Attenuation Characteristics of Circular Dielectric Waveguide at Millimeter Wavelengths

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***Abstract*—Experiments were done to determine the practicality of using circular dielectric waveguide for a low-loss transmission line at millimeter wavelengths. The lowest order mode on a circular dielectric guide will propagate regardless of how small the guide diameter is. The attenuation factor of this mode is proportional to the diameter. Thus it is possible to make the attenuation factor of a circular dielectric guide arbitrarily small by reducing its diameter.**

Loss measurements for several different diameters of polystyrene and Teflon rods were made at 72.70 GHz. The measurements were made by directly probing long sections of dielectric guide and plotting the average power as a function of length on an $X-Y$ plotter. Dielectric constants were measured from the standing wave patterns of polystyrene, Teflon, and fused quartz rods at 71.0 GHz.

Teflon rods exhibited attenuation factors from 0.8 dB/m to 2.2 dB/m, depending on the diameter. This is an improvement over silver waveguide at this frequency. Polystyrene rods were found to have attenuation factors ranging from 3.9 dB/m to 12.5 dB/m, again depending on the diameter of the rod. The measured dielectric constants are consistent with previously published data.

The various attenuation factors are related to the intrinsic loss tangent of the dielectric using the theory of the HE_{11} mode. Values of $\tan\delta$ derived from measurements of different rods are consistent, indicating that the experimental results are valid.

The problem of radiation from dielectric rods is discussed. The experimental results are not conclusive, but it appears likely that radiation loss is negligible.

I. Introduction

IN THE MILLIMETER wave region, the attenuation of a conventional metal waveguide is sufficiently high to make many engineering applications impractical. The principle source of this attenuation is the ohmic loss of the induced surface currents in the walls of the guide. In the case of a dielectric waveguide, there are no conducting surfaces, and wall losses are not a factor. Instead, the main sources of attenuation are molecular absorption within the dielectric and radiation from bends and surface imperfections.

Since there are no conductors present, the electric field at the surface of a dielectric waveguide is not constrained to be zero. Part of the energy in a guided mode propagates inside and part propagates outside the dielectric. Reducing the cross-sectional area of the dielectric will increase the relative amount of energy propagating outside the guide. If the external medium (e.g., air) has negligible loss, the net attenuation factor of the guide will decrease.

In the case of a circular dielectric waveguide, the lowest order guided mode has no cutoff, and will propagate regardless of how small the guide diameter is. In principle, at least, it is possible to make the attenuation factor of the circular dielectric guide arbitrarily small by reducing its diameter.

In an effort to determine the practicality of using dielectric guide at millimeter wavelengths, the attenuation factors of several polystyrene and Teflon rods were measured at a frequency of 72.70 GHz. For most cases, the rods were propagating a single mode (HE_{11}). This made it possible to relate the attenuation as a function of rod diameter to the intrinsic loss tangent of the dielectric material. The consistency of the results confirmed that existing theory is adequate to predict the performance of a dielectric waveguide whose dielectric constant, loss tangent, and diameter are known.

II. Experimental Results

The attenuation constants were measured by directly probing a long length of dielectric guide. By plotting the average power on the guide as a function of position along its length, information about the attenuation, standing wave ratio, and guide wavelength is easily obtained. Fig. (1) is a photograph of the experimental apparatus. A CW signal is generated and transmitted through a rectangular waveguide to the dielectric rod, which is clamped into a waveguide flange using four setscrews. A motor driven probe is synchronized with the time base on an $X-Y$ plotter. The probe itself is a piece of WR-15 waveguide with a hole drilled parallel to the E plane. The dielectric rod passes through the hole, coupling its longitudinal E field to the waveguide and from there to a diode detector.

