

sion of an Abel inversion routine to unfold the radial density profile for edge-on illumination of a plasma with cylindrical symmetry.

#### ACKNOWLEDGMENT

The author would like to acknowledge many valuable discussions with Prof. M. Kristiansen during the inception of this project. A note of appreciation is also extended to H. C. Kirbie for his assistance in assembling the interferometer.

#### REFERENCES

- [1] F. C. Jahoda, E. M. Little, W. E. Quinn, R. L. Ribe, and G. A. Sawyer, *J. Appl. Phys.*, vol. 35, p. 2351, 1964.
- [2] F. C. Jahoda, R. A. Jeffries, and G. A. Sawyer, *Appl. Opt.*, vol. 6, p. 1407, 1967.
- [3] F. C. Jahoda, *Appl. Phys. Lett.*, vol. 14, p. 341, 1969.
- [4] G. M. Molen, H. C. Kirbie, and M. J. Bernstein, *Bull. Amer. Phys. Soc.*, vol. 19, p. 945, 1974.
- [5] J. B. Gerardo and J. T. Verdeyen, *Proc. IEEE*, vol. 52, p. 690, 1964.
- [6] G. M. Molen, M. Kristiansen, M. O. Hagler, and R. D. Bengtson, *Appl. Phys. Lett.*, vol. 24, p. 583, 1974.

# A Frequency-Lock System for Improved Quartz Crystal Oscillator Performance

FRED L. WALLS AND SAMUEL R. STEIN

**Abstract**—The intrinsic noise of the best quartz crystal resonators is significantly less than the noise observed in oscillators employing these resonators. Several problem areas common to traditional designs are pointed out and a new approach is suggested for their solution. Two circuits are described which frequency lock a spectrally pure quartz crystal oscillator to an independent quartz crystal resonator. The performance of the composite system is predicted based on the measured performance of its components.

#### INTRODUCTION

IN RECENT YEARS, tremendous advances have been made in the manufacture of ultrastable quartz resonators. Moreover, it is quite likely that further improvements in this area will be made within the next two years. Especially promising are the SC cut, TTC cut, and the various electrodeless AT and SC cut resonators [1]–[4]. The purpose of this paper is to point out some problems in the electronics design of traditional quartz-crystal oscillators and to introduce some new circuit concepts which will significantly reduce these problems.

#### PROBLEM AREAS

The traditional circuitry for a crystal-controlled oscillator uses the resonator inside of the oscillating loop as shown schematically in Fig. 1. A necessary condition for oscillation

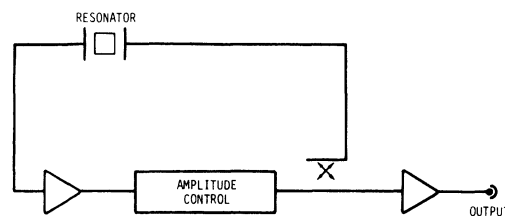


Fig. 1. Traditional quartz-crystal-controlled oscillator.

is that the phase shift around the loop be a multiple of  $2\pi$  rad. A small phase fluctuation  $\phi$  away from this state causes a fractional frequency change

$$y = \Delta\nu/\nu = \phi/2Q$$

where  $Q$  is the loaded quality factor of the resonator. In order to achieve a long-term fractional frequency stability of  $10^{-13}$  with a resonator having a loaded  $Q$  of  $2.5 \times 10^6$ , the phase variations must be less than  $5 \times 10^{-7}$  rad. For standard coaxial cable with phase stability of approximately 100 ppm/°C, this corresponds to a temperature change of 1°C over a 5 cm length. Since nearly all components are phase sensitive, it is doubtful that the required stability around the oscillating loop can be achieved for extended periods of time.

The phase shift around the loop is also perturbed by output loading and pickup of stray signals. For example, a 20-percent change in the load resistance from the matched condition produces a reflected signal back into the oscillator

Manuscript received February 9, 1978.

