On-Line Accuracy Assessment for the Dual Six-Port ANA: Treatment of Systematic Errors

CLETUS A. HOER

Abstract—Expressions are derived for calculating estimates of the systematic errors in dual six-port or four-port measurements of reflection coefficient and scattering parameters due to imperfections in the transmission-line standard used to calibrate the system. A new mathematical model for a four-port reflectometer makes it easier to visualize and analyze these errors. In this new model, two of the three parameters needed to characterize a four-port can be determined without standards. All imperfections in the standard perturb only the third parameter which acts as an impedance transformer.

I. INTRODUCTION

A block diagram of a dual six-port Automatic Network Analyzer (ANA) is shown in Fig. 1(a). Each six-port reflectometer is a linear passive network capable of measuring power and complex reflection coefficient at its measurement port (1 or 2 in Fig. 1(a)) in terms of power measurements made at four sidearm ports when an RF signal is applied to the remaining port. The S parameters of two-port devices are measured by inserting the two-port between the two six-ports which measure the ratios b_1/a_1 and b_2/a_2 on either side of the two-port [1]. A one-port termination is measured by connecting it to either test port 1 or 2. In this case, the ratio b_1/a_1 or b_2/a_2 is the reflection coefficient of the termination.

One way of calibrating a dual six-port ANA having sexless test-port connectors is shown in Fig. 1(b). The measurement planes are connected together, then one or more highly reflecting terminations are connected to one six-port, and then to the other six-port. Finally, a length of precision transmission line is connected between the two six-ports. This "thru-reflect-line" (TRL) [2] calibration technique assumes that S_{11} and S_{22} of the line are zero. The transmission coefficient S_{12} of the line, and the reflection coefficients Γ of the highly reflecting terminations do not need to be known. These parameters are determined along with the parameters describing the two six-ports.

The primary purpose of this paper is to determine the systematic errors in the measurement of Γ and S_{ij} when S_{11} and S_{22} of the standard line are not zero. These systematic errors will be considered to arise from two separate causes: 1) the line and connectors are essentially uniform and nonreflecting, but have dimensions different from those specified by some standards documents; and

Fig. 1. (a) and (b) Block diagram of a dual six-port ANA and connections made to calibrate it.

2) the line is perfect but reflections exist due to discontinuities at the connectors. Since, as described below, a six-port can be mathematically reduced to a four-port, the results apply equally well to a dual four-port as to a dual six-port.

A secondary purpose of this paper is to describe a new mathematical model for a four-port, making the analysis of these errors easier.

II. MATHEMATICAL MODEL

A mathematical model for a six-port or four-port reflectometer that is useful in analyzing systematic errors is shown in Fig. 2. The six-port sidearm power readings P_i are related to w through (1) where ρ_i and λ_i are parameters that depend on the six-port design, and P_0 is one of the sidearm readings chosen as a reference [3]. Once the ρ_i and λ_i are determined, the six-port is effectively reduced to a four-port whose sidearm ratio w is calculated from a set of power readings.

For a general four-port, the relationship between w and

Mathematical Model

Fig. 2. Mathematical model of a six-port showing different levels of dependence on the quality of the connectors and the standard. Equation (1) is independent of the quality of connectors and standards; (2) depends on the connector quality but is independent of the standard; and (3) depends on quality connectors and standard.

Γ involves three complex parameters. This relationship is often expressed as in (4) where A , B , and C are the three parameters referred to. We have recently discovered that these three parameters can be redefined and chosen in such a way that two of them, α and β in Fig. 2, can be determined without any standards. A standard is used only to obtain the last parameter, labeled C . Therefore, any imperfections in the standard affect only this single parameter C .

This leads one to define a new parameter g which is linearly related to w through α and β as in (2). Then g is related to Γ through (3) which contains the single parameter C .

The ρ_i and λ_i are determined from a collection of power readings obtained from five or more terminations having unknown Γ . Their determination does not require any standard or even quality connectors. The parameters α and β may be determined from the “thru” and “reflect” measurements alone. Therefore, they are sensitive to imperfections in the test-port connectors and in the connectors on the terminations of unknown Γ used in the “reflect” measurement, but they are not sensitive to imperfections in the standard line.

The last measurement with the standard line inserted provides a determination of C which is dependent on the quality of the line and the quality of the connectors. Defining the four-port parameters in this manner makes determining and understanding sources of error easier.

