choice. This idea has not been adopted by any textbook on EM. The idea may sound odd and untraditional, but if adopted and carefully practiced, it will surely help the students learn a lot more. The multiple-choice questions and the problems to be solved by the students should be carefully organized just as the main text. The problems in general should be presented in the same order as the materials in the main text and should be graded in difficulty, with relatively easy problems preceding hard ones. Enough problems should be provided to allow the instructor to choose some as illustrative examples.

Some learning aids which cannot be provided in the main text can be appended. Such aids may include mathematical formulas and identities, physical constants, and a brief review of materials such as vector analysis and complex variables, if not covered in the main text.

The question of whether an EM textbook follows an inductive or deductive approach should not pose a major problem to the student. An inductive approach presents the materials in historical order, starting with the experimental laws and finally synthesizing them in the form of Maxwell's questions. On the other hand, a deductive approach starts with Maxwell's equations, applies them to waves, and then deals with static and quasi-static situations as special cases. Each of these approaches has advantages and disadvantages. Whichever approach is adopted, it should be borne in mind that most of the students are getting exposed to EM for the first time and care must be exercised not to take too many things for granted.

IV. Instructor's Teaching Methods

The process of training is just as hard as that of learning. Just as the student of EM may be faced with some problems, the instructor of the course may find it hard to deliver the materials. Therefore, the instructor must be prepared to devise means whereby his ideas are effectively passed down to the students and the students can get the best out of the course.

One of the basic goals for the instructor who aims at producing quality students is that the students should get as much from the course as possible. The instructor may motivate the students to achieve this goal in several ways. Proper choice of the recommended textbook, orderly presentation of materials, emphasizing key points, giving examples, homework, and possibly quizzes may help.

Selecting a suitable EM textbook from the avalanche of new publications regularly appearing on the market should be carefully done by the instructor or his department. The selected textbook should contain the necessary basic principles and reflect the point of view of the instructor; otherwise, he may have to augment the textbook rather than follow it. An experiment on teaching effectiveness carried out by Zelby [7] indicates that his students considered him a good teacher when the textbook was followed and a poor one when he followed a point of view different from that of the textbook.

The importance of orderliness in presenting the materials cannot be overemphasized. According to Zelby, students seem to prefer the security of the textbook. It is easier for the student if the order of class topics follows that of the textbook, but the instructor may supplement areas where the book is weak. Also, it is necessary that the instructor link one subject or chapter to the next so that there is a flow of thought and the students can easily have a firm understanding of the subject matter.

Emphasizing the key points and ideas is essential to the students. This draws their observation to those points and particular attention is paid to them. It also helps the students to sort out the essentials from the nonessentials. Emphasizing the importance of vector analysis, for example, at the beginning helps the student pay special attention to the chapter where vector analysis is covered. Placing some emphasis on Maxwell's equations helps the student see the elegance of those equations.

Solving some typical examples in the class to illustrate basic concepts developed usually helps the student more than anything else. Since time does not permit solving many problems in class, the examples should be carefully selected so that they are neither too simple nor too hard to follow. The key lessons in the examples should be clearly pointed out and emphasized.

No amount of input by the instructor will make students learn until they start doing things by themselves. Carefully selected homework problems can help them learn. The homework may or may not be submitted for grading depending on the instructor. If the instructor gives quizzes regularly enough, submitting homework for grading may not be necessary.

If the instructor desires, the Schaum Outline series on *Electromagnetics* [8] and the *Mathematical Handbook* [9] can be suggested to the students as learning aids.

V. Conclusions

To alleviate the problems encountered by undergraduate students studying EM, the three problem areas identified as dealing with students, textbook, and instructor should be attacked simultaneously if possible. It is strongly believed that if the ideas suggested in this paper are carefully followed, electrical engineering students will feel more comfortable with the EM course. They will not only derive interest in the course, they will be highly challenged and motivated. A good textbook will be greatly appreciated by the students and the instructor. The instructor also will have a sense of accomplishment when the students perform very well.

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Simplified Biot-Savart Law for Planar Circuits

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***Abstract*—A simplified formula for the magnetic fields of line currents is obtained for the special case when the line currents and the observation point are confined to a plane. Its application is demonstrated by several examples.**

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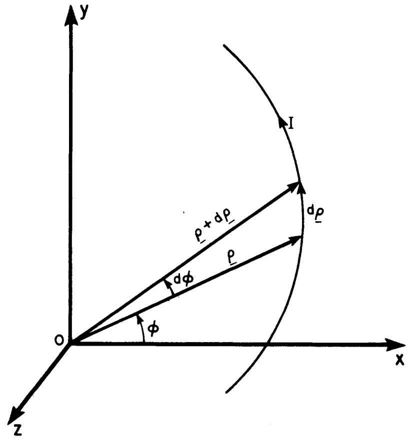

Fig. 1. Planar current carrying wire.

