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Optimum Threshold Signal Detection in Broad-Band Impulsive Noise Employing Both Time and Spatial Sampling

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Abstract—This paper investigates the optimum detection of a weak signal, where this signal is dominated by broad-band impulsive noise. It has been suggested that by using a form of space diversity reception (many receiving sites), it would be possible to "look between the large noise pulses" and thereby detect a signal well buried in the noise. In this paper we obtain and determine the performance of the locally optimum (or threshold) detector for any form of interference and apply these results to an example case of atmospheric noise for the single receiver case and for the diversity case of many receivers. Substantial improvement or processing gain can be obtained with one receiver with substantial, but less, additional improvement obtainable by then going to many additional receivers.

I. INTRODUCTION

The man-made EM environment and much of the natural one as well is basically "impulsive"; i.e., is random in nature and characterized by significant probabilities of large interference levels, unlike the normal noise processes inherent in transmitting and receiving elements. This impulsive character of the interference can drastically degrade the performance of conventional systems, which are designed to operate most effectively against the usually assumed white Gaussian noise processes. It is well known that "normal" detection techniques, i.e., those known to be optimum in white Gaussian noise, are very inefficient in impulsive noise and can be greatly improved upon.

It has been suggested that by using a form of space diversity reception, it would be possible to "look between the large noise pulses" and thereby detect a signal well buried in the noise. Here, we obtain and determine the performance of the locally optimum detector for any form of interference where the signal to be detected is completely known (purely

coherent). By "locally optimum" we mean that detector which asymptotically approaches true optimality as the signal becomes small ($\rightarrow 0$). In the literature such detectors are commonly called "locally optimum Bayes detectors" or LOBD's.

After obtaining the above results for the single receiver case, we extend the results to the case where space diversity is used. The form of space diversity chosen is one in which the noise value which has the minimum absolute value from among the n simultaneous values received at n receiver sites is chosen for use in an optimum processing algorithm. For illustration of the results, numerical examples are given for a sample case of atmospheric noise.

The next section summarizes the broad-band impulsive noise model used to obtain the numerical examples.

II. THE IMPULSIVE NOISE MODEL

Recent work by Middleton has led to the development of a physical-statistical model for radio noise. This model has been used to develop optimum detection algorithms for a wide range of communications problems [1], [2]. It is this model which we use here for our signal detection (pure detection, not communication) problem. The Middleton model is the only general one proposed to date in which the parameters of the model are determined *explicitly* by the underlying physical mechanisms (e.g., source density, beam-patterns, propagation conditions, emission waveforms, etc.). It is also the first model which treats narrow-band interference processes (termed class A), as well as the traditional broad-band processes (class B). It is the class B model of interest to us here. The model is also canonical in nature in that the mathematical forms do not change with changing physical conditions. For a large number of comparisons of the model with measurements and for the details of the derivation of the model, see Middleton [3]–[6] and Spaulding [7]. We only summarize the results of the model that we need here.

If we denote our received noise process by $Z(t)$, then the probability density function (pdf) for the received instantaneous amplitude is

$$p_Z(z) = \frac{e^{-z^2/\Omega}}{\pi\sqrt{\Omega}} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} A_\alpha^m \Gamma\left(\frac{m\alpha+1}{2}\right) \cdot {}_1F_1\left(-\frac{m\alpha}{2}; \frac{1}{2}; \frac{z^2}{\Omega}\right) \quad -\infty \leq z \leq \infty \quad (1)$$

where ${}_1F_1$ is a confluent hypergeometric function. The model has three parameters: α , A_α , and Ω . (A more detailed and complete model involving additional parameters has been developed, but (1) above is quite sufficient for our purposes.) The parameters α and A_α are intimately involved in the physical processes causing the interference. Again, definitions and details are contained in the references. The parameter Ω is a normalizing parameter. In the references the normalization is $\Omega = 1$, which normalizes the process to the energy contained in the Gaussian portion of the noise. Here, we use a value of Ω which normalizes the process (Z values) to be *measured* energy in the process. We cannot normalize to the computed energy, since for (1), the second moment (or any moment) does not exist (i.e., is infinite). This is a typical problem with many

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Fig. 1. Measured ELF atmospheric noise and envelope distribution from Middleton's Class B noise model.

such models for broad-band impulsive noise. While the more complete model removes this problem, use of (1) in conjunction with measured data will in no way limit us.