III. TRANSFORMING Γ

To give some meaning to C in (3), consider the following equations for transforming a reflection coefficient from one normalizing impedance to another. If Γ_{ζ_0} is the reflection coefficient measured relative to a normalizing or characteristic impedance ζ_0 , and Γ_{Z_0} is the corresponding reflection coefficient relative to Z_0 , then one can trans-

form from one impedance to the other with [4]

$$\Gamma_{Z_0} = \frac{\Gamma_{\zeta_0} - \gamma_0}{1 - \gamma_0 \Gamma_{\zeta_0}} \quad (5a)$$

or

$$\Gamma_{\zeta_0} = \frac{\gamma_0 + \Gamma_{Z_0}}{1 + \gamma_0 \Gamma_{Z_0}} \quad (5b)$$

where

$$\gamma_0 \equiv \frac{Z_0 - \zeta_0}{Z_0 + \zeta_0} \quad (6)$$

Comparing (5b) with (3) shows that C in the four-port model plays the role of an impedance transform like γ_0 in (6).

IV. TRANSFORMING S_{ij}

Equations for transforming S parameters S_{ij} of a two-port from one set of normalizing impedances (ζ_{01} , ζ_{02}) to S'_{ij} which are relative to another set (Z_{01} , Z_{02}) are given by Woods [4] as follows. If

$$\gamma_{01} \equiv \frac{Z_{01} - \zeta_{01}}{Z_{01} + \zeta_{01}} \quad (7)$$

$$\gamma_{02} \equiv \frac{Z_{02} - \zeta_{02}}{Z_{02} + \zeta_{02}} \quad (8)$$

then

$$S'_{11} = (S_{11} - \gamma_{01} - \gamma_{02}\delta + \gamma_{01}\gamma_{02}S_{22})/D \quad (9)$$

$$S'_{22} = (S_{22} - \gamma_{02} - \gamma_{01}\delta + \gamma_{01}\gamma_{02}S_{11})/D \quad (10)$$

$$S'_{12} = S_{12}(1 - \gamma_{01})(1 - \gamma_{02})/D \quad (11)$$

$$S'_{21} = S_{21}(1 - \gamma_{01})(1 - \gamma_{02})/D \quad (12)$$

where

$$D \equiv 1 - \gamma_{01}S_{11} - \gamma_{02}S_{22} + \gamma_{01}\gamma_{02}\delta \quad (13)$$

$$\delta \equiv S_{11}S_{22} - S_{12}S_{21}. \quad (14)$$

V. CALIBRATING THE FOUR-PORTS

Now, assume that if six-ports are used, they have been mathematically reduced to four-ports so that only a dual four-port needs to be considered in the rest of this paper. Also assume that α and β of both four-ports have been determined from the thru and reflect measurements so that only C_1 and C_2 in Fig. 3(a) are left to be determined from the line measurement.

In the line part of the TRL calibration, where C_1 and C_2 are determined, it is assumed that the S parameters for the line are $S_{11} = S_{22} = 0$ and $S_{12} = e^{-\gamma l}$, where γ is the propagation constant of the line and l is the length of the line. These assumptions imply that one has chosen a normalizing impedance which is the actual characteristic impedance, say ζ_0 , of the particular line used in the calibration. All Γ and S parameter measurements made by

Fig. 3. (a) and (b) Adding ΔC to transform Γ and S_{ij} , which are relative to ζ_0 to Γ' and S'_{ij} , which are relative to Z_0 .

either or both four-ports after this calibration are relative to ζ_0 , whatever that might be.

VI. DETERMINING ζ_0 OF THE LINE

Since γl of the line is determined from the TRL solution, it can be used in determining ζ_0 of the line. For a coaxial transmission line [5]

$$\gamma = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)} \quad (15)$$

$$\zeta_0 = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}} \quad (16)$$

where R_0 and L_0 are the series resistance and inductance per unit length of the line, respectively, and G_0 and C_0 are the parallel conductance and capacitance per unit length of the line, respectively. For an air line, $|G_0| \ll |\omega C_0|$. Then (15) and (16) combine to give

$$\zeta_0 = \frac{\gamma}{G_0 + j\omega C_0} \approx \frac{\gamma l}{j\omega C_0 l} \quad (17)$$

Since γl is known, only $C_0 l$ needs to be determined. The capacitance per unit length can be calculated from [5]

$$C_0 = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}, \quad \text{F/m} \quad (18)$$

where

- a outer diameter of inner conductor,
- b inner diameter of outer conductor,
- ϵ_r relative dielectric constant of the dielectric, and
- ϵ_0 dielectric constant of free space = 8.854×10^{-12} F/m.