Fig. 2 shows the experimental plots for Teflon and Rexolite rods of various diameters. (Rexolite 1422 is the trade name for a type of cross-linked polystyrene.) In most cases the exponential decay of the signal is easily observed. The spurious interference is a result of reflec-

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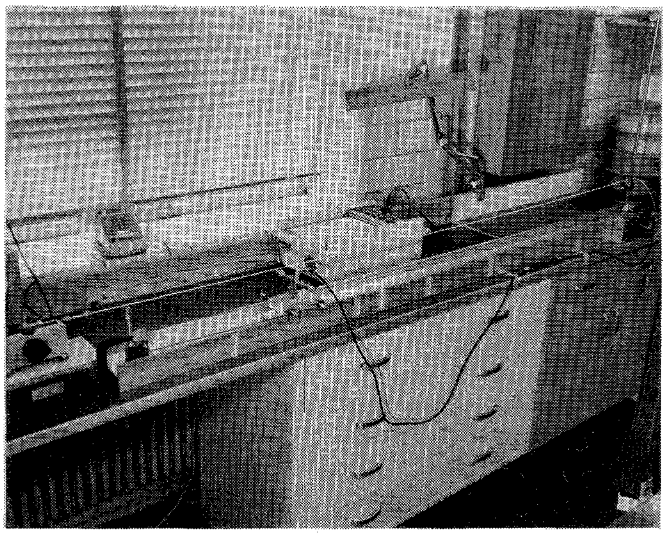

Fig. 1. The experimental apparatus.

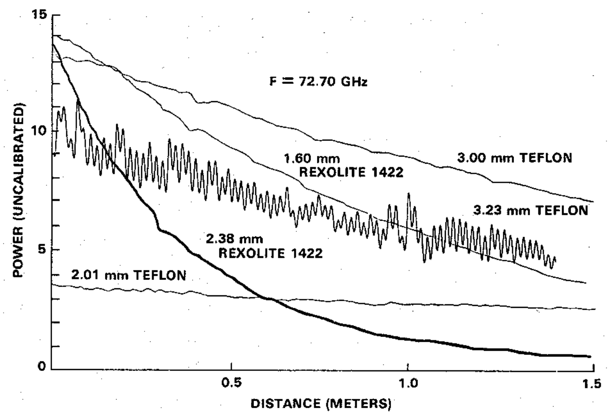

Fig. 2. Average power as a function of length along various dielectric rods.

tions from the ends of the dielectric line and the metal probe. If the diameter of the rod is small enough, only one mode will propagate. However, as the diameter increases beyond a certain critical size [1], higher order modes begin to propagate. The various modes have different attenuation characteristics, and interact differently with the probe. The interference pattern for the 3.23-mm Teflon is presumably due to these multimode interactions.

For loss measurements, longer samples provide more accurate results, especially for the small-diameter low-loss rods. In addition, the long length allows the incident wave to decay sufficiently so that the signal reflected from the end of the rod will not set up a standing wave, which would complicate the interpretation of experimental results.

One can intentionally set up a standing wave by placing a metal reflector at the end of a short section of rod. By measuring the guide wavelength from the standing wave pattern, the theory of the HE_{11} mode can be used to determine the dielectric constant of the rod [1]. Conversely, if the dielectric constant of the rod is known, measurement of the guide wavelength will verify that the HE_{11} mode is actually propagating. This was done at 71.0 GHz for Teflon, polystyrene, and fused quartz rods. The results are summarized in Table I. A discussion of the theory is in the Appendix.

TABLE I
MEASUREMENT OF DIELECTRIC CONSTANT AT 71.0 GHz

MATERIAL	fused quartz	polystyrene	teflon
Rod diam.	1.09 mm	2.38 mm	2.01 mm
d_c	1.95 mm	2.62 mm	3.24 mm
d/λ_0	.259	.563	.477
λ/λ_0	.980	.811	.947
ε_r	3.75	2.53	2.00
ε_r(25GHz) [7]	3.78	2.56	2.08

Note: d_c = critical diameter at which higher order modes begin to propagate, d = rod diameter, λ_0 = free space wavelength, λ = guide wavelength, and ϵ_r = experimentally determined value for the relative dielectric constant; the close agreement between the experimental value for ϵ and the previously known value for ϵ at 25 GHz confirms that the HE_{11} mode is indeed propagating.