The authors are with the Time and Frequency Division, National Bureau of Standards, Boulder, CO 80302.

output whose amplitude is 10 percent of the output voltage. The reflection changes the phase in the oscillating loop by an amount

$$\phi = (1/10)\beta \cos \theta$$

where  $\beta$  is the isolation from the output to the oscillating loop expressed as a voltage ratio and  $\theta$  is the phase of the reflected signal relative to the unperturbed loop signal. In order to assure that  $|\phi|$  is less than  $5 \times 10^{-7}$  rad for arbitrary  $\theta$ ,  $\beta$  must be less than  $5 \times 10^{-6}$  or  $-106$  dB.

At the present time, crystal-controlled oscillators designed for good long term stability use either fifth overtone AT cut or FC cut resonators driven at approximately 0.1 to 1  $\mu$ W of crystal dissipation [4], [5]. The relatively low power dissipation is used in order to avoid excessively large frequency shifts. This effect, due to nonlinear interactions within the crystal, is commonly called the amplitude to frequency effect (AF), and is typically of the order  $\Delta\nu/\nu = 10^{-9}/\mu$ W of crystal dissipation for 5-MHz fifth overtone AT cut crystals. Therefore, it is necessary to control the crystal dissipation to  $10^{-4}$   $\mu$ W in order to achieve stabilities of  $10^{-13}$ . This required power stability then dictates a crystal dissipation of order 0.1 to 1  $\mu$ W. The relatively low value of crystal dissipation required to achieve the good long term stability limits the signal-to-noise ratio or spectral purity to a relatively poor value compared to a hard-driven oscillator. However, the AF effect is reported to be about 100 times smaller in the new doubly rotated cuts like the SC, TTC, and electrodeless SC cut resonators [2], [3], and [6].

Finally, the frequency of the oscillator is usually adjusted with a tuning capacitance of approximately 25 pF for a 5 MHz unit. If the crystal has a  $Q$  of  $2.5 \times 10^6$  and a motional resistance,  $R_s = 70 \Omega$ , then the fractional frequency change due to a small change in the tuning capacitor  $C_L$  is

$$y = \frac{1}{2} \left( \frac{1}{\omega RQ} \right) \left( \frac{1}{C_L} \right) \frac{\Delta C_L}{C_L} \cong 4 \times 10^{-6} \frac{\Delta C_L}{C_L}$$

which can be derived from considering the  $LCR$  equivalent circuit of a quartz crystal. Thus in order to achieve a long-term stability of  $10^{-13}$  the tuning capacitor must be constant to  $6 \times 10^{-7}$  pF, which is very difficult at best.

#### POSSIBLE SOLUTIONS

It is possible to attack all of these problems with a single circuit which uses the crystal in a frequency discriminator configuration instead of within an oscillating loop. Two possible realizations will be suggested which share the following principles: A crystal is operated at the optimum power dissipation for long-term stability while the required short-term stability is achieved with a second oscillator; the effect of the load capacitor is decreased an order of magnitude by increasing its value a factor of 10; load effects are minimized by using very-low-noise high-isolation buffer amplifiers; the effect of phase variations on the critical circuitry is reduced by using frequency or phase modulation in conjunction with a frequency-lock loop. The principles of operation are similar to the superconducting

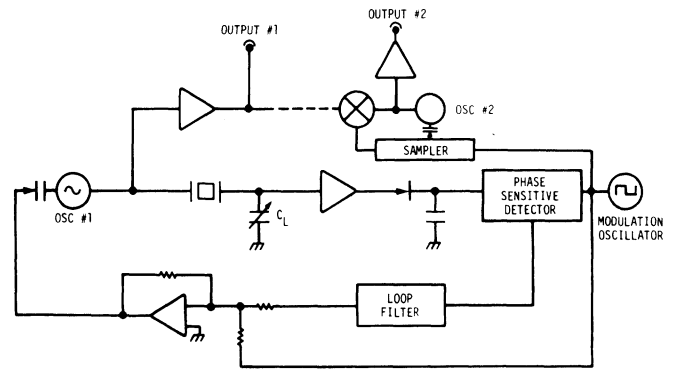


Fig. 2. Square-wave frequency-modulation system for locking a quartz-crystal oscillator with high spectral purity to a passive quartz resonator.

cavity-stabilized oscillator which is described in detail in the literature [7], [8].