Since C_0 is independent of frequency, $C_0 l$ is the total capacitance of the line at low frequencies and can be accurately measured with a precision capacitance bridge at say 1 kHz. This provides another method for obtaining $C_0 l$ if the line is sufficiently long to have a large enough capacitance to be measured accurately.

VII. UNCERTAINTY IN DETERMINING ζ_0

Because of losses in a real line, ζ_0 will always be complex with a slightly negative phase angle. Also, because of imperfections in the line such as inaccurate or non-uniform dimensions, eccentricity, sag, etc., ζ_0 will differ from the characteristic impedance Z_0 of a "perfect" line defined by some documents specifying the dimensions, conductivity, and Z_0 of a standard line.

If ζ_0 is obtained from (17), the worst-case uncertainty in ζ_0 can be obtained from the following derivative of (17) and propagation-of-error formulae derived by Ku [6]:

$$\frac{d\zeta_0}{\zeta_0} = \frac{d(\gamma l)}{\gamma l} - \frac{d(C_0 l)}{C_0 l} \quad (19)$$

In the National Bureau of Standard's (NBS) dual six-port, the error $d(\gamma l)$ in γl is assumed to be random. In (19) this error is taken to be a systematic error and is obtained from three times the standard deviation of γl estimated from the TRL calibration [7]. A sample calculation is given in Section XI.

If the total capacitance $C_0 l$ is measured with a capacitance bridge, then $d(C_0 l)$ is obtained from the bridge specifications. If C_0 is calculated from (18), then the uncertainty in $C_0 l$ obtained by differentiating (18) is

$$\frac{d(C_0 l)}{C_0 l} = \frac{dl}{l} + \frac{d\epsilon_r}{\epsilon_r} + \frac{da/a - db/b}{\ln(b/a)} \quad (20)$$

The uncertainties da and db , with which a and b are measured, may have to be inflated to take into account other imperfections such as nonuniformity, sag, eccentricity, etc.

VIII. TRANSFORMING FROM ζ_0 TO Z_0

Let Z_0 be the characteristic impedance specified by some standards documents as that of a perfect line standard relative to which Γ and S_{ij} are to be measured. For example, Z_0 might be 50 Ω . Then, each Γ in Fig. 3(a) which is relative to ζ_0 can be transformed to a Γ' relative to Z_0 by adding a transform ΔC to each four-port as shown in Fig. 3(b), where

$$\Delta C = \frac{Z_0 - \zeta_0}{Z_0 + \zeta_0} \quad (21)$$

Measurements of Γ and S_{ij} relative to ζ_0 can be transformed to Γ' and S'_{ij} relative to Z_0 using (5a) and (9)–(14), replacing γ_0 (or γ_{01} and γ_{02}) with ΔC . Equation (5a) becomes

$$\Gamma' = \frac{\Gamma - \Delta C}{1 - \Delta C \Gamma} \quad (22)$$

When $|\Delta C|^2 \ll 1$, (9)–(14) reduce to

$$S'_{11} = S_{11} - (1 - S_{11}^2 - S_{12}S_{21})\Delta C/D \quad (23)$$

$$S'_{22} = S_{22} - (1 - S_{22}^2 - S_{12}S_{21})\Delta C/D \quad (24)$$

$$S'_{12} = S_{12}(1 - \Delta C^2)/D \quad (25)$$

$$S'_{21} = S_{21}(1 - \Delta C^2)/D \quad (26)$$

where

$$D = 1 - (S_{11} + S_{22}) \Delta C. \quad (27)$$

From (22) we can write

$$\Delta \Gamma \equiv \Gamma' - \Gamma = \frac{(1 - \Gamma^2) \Delta C}{1 - \Gamma \Delta C}. \quad (28)$$

Note that $\Delta \Gamma = 0$ when $\Gamma = -1$ or $+1$ for a short or open. In other words, a short or open always transforms to a short or open, independent of the value of the transform ΔC .