The result corresponding to (1) for the envelope cumulative distribution (APD) is

$$P(\epsilon > \epsilon_0) = e^{-\epsilon_0^2/\Omega} \left[1 - \frac{\epsilon_0^2}{\Omega} \sum_{m=1}^{\infty} \frac{(-1)^m}{m!} A_{\alpha}^m \right. \\ \left. \times \Gamma\left(1 + \frac{m\alpha}{2}\right) {}_1F_1\left(1 - \frac{m\alpha}{2}; 2; \frac{\epsilon_0^2}{\Omega}\right) \right] \quad (2) \\ 0 \leq \epsilon \leq \infty$$

It is the envelope distribution in the above form which is usually measured and which we use for validation of the model by comparison with measurements. The references contain many such comparisons.

Our next step is to choose a typical noise sample which we can use throughout the analysis to obtain typical real results. These results are then, of course, for illustration and once the computational procedures are developed, results for any noise situation can be computed quickly. Fig. 1 shows some measured data taken from Evans and Griffiths [8]. These data were recorded at Saipan in the bandwidth 5-320 Hz. Also shown on Fig. 1 is the distribution from (2) for $A_{\alpha} = 1.0$ and $\alpha = 1.2$. The normalizing parameter Ω was obtained by actual integration of the distribution with the assumption that the noise saturated at high levels as indicated by the dashed curve at approximately 45 dB. The resulting value of Ω is 0.00079433. The measured data do not indicate where the noise saturates; and while our choice is rather arbitrary, it is chosen high so that large processing gains can be achieved, thereby giving optimistic rather than pessimistic results.

III. THE OPTIMUM THRESHOLD DETECTOR AND ITS PERFORMANCE

In this section we will follow standard statistical-decision theory approaches to derive our optimum processing algorithm

(receiver) and to determine its performance. Our problem is to decide optimally between two hypotheses.

$$H_0: X(t) = Z(t), \quad 0 \leq t < T$$

$$H_1: X(t) = Z(t) + S(t), \quad 0 \leq t < T. \quad (3)$$

In (3) $X(t)$ is our received waveform in detection time T and this waveform is either received noise along $Z(t)$ or noise plus the signal $S(t)$, which we want to detect.

To obtain our receiver structure we follow the standard procedure of replacing all waveforms by vectors of samples from the waveforms ($X(t) \rightarrow (X)$, etc.) and forming the likelihood ratio $\Lambda(X)$. When $Z(t)$ is non-Gaussian, we operate so as to generate (essentially) independent samples z_i , so that only first order pdf's are required in our analysis. This is a necessary simplification which leads to upper bounds on optimality. We discuss this assumption of independence and its practical implications later. We now use the LOBD or threshold operation which we know becomes asymptotically optimum as our signal $S(t) \rightarrow 0$ and/or $N \rightarrow \infty$. Increasing N , the number of samples corresponds to increasing the detection time T , since we cannot for any noise process sample more rapidly than the bandwidth (b samples/s for b Hz) and maintain independence.

Using a vector Taylor expansion about our signal S and for coherent detection using only the first-order terms, since as $S(t)$ becomes small the higher order terms become insignificant when compared to the first-order terms, we obtain

$$\Lambda(X) \approx \frac{p_Z(X) - \sum_{i=1}^N \frac{\partial p_Z(X)}{\partial x_i} s_i}{p_Z(X)} \underset{H_0}{\overset{H_1}{\geq}} \lambda \quad (4)$$

where λ is a threshold determined by specifying a probability of false alarm P_F . Note that we have decided that our signal $S(t)$ is completely known to us and all we do not know is its presence or absence. This coherent problem results in the best possible performance.

Since we have independent samples so that the N th order pdf $p_Z(X)$ is given by the product of the N first-order pdf's, we obtain from (4)

$$\Lambda(X) = 1 - \sum_{i=1}^N \frac{d}{dx_i} \ln p_Z(x_i) s_i \underset{H_0}{\overset{H_1}{\geq}} \lambda. \quad (5)$$

Expression (5) is a well-known result for the LOBD in non-Gaussian noise and Fig. 2 then is our receiver.

