - b^2 h^2}{2agbh}, \quad \phi_h < 0 \\ \cos(\phi_k) &= \frac{b^2 h^2 - a^2 g^2 - c^2 k^2}{2agck}, \quad \phi_k > 0. \end{aligned} \tag{12}$$

In (11) and (12) and in (14) below, the letters, a, b, c, d, e, f, g, h, and k represent the moduli of the elements of M_3. Note that these phases depend only on the moduli of the matrix elements of M_3. The moduli of the matrix elements are directly related to splitting ratios. For example, if power is applied to input arm 1 and only to input arm 1, the power measured at the three output arms will be in the ratio of $a^2 : d^2 : g^2$. The moduli of all the elements of M_3 can be determined from splitting ratio data.

The operation of the interferometer of Fig. 1 can be described in terms of a product transfer matrix of the form

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & -A \end{bmatrix} \begin{bmatrix} 0 \\ P_0^{1/2} \\ 0 \end{bmatrix}. \tag{13}$$

Here ϕ is the phase difference between the signal and reference arms [5]. For the purposes of this calculation the two arms of the interferometer have been connected to input leads 2 and 3 of the 3×3 coupler. If the laser is injected into the top arm of the 2×2 coupler, the optical power which emerges from the three output arms is

$$\begin{aligned} P_1 &= \alpha\alpha^* = (A^2 b^2 + B^2 c^2 + 2ABbc\, m \cos\phi) P_0 \\ P_2 &= \beta\beta^* = [A^2 e^2 + B^2 f^2 + 2ABef\, m \cos(\phi + \phi_f - \phi_e)] P_0 \\ P_3 &= \gamma\gamma^* = [A^2 h^2 + B^2 k^2 + 2ABhk\, m \cos(\phi + \phi_k - \phi_h)] P_0. \end{aligned} \tag{14}$$

The parameter m is unity for the single polarization analysis. In the more general case for which the two couplers are polarization invariant, but the long fiber arms are birefringent, $m < 1$. Since the fibers are single mode, the optical field at any point in the fiber arm is fully polarized. The state of polarization can be specified by two points on the Poincaré sphere [6], one for each interferometer arm. The parameter m is given by $\cos(\Theta/2)$ where Θ is the angle between radii drawn to the two points on the Poincaré sphere. The interpretation of the output of this type of fiber interferometer is not straightforward because in some cases m as well as ϕ can be a function of path length difference between the arms. It should also be noted that m can change as the direction of the input polarization changes.

In the case of the completely symmetric 3×3 coupler, the square of the modulus of each element of M_3 is equal to $\frac{1}{3}$. For this case, $\phi_f - \phi_e = 120^\circ$ and $\phi_k = \phi_h = -120^\circ$. From (14) it can be seen that the phase dependent terms in P_1, P_2, and P_3 are 120° out of phase with each other. Two signals 90°

out of phase can be readily constructed as $P_2 - P_3$ and $2P_1 - P_2 - P_3$.

In summary, it has been shown that four parameters are required to characterize the power transfer properties of the lossless 3×3 coupler. These parameters can be obtained from splitting ratio data. It has further been shown how the performance of a fiber interferometer incorporating a 3×3 coupler can be predicted from these parameters.

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Richard G. Priest, for a photograph and a biography, see p. 511 of the April 1982 issue of this TRANSACTIONS.

2 × 2 Optical Waveguide Matrix Switch Using Nematic Liquid Crystal

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***Abstract*—A 2 X 2 nonblocking optical matrix switch, composed of four elemental switches formed in a slab waveguide, is described. Switching action is based on total internal reflection caused by an electrically controlled change in refractive index of the liquid crystal. Propagation loss was remarkably reduced to 2.3 dB/cm, both by using liquid crystal as a cladding layer and by operating at the 1.31 μm wavelength. The experimental matrix switch exhibited 7.3–7.7 dB insertion loss, −17.3 to −18.2 dB crosstalk, and a 14° switching angle.**

I. INTRODUCTION

OPTICAL switches play an important role in optical communication systems. Especially, matrix switches are indispensable for exchanging optical information signals in local offices. Several bulk and waveguide matrix switches have been studied so far [1]-[9]. The bulk mechanical matrix switch using movable prisms exhibited excellent characteristics such as low insertion loss and low crosstalk [1], [2]. However, the bulk matrix switch has limitations in production, compactness, and speed. Waveguide matrix switches are expected to be superior to the bulk matrix switch in performance, such as in alignment free structure, easy integration of a large number of elemental switches on a common substrate, and speed. Waveguide matrix switches are classified into the following two types: nonblocking switches based on total internal reflection [3]-[6]; and blocking switches based on total internal reflection [7] or a directional coupler [8]. The nonblocking matrix switch is a more suitable switch for switching networks in local offices than a blocking matrix switch because the nonblocking matrix switch can accomplish input-output connection upon request by actuating independently only one elemental switch corresponding to the input and output ports. A nonblocking total internal reflection matrix switch using mechanically moving *GGG* chips exhibited a large switching angle of 22°, which is very useful for constructing the matrix switch [3]. However, the use of the moving *GGG* chips restricts the realization of a large scale matrix switch because it is difficult to compactly arrange the *GGG* chips in a matrix fashion. The nonblocking total internal reflection matrix switch using the $LiNbO_3$ waveguide is advantageous for switching times up to nanosecond. However, it is difficult to integrate a large number of elemental switches because of the small switching angle of less than 4°. The small switching angle arises from a small refractive index change, induced by an external electric field in $LiNbO_3$.

It is well known that the refractive index change in nematic liquid crystal is orders of magnitude larger than that in $LiNbO_3$. The large refractive index change in liquid crystal is convenient for constructing a compact matrix switch. Recently, a bulk nonblocking 4×4 matrix switch using nematic liquid crystal has been developed [9]. Although a waveguide switch using nematic liquid crystal has also been reported, it cannot be applied to the matrix switch because of the "on-off" switching action [10].

This paper describes an electrically controlled nonblocking waveguide matrix switch, based on total internal reflection, which is composed of a SiO_2-Ta_2O_5 waveguide film and a nematic liquid crystal cladding layer. An optimum design for achieving a large switching angle is presented. Experimental 2×2 matrix switches, which were fabricated on the basis of experimental results on the fundamental 1×2 switch, are demonstrated.

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