Using a theory first developed by Elsasser [2], the bulk attenuation factor of the HE_{11} mode on a dielectric rod can be related to the intrinsic loss tangent of the dielectric material. Elsasser's theory is outlined in an Appendix, and only the results are presented here.

The attenuation factor α is related to the dielectric constant and loss tangent by the equation

$$\alpha = \frac{2\pi\epsilon_r \tan\delta}{\lambda_0} R \ (4.343) \ \mathrm{dB/m} \tag{1}$$

where

- ϵ_r relative dielectric constant,
- λ_0 free space wavelength of signal in meters,
- $\tan\delta$ loss tangent of material,
- R scaling factor that takes into account the relative amount of energy that is propagating outside of the dielectric.

The factor of 4.343 is a power conversion factor from nepers to decibels.

Equation (1) is derived by using Poynting's theorem to determine the fractional power loss in a differential section of dielectric guide. The factor R is derived by integrating the field equations for the HE_{11} mode.

Fig. 3 shows the factor R as a function of rod diameter for Teflon ($\epsilon_r = 2.08$) and polystyrene ($\epsilon_r = 2.56$). As the diameter is increased, R approaches the limiting factor $1/\sqrt{\epsilon_r}$. Equation (1) then becomes the expression for the attenuation of a plane wave in an infinite dielectric medium.

Table II is a tabulation of the experimentally determined attenuation factors for several rods. Using (1), a value for the intrinsic loss tangent of the material is computed. Comparison of the different values for $\tan\delta$ shows that the theory is consistent with experimental results.

At 72.70 GHz, the theoretical attenuation for a section of WR-15 silver waveguide is 1.6 dB/m. Only the 2-mm

TABLE II
ATTENUATION FACTORS OF VARIOUS DIELECTRIC WAVEGUIDES AT 72.70 GHz

MATERIAL	DIAMETER (mm)	d/λ_0	POWER ATTENUATION (dB/meter)	R	tan δ
Rexolite 1422	1.60	.3877	3.93	.2251	.00103
"	2.38	.5768	9.06	.5438	.00098
"	3.20	.7755	10.10	.6233	00096
Polystyrene	3.32	.8045	11.25	.6288	.00106
"	3.63	.8797	12.47	.6350	.00116
Teflon	2.01	.4871	.83	.3253	.00019
"	3.00	.7270	1.90	.6001	.00023
"	3.23	.7817	2.17	.6277	.00025

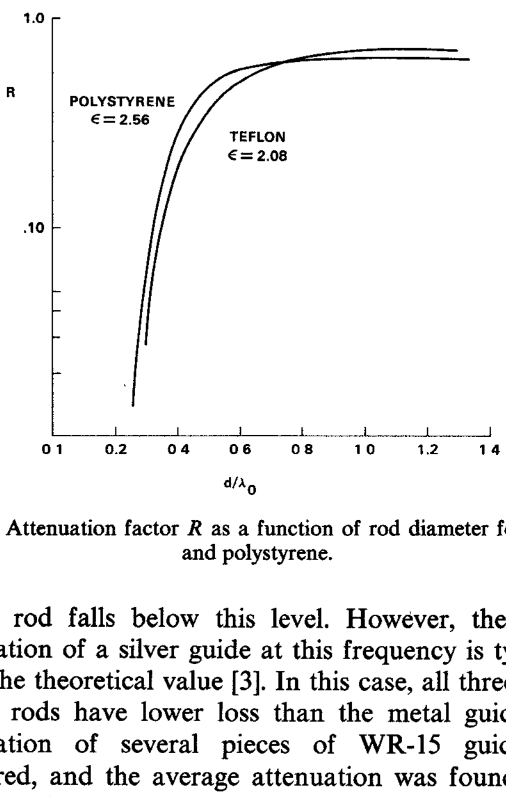

Fig. 3. Attenuation factor R as a function of rod diameter for Teflon and polystyrene.