IX. SYSTEMATIC ERRORS IN Γ' AND S'_{ij}

A. From ζ_0

If *all* measurements were made relative to the same line or an identical line (an impossibility) and if that line were accepted as the standard of reflection coefficient, then there would be no systematic error associated with the standard. Systematic error comes about when it is necessary to relate Γ and S_{ij} measured relative to ζ_0 to a different Z_0 of a line having specified dimensions. The systematic error is therefore determined by how well ΔC in (21) can be determined.

Since Z_0 in (21) is specified, say 50Ω , there is no error associated with Z_0 . The uncertainty ϵ in ΔC obtained by differentiating (21) is, therefore

$$\epsilon \equiv d(\Delta C) = \frac{2Z_0 d\zeta_0}{(Z_0 + \zeta_0)^2} \approx -\frac{d\zeta_0}{2\zeta_0} \quad (29)$$

where $d\zeta_0/\zeta_0$ may be obtained from (19). The corresponding uncertainties $\Delta \Gamma$ and ΔS_{ij} in Γ' and S'_{ij} are obtained from equations (22)–(27) with ΔC replaced by ϵ .

$$\Delta \Gamma = -\frac{(1 - \Gamma^2) \epsilon}{1 - \Gamma \epsilon}. \quad (30)$$

$$\Delta S_{11} = -(1 - S_{11}^2 - S_{12}S_{21}) \epsilon / D \quad (31)$$

$$\Delta S_{22} = -(1 - S_{22}^2 - S_{12}S_{21}) \epsilon / D \quad (32)$$

$$\Delta S_{12} = S_{12}(S_{11} + S_{22} - \epsilon) \epsilon / D \quad (33)$$

$$\Delta S_{21} = S_{21}(S_{12} + S_{22} - \epsilon) \epsilon / D \quad (34)$$

where

$$D = 1 - (S_{11} + S_{22}) \epsilon. \quad (35)$$

B. From Connectors

Reflections from connector discontinuities on either end of the line were not included in the analysis up to this point. An equation for estimating the effect of connector imperfections can be obtained from (23) and (24). When it is assumed that $S_{11} = S_{22} = 0$ during the calibration, (23) and (24) give

$$S'_{11} = S_{22} = -(1 - S_{12}^2) \Delta C. \quad (36)$$

Solving for ΔC :

$$\Delta C = \frac{-S'_{11}}{1 - S_{12}^2}. \quad (37)$$

This equation can be used to calculate an estimate of ΔC from an estimate of S'_{11} caused by connector discontinuities, where $S_{12} = e^{-\gamma l}$ is the transmission through the line. This value of ΔC is used in (30)–(35) in place of ϵ to estimate systematic errors caused by connector imperfections.

A useful approximation for (37) can be obtained by assuming that the line is lossless. Then $S_{12} = e^{j\psi}$ and (31) becomes

$$\Delta C = \frac{-S'_{11}}{2 \sin \psi} e^{-j\theta} \quad (38)$$

where $\theta = \psi - 90^\circ$ and ψ is the phase shift through the line. If the estimated reflection $|S'_{11}|$ is assumed to be constant with frequency, then (38) states that $|\Delta C|$ is a minimum when $\psi = 90^\circ$ and increases as ψ becomes less than or greater than 90° . When $\psi = 90^\circ$, $\Delta C = -S'_{11}/2$.

Fig. 4 shows the value of $\epsilon \equiv |\Delta C|$ relative to the value when $\psi = 90^\circ$ as a function of ψ , assuming that S'_{11} is independent of frequency.

X. WORST-CASE ERRORS

If nothing is known about the phase angle of $d(\Delta C)$ in (29) or ΔC is (37) and (38), which is often the case, then (30)–(35) can be used to estimate the maximum error in Γ and S_{ij} . Let ϵ represent $d(\Delta C)$ in (29) or ΔC in (37) and (38). Then from (30), the maximum systematic error $|\Delta \Gamma|$ in Γ due to ϵ is

$$|\Delta \Gamma| = \frac{|1 - \Gamma^2| |\epsilon|}{1 - |\Gamma| |\epsilon|}. \quad (39)$$