The first circuit is shown in Fig. 2. The quartz crystal is interrogated with a square-wave frequency modulated signal from oscillator 1. Transmission through the resonator converts the frequency modulation to amplitude modulation if the average signal frequency differs from the resonant frequency. A diode is used to detect the modulation envelope whose phase depends upon whether the average signal frequency is above or below the resonance. Thus a phase sensitive detector can be used to generate a feedback signal which will frequency-lock oscillator 1 to the quartz crystal.

The power dissipated in the quartz crystal should be set to approximately  $10^{-7}$  W because of the amplitude to frequency effect. Since the crystal has a bandwidth of about 2 Hz, the spectral purity for Fourier frequencies above 1 Hz (the approximate cutoff frequency of the servo scheme) is determined by the performance of oscillator 1. The tuning capacitor  $C_L$  is increased an order of magnitude to 250 pF in order to reduce the sensitivity to the load capacitance and to swamp the stray capacitances which contribute to the total load capacitance. However, this implies that the resonator must be manufactured to closer tolerances than for traditional designs. The total electrical length of the transmission line connecting oscillator 1 to the diode demodulator via the crystal does not directly affect the frequency at which the system locks. However, dispersion in the transmission line does cause the lock frequency to deviate from the crystal resonance. Since the modulation frequency and its deviation is limited to about 1 Hz by the crystal  $Q$ , dispersive effects are smaller by a factor of  $2 \times 10^{-7}$  compared to the phase-stability problem of the traditional design. Finally, buffer amplifiers with isolation in excess of 120 dB and white phase noise floors  $S_\phi < -170$  dB are easily constructed thereby reducing to insignificance the problem of coupling spurious signals into the resonator [9].

There are two major disadvantages to the circuit of Fig. 2. First, since the frequency of the modulation must be small compared to 2 Hz, the attack time of the feedback loop needs to be longer than about 0.16 s. This means that the stability of oscillator 1 may be worse than the passive crystal before the attack time is reached. Second, the direct output of the circuit has phase excursions of about 1 rad. Consequently, to

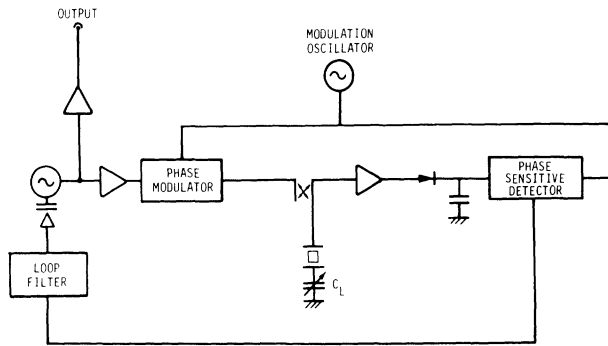


Fig. 3. Phase modulation system for locking an oscillator to a passive quartz resonator.

asynchronously realize a frequency stability of  $10^{-13}$  it is necessary to average for  $3 \times 10^5$  s or about 3 days. The latter problem may be overcome by phase-locking a oscillator 2 to oscillator 1 with a synchronous PLL. The loop filter for the PLL must average the phase difference between oscillators 1 and 2 over complete cycles of the modulation frequency in order to cancel most of the unwanted phase modulation.