Teflon rod falls below this level. However, the actual attenuation of a silver guide at this frequency is typically twice the theoretical value [3]. In this case, all three of the Teflon rods have lower loss than the metal guide. The attenuation of several pieces of WR-15 guide was measured, and the average attenuation was found to be over 5 dB/m, presumably due to surface deterioration. A comparable aging of the surface of Teflon would not produce a comparable loss since the primary loss mechanism is bulk rather than surface absorption.

It is often assumed that a dielectric waveguide will radiate a significant amount of energy. Examination of basic theory indicates that a cylindrical guide with no surface imperfections will not radiate. Radiation occurs when surface imperfections or bends introduce higher order, unbound modes. In connection with the development of fiber optics communications systems, Marcuse [4], [5] and Marcatili [6] have investigated the problem of radiation from dielectric guides. In particular, Marcuse has shown that the amount of radiation depends not only on the magnitude of the surface imperfections, but on the spatial frequencies of the imperfections. He relates the amount of coupling between guided and radiation modes to the Fourier transform of the wall distortion function of the dielectric guide[1] [4]. In a separate paper he investigates the amount of radiation loss from the HE_{11} mode of a Teflon guide with random surface imperfections [5]. Without investigating the surface profile of the dielectric rod in great detail, it is impossible to predict how much radiation loss will occur. However, the theory indicates that radiation loss due to surface imperfections need not be substantial for a millimeter wavelength dielectric guide.

If radiation loss were significant, then the observed values of tan δ in Table II would be substantially higher than the intrinsic loss tangent of the dielectric. Data for tan δ at 70 GHz is not available, but comparison of the results with lower frequency data for tan δ suggests that radiation loss is not significant [7]. But because of the wide variations in dielectric properties of different samples of a given material at these frequencies, it is impossible to draw any definite conclusions.

The net results of this study indicate that at 70 GHz, dielectric waveguide can be made to have lower loss than conventional metal waveguide. Since the loss tangent of most materials decreases sharply as frequency is reduced, it is probable that the dielectric guide will have losses comparable to or lower than those of metal waveguide at lower millimeter wave frequencies as well. And there is little doubt that the quality of the dielectric materials can be improved, reducing the losses even further.

There are many mechanical, chemical, and electrical problems to be solved before dielectric waveguide can be made to replace metal guide. In particular, it will require some mechanical ingenuity to physically support the guide without interfering with its electrical properties. However, the difference in price and weight between silver and plastic may well justify the effort to overcome these problems.

[1]It should be noted that optical fiber theory deals primarily with multimode propagation in fibers whose diameters are orders of magnitude greater than the wavelength of light they transmit. To minimize dispersion resulting from the presence of so many modes, the fibers are typically constructed with a graded index of refraction. The single-mode constant index of refraction dielectric guide described in this paper represents the simplest case of optical fiber theory.

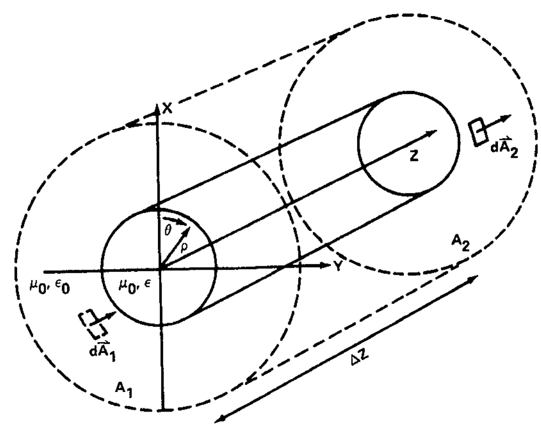

Fig. 4. Section of a dielectric waveguide.