We see that the receiver is the standard memoryless Gaussian (i.e., degenerate matched filter) receiver preceded by a particular nonlinearity given by $-d/dx \ln p_Z(x)$. Note that this is a completely canonical result in that we have not yet specified (in the above deviation) what $p_Z(x)$ is or what $S(t)$ is except that it is completely known. Fig. 3 [or (5)] is our optimum receiver (or processor) which takes our received waveform samples x_i and uses them as shown to determine our decision variable δ , which is then compared against a threshold $\lambda - 1$ or λ' . Now, to determine the performance we need the pdf of δ (where δ approaches $\Lambda - 1$ for small S). Since N is to be large (in order to achieve any kind of processing gain) and because of the nonlinearity which limits the amplitude excursions of the noise (as we will see later), we can apply the Central Limit Theorem. This means that we only need to compute the mean and variance of δ under each of the two hypotheses. It should be noted that the nonlinearity does not "Gaussianize" the noise.

Fig. 2. LOBD for Neyman-Pearson detection of a purely coherent signal.

Fig. 3. For fixed detection time, probability of false alarm, and probability of detection, the dB improvement (dB less signal required) over Gaussian noise by the optimum detector, atmospheric noise of Fig. 1, versus number of receiving sites n .

We start with y_i , the output of the nonlinearity for input x_i (see Fig. 3). Suppose H_1 is true; then

$$\begin{aligned} E[y_i | H_1] &= \int_{-\infty}^{\infty} y_i p(y_i) dy_i \\ &= \int_{-\infty}^{\infty} \left[-\frac{d}{dz} \ln p_Z(z) \right] p_Z(z - s_i) dz. \end{aligned}$$

Using, as above, a Taylor expansion for $p_Z(z - s_i)$ and ignoring higher order terms in s_i , we obtain

$$E[y_i | H_1] = s_i L \quad (6)$$

where

$$\begin{aligned} L &= \int_{-\infty}^{\infty} \left[-\frac{d}{dz} \ln p_Z(z) \right]^2 p_Z(z) dz \\ &= \int_{-\infty}^{\infty} \frac{[p_Z'(z)]^2}{p_Z(z)} dz. \end{aligned} \quad (7)$$

As we shall see, the parameter L determines the processing gain achievable for any $P_Z(z)$, including Gaussian noise.

In similar fashion, we obtain

$$\text{Var}[y_i | H_1] = L - s_i^2 L^2 \quad (8)$$

so that for δ , then

$$E[\delta | H_1] = L \sum_{i=1}^N s_i^2$$

and

$$\text{Var}[\delta | H_1] = \sum_{i=1}^N s_i^2 [L - s_i^2 L^2]. \quad (9)$$

Likewise, for H_0 being true, we obtain

$$E[\delta | H_0] = 0$$

and

$$\text{Var}[\delta | H_0] = \sum_{i=1}^N s_i^2 L. \quad (10)$$

Since we are using the Central Limit Theorem, the δ is normally distributed with the appropriate mean and variance for H_0 and H_1 . Our performance is given by

$$\begin{aligned} \beta &= P_F = \int_{\lambda'}^{\infty} p(\delta | H_0) d\delta, \\ P_D &= \int_{\lambda'}^{\infty} p(\delta | H_1) d\delta \end{aligned} \quad (11)$$

where P_D is the probability of detection.

We now must choose a signal. Let

$$S(t) = \sqrt{2S} \cos \omega_0 t, \quad \omega_0 T \gg 1. \quad (12)$$

In (12) S is the signal power and since the noise has been normalized to 1, S also is our signal-to-noise ratio (SNR). In nonlinear detection problems performance quite often turns out to be a function of the absolute noise level, as well as SNR. This is not the case here and our normalization is appropriate. From (12) $s_i = \sqrt{2S} \cos \omega_0 t_i$, so

$$\sum_{i=1}^N s_i^2 \approx NS \quad \text{and} \quad \sum_{i=1}^N s_i^4 \approx \frac{3S^2 N}{2}. \quad (13)$$

Of course, if our actual signal is different than our choice in (12), the results we need from (13) could be somewhat different, but easy to determine provided the signal is exactly known.

We must now set β to some acceptable level and solve for λ' via (11). Let $\beta = 10^{-3}$, then after some algebra our probability of detection (12) is given by

$$\begin{aligned} P_D &= \frac{1}{2} \text{erfc}(t), \quad t > 0 \\ &= 1 - \frac{1}{2} \text{erfc}(|t|), \quad t \leq 0 \end{aligned} \quad (14)$$

where

$$t = \frac{3.09\sqrt{LSN} - LSN}{\sqrt{2LSN - 3L^2 S^2 N}} \approx \text{erfc}^{-1}(2\beta) - \sqrt{\frac{LSN}{2}}, \quad (L^2 S^2 \ll 1). \quad (15)$$

Note from (15) that performance depends on the SNR S , the detection time T (given by N), and the parameter L [defined in (7)]. Also, note that the amount of "processing gain" attainable for a given S and N is given by L ; that is, the larger L is, the smaller S can be to give the same value of t and, therefore, the same performance P_D .