While recently teaching an undergraduate course in electromagnetics, the author observed that the usual vector form of the Biot–Savart law reduces to a scalar integral for the special case in which the line currents and the observation point are confined to a plane. This considerably simplifies the solution of standard textbook problems. The purpose of this communication is to present the method and illustrate it by examples. For the geometry of Fig. 1, the magnetic field at the origin, due to the line element $d\boldsymbol{\rho}$, is given by the vector form of the Biot–Savart law

$$dH = \frac{I\boldsymbol{\rho} \times d\boldsymbol{\rho}}{4\pi\rho^3} \tag{1}$$

where I is the current flowing in the line element and a cylindrical coordinate system ρ, ϕ, z has been used. A simple calculation shows that

$$\boldsymbol{\rho} \times d\boldsymbol{\rho} = \hat{z}\rho^2 d\phi \tag{2}$$

where $\hat{z}$ is a unit vector in the z-direction. Thus, the magnetic field has only a z-component and assuming that the current is uniform and of unit amplitude, one has

$$H = (\tfrac{1}{4}\pi) \int \frac{d\phi}{\rho} \tag{3}$$

where the integration is along the line current. This formula is much easier to use than its original form given by (1), and especially so if the polar equation of the line $\rho \equiv \rho(\phi)$ is given. We will now consider several cases.

The polar equation of an ellipse with the origin at a focus [Fig. 2(a)] assumes the form

$$\rho = \frac{a(1 - e^2)}{1 - e\cos\phi}, \quad \begin{matrix} 0 \le \phi \le 2\pi, \\ 0 \le e < 1 \end{matrix} \tag{4}$$

where a is the semimajor axis and e is the eccentricity. Substituting (4) in (3) and integrating, we find the magnetic field at the focus to be

$$H = [2a(1 - e^2)]^{-1} \tag{5}$$

which is far simpler than the result given in [1]. By shifting the origin to the center of the ellipse, one can show that the magnetic field at the center is $L/4A$ where L and A are, respectively, the perimeter and the area of the ellipse [2]. The other conic sections can be disposed of similarly. The equation of one branch of a hyperbola with the origin at the interior focus [Fig. 2(b)] is given by

$$\rho = \frac{a(e^2 - 1)}{1 - e\cos\phi}, \quad \begin{matrix} \cos(1/e) < \phi < 2\pi - \cos(1/e), \\ 1 < e < \infty \end{matrix} \tag{6}$$

and the magnetic field at the focus is

$$H = [2\pi a(e^2 - 1)]^{-1} \times [\pi - \cos(1/e) + (e^2 - 1)^{1/2}]. \tag{7}$$

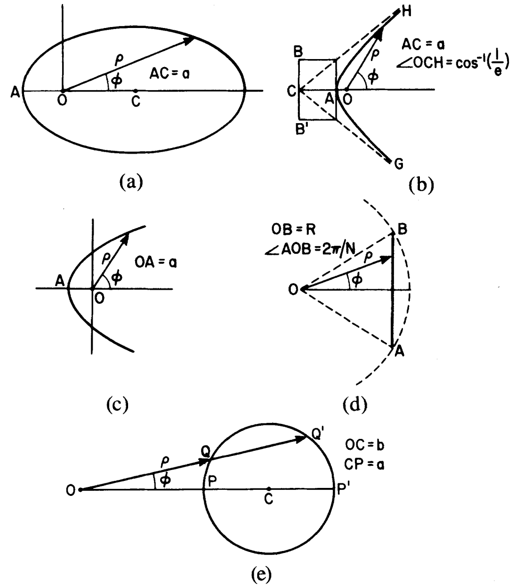

Fig. 2. Geometry of planar circuit examples. (a) Ellipse. (b) Hyperbola. (c) Parabola. (d) Polygonal side. (e) Circular loop.

On the other hand, for a parabola [Fig. 2(c)], one has the polar form

$$\rho = \frac{2a}{1 - \cos\phi}, \quad 0 < \phi < 2\pi \tag{8}$$

and the magnetic field at the focus is simply $(\frac{1}{4}a)$. Next, consider a regular polygon of N sides inscribed in a circle of radius R [3]. Taking the center of the polygon as the origin, the polar equation of a side can be written as [Fig. 2(d)]

$$\rho = R\cos(\pi/N)\sec\phi, \quad -\pi/N \le \phi \le \pi/N \tag{9}$$

where the line $\phi = 0$ is the bisector of the side. The magnetic field at the center due to the N sides is given by

$$\begin{aligned} H &= (N/4\pi) \int_{-\pi/N}^{\pi/N} \frac{d\phi}{R\cos(\pi/N)\sec\phi} \\ &= [2\pi R]^{-1} N \tan(\pi/N). \end{aligned} \tag{10}$$

As a final example, consider a circular loop of radius a. For an exterior point in the plane of the loop and at a distance b from the center [Fig. 2(e)], one has

$$\rho = b\cos\phi \pm (a^2 - b^2\sin^2\phi)^{1/2}, \quad |\phi| \le \sin^{-1}(a/b) \tag{11}$$

where the exterior point has been chosen as the origin and $\phi = 0$ is the line through the center of the loop, and the upper and lower signs pertain to the distant and nearer arcs of the circle. Substituting (11) in (3), one can show that

$$H = (1/\pi b)\,[K(k) - (1 - k^2)^{-1} E(k)] \tag{12}$$

where K and E are complete elliptic integrals of the first and the second kind of modulus k, where $k = a/b$. For interior points, one obtains

$$H = E(k)\,[\pi a(1 - k^2)]^{-1} \tag{13}$$

where now $k = b/a$. In conclusion, the present formulation yields solutions to many planar circuit problems with considerably less effort than traditional methods.

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