This system appears to be quite complex and costly. A simpler system can be implemented which is based upon the same general principles, but which overcomes the two deficiencies just described. The simplified block diagram is shown in Fig. 3. Once again, the crystal dissipates about  $10^{-7}$  W,  $C_L$  is approximately 250 pF and extensive use is made of low-noise isolation amplifiers. One major difference from the previous circuit is that the required modulation is accomplished by phase modulating the oscillator output rather than frequency modulating the oscillator itself. Consequently, a system output can be provided which is uncontaminated by the internal modulation frequency. The second significant difference is that the diode detector produces the modulation envelope of the signal which is reflected from the resonator rather than the signal transmitted through it. Thus the modulation frequency can greatly exceed the crystal bandwidth. The signal generated by this technique is proportional to the imaginary part of the reflection coefficient of the resonator which is, itself, linearly proportional to the frequency deviation from the center of the resonance. The heuristic explanation of this behavior is that the carrier reflects from the resonance but the sidebands are so far removed from the center that they effectively reflect from a short circuit (more detail is given in [7]). The advantage of the high modulation frequency which is possible in this system (for example, 200 Hz) is that the attack time of the frequency lock loop can be decreased by a factor of 200 compared to the case of the circuit in Fig. 2. This would make it possible to use one inexpensive component oscillator rather than the two high-quality oscillators needed in the previous example.

#### EXPECTED PERFORMANCE

The performance expected from the system suggested in Fig. 3 can be estimated from the measurements on the various components. Curve A of Fig. 4 shows the measured

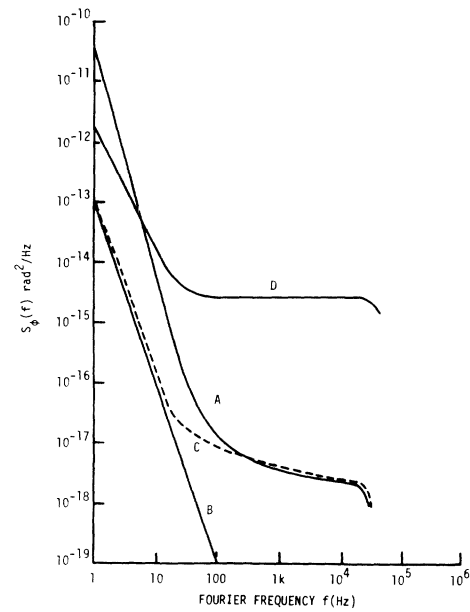


Fig. 4. Spectral density of phase noise for (A) a commercial high power 5 MHz oscillator, (B) a quartz crystal resonator measured passively, (C) the proposed system for locking A to B with 100 Hz unity gain frequency, and (D) a commercial low-power quartz-crystal oscillator.

spectral density of phase fluctuations  $S_\phi(f)$  for a commercially available high power 5-MHz oscillator that would be suitable for the local oscillator. Note that its phase noise rises very rapidly close to the carrier, due primarily to the high drive level.

The frequency stability of commercial high-quality AT-cut quartz resonators has been measured using a passive phase bridge technique which has been previously discussed [10]. By evaluating three or more resonators in various pairs one can independently determine the stability of each resonator. Curve B of Fig. 4 shows the equivalent  $S_\phi(f)$  for the best two of the four samples tested.

The use of a frequency-lock loop with a second-order loop filter (see, for example, [11]) should make it possible to achieve a system output with the phase noise shown in curve C of Fig. 4.

For comparison, the spectral density of phase for a commercial high quality, low-power 5-MHz quartz controlled oscillator is shown in curve D of Fig. 4. Note that the curve C, the system output, is significantly superior to curve D at all Fourier frequencies.

The corresponding time domain stabilities can be calculated from Fig. 4, and, assuming that the contribution to the phase noise from spurious pickup is insignificant, are given in Fig. 5. Again, note that the system output, curve C, is projected to yield excellent short term stability and long term stability. The frequency stability of such a system should exceed that of all available commercial standards for measurement times below 1000 s.