The corresponding maximum error $\Delta \psi$ in the phase angle ψ of Γ is obtained from the diagram in Fig. 5 as

$$\Delta \psi = \sin^{-1} \frac{|\Delta \Gamma|}{|\Gamma|}. \quad (40)$$

The maximum systematic error $|\Delta S_{ij}|$ in each S_{ij} is obtained from (31)–(35) in the same way, that $|\Delta \Gamma|$ in (39) was obtained from (30). The corresponding maximum error $\Delta \psi_{ij}$ in the phase angle ψ_{ij} of S_{ij} is

$$\Delta \psi_{ij} = \sin^{-1} \frac{|\Delta S_{ij}|}{|S_{ij}|}. \quad (41)$$

XI. AN EXAMPLE

In this example, we calculate ϵ from (29), (19), and (20) for a 1.5-cm length of precision 7-mm air line. Assume that the diameters are measured to an uncertainty of ± 0.00013 cm, and that they are uniform to this value along their length. Assume that the length is measured to ± 0.00025 cm. Also assume that the relative dielectric constant of air is known to 0.01 percent. Then the worst-case error in $C_0 l$ obtained from (20) is

$$\begin{aligned} \frac{d(C_0 l)}{C l} &= \left\{ \frac{2.5}{1.5} + \frac{1}{1} + \frac{1.3}{\ln(7/3)} \left[\frac{1}{0.7} + \frac{1}{0.3} \right] \right\} \\ &\times 10^{-4} = 0.0010. \end{aligned} \quad (42)$$

Fig. 4. Relative change in systematic error as a function of the phase shift through the standard line.

Fig. 5. Relationship between the maximum error $|\Delta\Gamma|$ in the magnitude of Γ and the maximum error $\Delta\psi$ in the phase angle of Γ .

To this value must be added the uncertainty in propagation constant γl of the line to obtain $d\xi_0/\xi_0$ in (19). The Appendix gives the following expression for this uncertainty in terms of the uncertainty dS_{12} decibels in the magnitude of S_{12} of the line, and the uncertainty $d\psi_{12}$ in the phase shift ψ_{12} through the line:

$$\frac{d(\gamma l)}{\gamma l} = -\frac{d\psi_{12}}{\psi_{12}} - j \frac{dS_{12} \text{ (dB)}}{8.69\psi_{12}}. \quad (43)$$

Typical standard deviations for the random errors in these values obtained from the TRL calibration of the NBS dual six-port at 2 GHz are

$$dS_{12} \text{ (dB)} = 0.0008 \text{ dB}$$

$$d\psi_{12} = 0.003^\circ = 5 \times 10^{-5} \text{ rad}$$

where

$$\psi_{12} = 36^\circ = 0.63 \text{ rad.}$$

The systematic errors in these values are negligible. Using three times these values (3σ values) in (43) gives

$$\frac{d(\gamma l)}{\gamma l} = (2.4 + j4.5) 10^{-4}. \quad (44)$$

Combining (42) and (44) in (19) and (29) using prop-

agation of error formulae derived by Ku [6] gives for a worst-case error

$$\epsilon = 0.00062 + j0.00023 \quad (45)$$

and

$$|\epsilon| = 0.00066. \quad (46)$$

To this must be added an estimate of the uncertainty due to connector imperfections. Assuming $|S'_{11}| = 0.001$ in (38) gives

$$|\Delta C| = 0.00085. \quad (47)$$

Adding (46) and (47) gives an overall worst-case value for ϵ of

$$|\epsilon| = 0.0015. \quad (48)$$

From (30), the above value of ϵ creates a systematic error in measuring a matched termination of $\Delta\Gamma = \epsilon$. The maximum error in measuring reflection coefficients will be the error in a termination whose value is $\Gamma = \pm j$. For this termination, (30) or (39) give $|\Delta\Gamma| \approx 2|\epsilon| = 0.003$. Similar values are obtained for ΔS_{11} and ΔS_{22} from (31) and (32).