III. Appendix

A section of the dielectric waveguide is sketched in Fig. 4. The field solutions for the various guided modes are found by solving Maxwell's equations subject to the appropriate boundary conditions (i.e., continuity of tangential $\bar{E}$ and $\bar{H}$ at $r=a$).

The internal fields are described by Bessel functions, which are bounded at the origin. Outside the dielectric, the electric and magnetic fields behave as Hankel functions, which decay exponentially as r approaches ∞. Matching the boundary conditions yields the guidance condition for the dielectric rod. This is described in detail by Stratton [8]. Using Elsasser's notation [2], the guidance condition is

$$(\epsilon_r f+g)(f+g)-n^2(\epsilon_r/p^2+1/q^2)(1/p^2+1/q^2)=0 \quad (2)$$

where

$$f=\frac{J_n'(p)}{pJ_n(p)} \qquad g=\frac{H_n^{2\prime}(q)}{qH_n^2(q)}. \quad (3)$$

$J_n(p)$ and $H_n^2(q)$ are the ordinary Bessel function and the Hankel function of the second kind, respectively. The primes denote differentiation. ϵ_r is the relative dielectric constant ϵ_2/ϵ_1. For a dielectric rod suspended in air, ϵ_1 can be replaced by ϵ_0, the permittivity of free space. n is the mode index coefficient, and is an integer. p and q are described by the relationships

$$d/\lambda_0=\frac{1}{\pi}\left(\frac{p^2+q^2}{\epsilon_r-1}\right)^{1/2} \quad (4)$$

$$\lambda/\lambda_0=\left[\frac{(1/p^2+1/q^2)}{\epsilon_r/p^2+1/q^2}\right]^{1/2}. \quad (5)$$

For $n=0$, (2) has the solutions

$$(\epsilon_r f+g)=0 \quad (6a)$$

$$(f+g)=0. \quad (6b)$$

These solutions describe the TE_{01} and TM_{01} modes of the guide. For $n=1$, (2) describes the HE_{11} mode. Unlike the $n=0$ modes, the HE_{11} mode has both a longitudinal $\bar{E}$ and a longitudinal $\bar{H}$ field. With the exception of the $n=0$ modes, all guided modes of a dielectric cylinder have this hybrid nature.

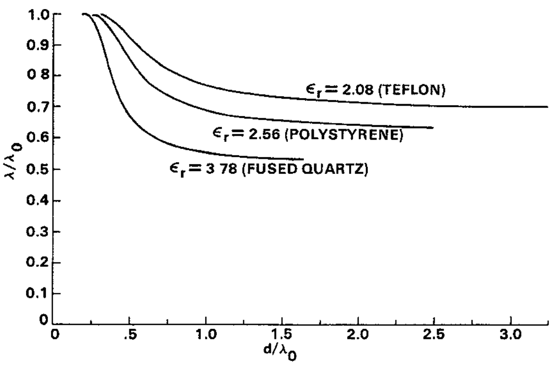

Fig. 5. Guidance characteristics of the HE_{11} mode.

The critical diameter below which only the HE_{11} mode can propagate is given by [1]

$$J_0(x)=0 \quad (7a)$$

$$x=\pi d_c(\epsilon_r-1)^{1/2}. \quad (7b)$$

Fig. 5 is a graphical solution of the guidance condition for the HE_{11} mode. Since the guidance condition is transcendental, numerical techniques were necessary. Solutions are shown for the values of dielectric constant corresponding to fused quartz ($\epsilon_r=3.78$), polystyrene (2.56), and Teflon (polytetrafluoroethylene, 2.08). The attenuation factor α is derived by applying Poynting's theorem to the section of waveguide in Fig. 4. If all of the power is propagating in the direction of the rod, then the net power flow into the volume element dV is

$$P_{\text{in}}=\int \bar{E}\times\bar{H}\cdot d\bar{A}_1. \quad (8a)$$

The power leaving the volume is

$$P_{\text{out}}=\int \bar{E}\times\bar{H}\cdot d\bar{A}_2. \quad (8b)$$

The energy dissipated is

$$\int \bar{E}\cdot\bar{J}\,dV, \qquad \text{where } \bar{J}=\sigma\bar{E}. \quad (9)$$

σ need not be an ohmic conductivity; in the case of a lossy dielectric, $\sigma=\omega\epsilon\tan\delta$.