The frequency stability beyond 1000 s cannot be estimated from present measurements on the crystal resonators. However, from the above analysis we expect that the frequency stability of such a system should be superior to any present crystal-controlled oscillator, and further, that as

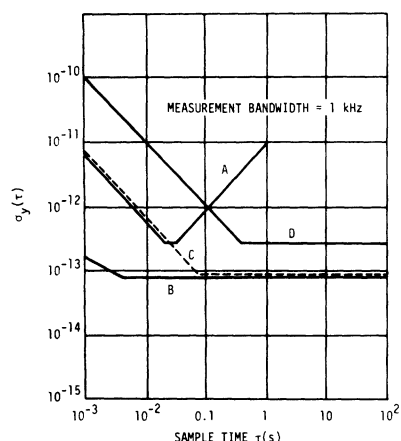


Fig. 5. The computed two-sample deviations corresponding to curves A, B, C, D of Fig. 4.

new quartz resonator types become available frequency drift and other long term frequency changes can be kept below  $10^{-12}$ /week. The new crystal resonators are likely to exhibit much better turn-off turn-on retrace, lower hysteresis due to temperature cycling, and much lower acceleration sensitivity [2], [3].

#### REFERENCES

- [1] E. P. EerNisse, "Calculations on the stress compensated (SC cut) quartz resonator," in *Proc. 30th Annu. Symp. Frequency Control*, pp. 8-11, 1976. (Available from Electronics Industries Assoc., 2001 Eye St., NW, Washington, DC 20006.)
- [2] J. A. Kusters, C. A. Adams, and H. Yosida, "TTC's-further developmental results," in *Proc. 31st Annu. Symp. Frequency Control*, pp. 3-7, 1977. (Available from Electronics Industries Assoc., 2001 Eye St., NW, Washington, DC 20006.)
- [3] R. Besson, "A new electrodeless resonator design," *Proc. 31st Annu. Symp. Frequency Control*, pp. 147-152, 1977. (Available from Electronics Industries Assoc., 2001 Eye St., NW, Washington, DC 20006.)
- [4] J. J. Gagnepain and F. L. Walls, "Fundamental properties of quartz crystal controlled oscillators," to be published.
- [5] L. S. Cutler, "A frequency standard of exceptional spectral purity and long term stability," *Frequency*, vol. 1, pp. 20-27, 1962.
- [6] J. J. Gagnepain, J. C. Poncot, and C. Pegeot, "Amplitude-frequency behavior of doubly rotated quartz resonators," in *Proc. 31st Annu. Symp. Frequency Control*, pp. 17-22, 1977. (Available from Electronics Industries Assoc., 2001 Eye St., NW, Washington, DC 20006.)
- [7] S. R. Stein and J. P. Turneaure, "The development of the superconducting cavity stabilized oscillator," in *Proc. 27th Annu. Symp. Frequency Control*, pp. 414-420, 1973. (Available from the National Technical Information Service, Sells Building, 5285 Port Royal Road, Springfield, VA 22161.)
- [8] S. R. Stein, "The superconducting-cavity stabilized oscillator and an experiment to detect time variation of the fundamental constants," Ph.D. dissertation, Stanford Univ., 1974. (Available from University Microfilms, 300 North Zeeb Road, Ann Arbor, Michigan 48106.)
- [9] F. L. Walls, S. R. Stein, J. E. Gray, and D. J. Glaze, "Design considerations in state-of-the-art signal processing and phase noise measurement systems," in *Proc. 30th Annu. Symp. Frequency Control*, pp. 269-274, 1976. (Available from Electronics Industries Assoc., 2001 Eye St., NW, Washington, DC 20006.)
- [10] F. L. Walls and A. E. Wainwright, "Measurement of the short-term stability of quartz crystal resonators and the implication for crystal oscillator design and applications," *IEEE Trans. Instrum. Meas.*, vol. IM-24, pp. 15-20, 1975.
- [11] F. L. Walls and S. R. Stein, "Servo techniques in oscillators and measurement systems," NBS Tech. Note 692. (Available from Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402, SD Cat. No. C13.46:692.)