Equations (33) and (34) show that the value of $|\epsilon| = 0.0015$ creates a negligible systematic error in measuring attenuation. First express ΔS_{12} in decibels by writing

$$\begin{aligned} \Delta S_{12} \text{ (dB)} &= 20 \log \left| \frac{S'_{12}}{S_{12}} \right| \\ &= 20 \log \left| 1 + \frac{\Delta S_{12}}{S_{12}} \right| \\ &= 8.69 \left| \frac{\Delta S_{12}}{S_{12}} \right| \text{ dB.} \end{aligned} \quad (49)$$

Assuming that $S_{11} = S_{22} = 0.05$, (33) in (49) gives a systematic error of 0.0013 dB in measuring attenuation of any value. If $S_{11} = S_{22} = 0.01$, this systematic error drops to 0.0003 dB. The corresponding maximum systematic errors in the phase shift through the two-port are obtained from (41) as 0.009° and 0.002° , respectively. These systematic errors in phase are also independent of the value of attenuation through the two-port.

XII. CONCLUSIONS

When a length of uniform transmission line is used as the standard in calibrating a dual six-port or four-port ANA, all measured values of Γ or S_{ij} are relative to the characteristic impedance ξ_0 of that line. These values of Γ and S_{ij} must be transformed to some specified characteristic impedance Z_0 . Systematic errors are created by uncertainties in this transformation and by imperfections in connectors. Equations for calculating estimates of these systematic errors have been derived and an example has been given.

The systematic error in Γ is a function of Γ , being zero for $\Gamma = \pm 1$, and maximum for $\Gamma = \pm j$. The systematic

error for $\Gamma = 0$ is one-half the maximum. The systematic errors in S_{11} and S_{22} of a two-port are functions of all the S parameters of the two-port. However, the systematic errors in S_{12} , when expressed in decibels and phase angle, are independent of the value of attenuation of the two-port, thereby becoming negligible compared to random errors when the attenuation becomes large.

APPENDIX A

To express $d(\gamma l)/\gamma l$ in (19) in terms of S_{12} of the line, let

$$\gamma l = x + jy \quad (\text{A1})$$

and

$$S_{12} = |S_{12}| e^{j\psi_{12}}. \quad (\text{A2})$$

Then

$$S_{12} = e^{-\gamma l} = e^{-x} e^{-jy}. \quad (\text{A3})$$

Expressing S_{12} in decibels

$$\begin{aligned} S_{12} \text{ (dB)} &= -20 \log_{10} |S_{12}| \\ &= -20 \log_{10} e^{-x} = 20x \log_{10} e \end{aligned} \quad (\text{A4})$$

or

$$x = \frac{S_{12} \text{ (dB)}}{8.69} \text{ Np}. \quad (\text{A5})$$

Also, from (A2) and (A3)

$$y = -\psi_{12} \text{ rad}. \quad (\text{A6})$$

From (A1)

$$\frac{d(\gamma l)}{\gamma l} = \frac{dx + jdy}{x + jy}. \quad (\text{A7})$$

For low-loss lines $x \ll y$, which reduces (A7) to

$$\frac{d(\gamma l)}{\gamma l} \approx \frac{dy}{y} - j \frac{dx}{y}. \quad (\text{A8})$$

Using (A5) and (A6) in (A8) gives the desired result:

$$\frac{d(\gamma l)}{\gamma l} = -\frac{d\psi_{12}}{\psi_{12}} - j \frac{dS_{12} \text{ (dB)}}{8.69\psi_{12}} \quad (\text{A9})$$

where $d\psi_{12}$ and ψ_{12} are in radians.

REFERENCES

- [1] C. A. Hoer, "A network analyzer incorporating two six-port reflectometers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, no. 12, pp. 1070-1074, Dec. 1977.
- [2] G. F. Engen and C. A. Hoer, "'Thru-Reflect-Line': An improved technique for calibrating the dual six-port ANA," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, no. 12, pp. 987-993, Dec. 1979.
- [3] G. F. Engen, "Calibrating the six-port reflectometer by means of sliding terminations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, no. 12, pp. 951-957, Dec. 1978.
- [4] D. Woods, "Coaxial two-port resistor and its application to rf measurements and standards," *Proc. Inst. Elec. Eng.*, vol. 118, no. 12, sec. 11, pp. 1859-1866, Dec. 1971.
- [5] S. Ramo, J. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1956, pp. 46, 89.
- [6] H. H. Ku, "Notes on the use of propagation of error formulas," *J. Res. Nat. Bur. Stands. (Eng., Instrum.)*, vol. 70C, no. 4, pp. 263-273, Oct.-Dec. 1966.
- [7] R. M. Judish, "On-line accuracy assessment for the dual six-port ANA: Statistical methods for random errors," pp. 507-513, this issue.