If $Z(t)$ is a white Gaussian process, the nonlinearity $-d/dz \ln p_Z(z)$ is simply z , meaning that the receiver of Fig. 2 reduces to that one which is well known to be optimum for Gaussian noise. It can also be shown that the integral (7) for L is minimized if $p_Z(z)$ is Gaussian and is equal to 1 iff $p_Z(z)$ is Gaussian.

Now consider (1), our noise model, with the parameter as per Fig. 1, our example of atmospheric noise. Numerical integration of the resulting integral for L (7) using the pdf given by (1) gives $L = 887.08$. This means that the LOBD receiver for our atmospheric noise requires $1/887.08$ (-29.5 dB) as much signal as the corresponding optimum receiver for Gaussian noise. That is, we have a 29.5 dB processing gain using one receiver. Note that this is the gain of *optimum* for atmospheric noise over *optimum* for Gaussian noise. If we used the optimum for Gaussian noise receivers with atmospheric noise, it would obviously perform more poorly than if the noise were Gaussian. This means that the optimum receiver for atmospheric noise would be *more* than 29.5 dB better than the Gaussian receiver used with the same atmospheric noise.

Now that we know L , we can easily compute P_D via (14) and (15). We delay doing so until the next section when we go on to n receiver sites, the current $n = 1$ situation being a special case.

IV. OPTIMUM RECEPTION WITH THE USE OF SPACE DIVERSITY

As mentioned previously, it has been suggested that signal detection could be greatly enhanced by using many receiving sites and thereby looking for the signal "between the noise pulses." We rather loosely call such a technique space diversity, although diversity procedures are usually used to overcome fading signal problems and not noise problems. In this section we want to compute what additional gain (additional to the optimum one receiver of the previous section) can be achieved by employing n receiving sites. Basically, the problem has been solved in the previous section, since the optimum receiver derived there was canonical, i.e., good for any interference situation. All we need to do is to determine our new noise distribution and then apply the previous techniques.

Suppose we have n sites so that at each instant of time t_i we have n noise samples—one from each site. We assume that these n noise samples, call them z_{ij} , $j = 1, n$, are independent. We also assume that by some means coherence is maintained so that all the corresponding signal samples are essentially the same; i.e., $s_{ij} = s_i$, s_i as before. Actually, all we have are waveform samples x_{ij} , which either are noise alone or signal plus noise. Suppose now we form our vector of waveform samples $\{x_i\}$, $i = 1, N$ by choosing each x_i from the n x_{ij} 's via

$$x_i = \min_j x_{ij}. \quad (16)$$

Using (16) guarantees us that we always obtain the sample with the minimum z_i , since all s_{ij} 's are the same. However, all (16) does is push us toward large negative values of z_i . Large negative values are just as bad as large positive values. What we really want is to choose our noise samples as follows: For z_i choose the z_{ij} which has the smallest *absolute* value. That is,

$$z_i = \pm \min_j |z_{ij}| \quad (17)$$

where the plus or minus depends on whether the appropriate z_{ij} is positive or negative. We proceed now using initially (16). This we know can give little improvement, but these results are required to proceed on to (17). We need the distribution of noise sample $\hat{z}_i$, where $\hat{z}_i$ is minimum z_{ij} , $j = 1, n$. For convenience, we temporarily drop the subscript i . If $p_Z(z)$ is the pdf of z , then the pdf of $\hat{z}$, denoted $p_{\hat{Z}}(\hat{z})_n$, is

$$p_{\hat{Z}}(\hat{z})_n = n p_Z(\hat{z}) \left[\int_{\hat{z}}^{\infty} p_Z(t) dt \right]^{n-1} \quad (18)$$

where $p_Z(z)$ is given by (1). Even though $p_{\hat{Z}}(\hat{z})_n$ is no longer symmetrical about $z = 0$, it turns out that performance for this pdf is still given by (14) and (15).