Conservation of power requires that

$$\int \bar{E}\times\bar{H}\cdot d\bar{A}_1-\int \bar{E}\times\bar{H}\cdot d\bar{A}_2=\int \bar{E}\cdot\bar{J}\,dV. \quad (10)$$

Defining $\Phi=\int\bar{E}\times\bar{H}\cdot d\bar{A}$, and taking the limit as $\Delta z\to 0$, (10) becomes

$$\frac{d\Phi}{dz}=\int \bar{E}\cdot\bar{J}\,dA. \quad (11)$$

Defining α to be the power attenuation factor (as opposed to voltage attenuation), we have

$$\alpha=\frac{1}{\Phi}\frac{d\Phi}{dz}. \quad (12)$$

This leaves

$$\alpha = \frac{\int \bar{E} \cdot \bar{J}\, dA}{\int \bar{E} \times \bar{H} \cdot d\bar{A}} = \frac{\int_0^{2\pi} \int_0^{\infty} \bar{E} \cdot \bar{J}\, \rho d\rho\, d\theta}{\int_0^{2\pi} \int_0^{\infty} (E_\rho H_\theta - E_\theta H_\rho)\, \rho d\rho\, d\theta}. \tag{13}$$

If the medium outside the dielectric is lossless, the current density J will be zero for $\rho > a$. Thus the integral in the numerator need only be evaluated between $0 < \rho < a$.

By introducing the free space impedance $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, and relating the conductivity σ to the loss tangent of the dielectric, (13) can be rewritten

$$\alpha = \sigma \eta R = \epsilon_r \tan\delta\, \omega \sqrt{\mu_0 \epsilon_0}\, R = \frac{2\pi}{\lambda_0} \epsilon_r \tan\delta\, R \tag{14}$$

where

$$R = \frac{\int_0^{2\pi} \int_0^{a} (|\bar{E}|^2/\eta_0)\, \rho d\rho\, d\theta}{\int_0^{2\pi} \int_0^{\infty} (E_\rho H_\theta - E_\theta H_\rho)\, \rho d\rho\, d\theta}. \tag{15}$$

R can be calculated for the HE_{11} mode from the following equations: [2]

$$R = \left| \frac{\dfrac{\epsilon - 1}{q^2} \dfrac{f^2 + (1/p^2) - (1/p^4)}{(1/p^2) + (1/q^2)} + (U^2 + V^2)X + \dfrac{4UV}{p^4}}{UX(\epsilon + V^2) + UY(1 + V^2) + \dfrac{2V}{p^4}(\epsilon + U^2) - \dfrac{2V}{q^4}(1 + U^2)} \right| \tag{16a}$$

$$\left\{(p^2 + q^2)/(\epsilon - 1)\right\}^{1/2} = (2\pi a/\lambda_0) \tag{16b}$$

$$U = \lambda_0/\lambda = \left\{(\epsilon q^2 + p^2)/(q^2 + p^2)\right\}^{1/2} \tag{16c}$$

$$V = \left\{(\epsilon f + g)/(f + g)\right\}^{1/2} \tag{16d}$$

$$X = f^2 + (2f + 1)/p^2 - 1/p^4 \tag{16e}$$

$$Y = -g^2 - (2g - 1)/q^2 + 1/q^4 \tag{16f}$$

$$f = \frac{J_n'(p)}{pJ_n(p)} \qquad g = \frac{H_n^{2\prime}(q)}{qH_n^2(q)}. \tag{16g}$$

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