In the operation of (17) we choose the noise sample which has the minimum absolute value. Call this sample $\tilde{z}$ and we want $p_{\tilde{Z}}(\tilde{z})_n$. Using symmetry, we obtain

$$p_{\tilde{Z}}(\tilde{z})_n = 2^{n-1} p_{\hat{Z}}(\hat{z})_n, \quad \hat{z} \geq 0. \quad (19)$$

Note that from (19) $p_{\tilde{Z}}(\tilde{z})_n$ has a discontinuous derivative at $\tilde{z} = 0$. This will have an interesting effect on our required nonlinearity.

All that remains is the computation of L where in general

$$L = \int_{-\infty}^{\infty} \frac{[p'(\cdot)]^2}{p(\cdot)} d\cdot. \quad (20)$$

For the case of the previous section, i.e., single receiver (the z case), the integrals is symmetric about $z = 0$. In the $\hat{z}$ case we do not have symmetry with, for $n \geq 2$, most of the contribution to L coming from the range $(-\infty, 0)$. In the $\tilde{z}$ case we again have symmetry and

$$L \text{ (for } \tilde{z}) = 2^n \int_0^{\infty} \frac{[p_{\hat{Z}}'(\hat{z})]^2}{p_{\hat{Z}}(\hat{z})} d\hat{z}. \quad (21)$$

Also, for our required nonlinearity

$$\frac{p_{\tilde{Z}}'(\tilde{z})}{p_{\tilde{Z}}(\tilde{z})} = \frac{p_{\hat{Z}}'(\hat{z})}{p_{\hat{Z}}(\hat{z})}, \quad \hat{z} \geq 0 \quad (22)$$

with symmetry about $\tilde{z} = 0$.

Using the above along with some extensive analysis to simply the resulting expressions and numerical integration of the various L integrals, Table I present the results.

The probability of detection is given by (14) and (15) and is based on the parameter t (15). As noted before, L gives directly the processing gain for a fixed detection time given via N . Note also that in our approximations that lead up to (15) above, LS must be small. This is required for a positive variance and is demonstrated in the denominator of (15) in that we cannot allow the quantity under the square root to be negative. In fact, for situations of interest LS is quite small ($\ll 1$) so that the LS term dominates the $L^2 S^2$ term in the denominator of (15). This means that performance improves essentially directly with the detection time N , as well as directly with L i.e., N ten times as large means a required signal ten times smaller, etc.

Fig. 3 shows our "Improvement factor," given by $10 \log L$ for the situations we have calculated above. Fig. 4 shows our optimum nonlinearity for various n ($n = 1$ being the normal one receiver case). On Fig. 4 we have denoted the nonlinearity by $l(z)$; that is

$$l(\tilde{z}) = -\frac{d}{d\tilde{z}} \ln p_{\tilde{Z}}(\tilde{z})_n. \quad (23)$$

Note for $n \geq 2$ the sharp discontinuity at $\tilde{z} = 0$. One might think that by using our minimum space sampling approach the resulting interference might approach Gaussian (with a much smaller noise power, of course.) This is not the case at all, since if it was, our nonlinearity (Fig. 4) would approach that for Gauss; namely, $l(\tilde{z}) = \tilde{z}$. Instead, we see the nonlinearity in the range $-0.1 < \tilde{z} < 0.1$ approaches the characteristic of a hard limiter coupled with rapid attenuation outside this

TABLE I
VALUES OF L FOR OUR CASES

n	$\hat{z}$		$\tilde{z}$	
	L	$10 \log L$	L	$10 \log L$
1	887	29.5	887	29.5
2	1287	31.1	2934	34.7
3	1508	31.8	6290	38.0
4	1628	32.1	10950	40.4
5	1689	32.3	16919	42.3
6	1713	32.3	24196	43.8
8	1700	32.3	42688	46.3
10	1643	32.3	66433	48.2

Fig. 4. The optimum nonlinearity for detection of known signal in atmospheric noise of Fig. 1. Noise samples are those having the minimum absolute value of n space samples (n receivers).

range. This is a rather interesting result. (For the effects of various *ad hoc* nonlinearities (suboptimum), see Miller and Thomas [9].) Fig. 5 shows the probability of detection P_D from (14) for Gaussian noise ($L = 1$) and our atmospheric noise case ($n = 1, L = 887$) for $N = 10, 100$, and 1000 and a probability of false alarm of 10^{-3} .

V. DISCUSSION

In the above analysis there are two items to be discussed: the effect of our choice of bandwidth and the effect of our assumption of independent samples. It has long been realized that for communication systems the bandwidth had to be increased over that of "normal" systems in order to gain improvement in impulsive noise. Wide-band limiting is an example. The same is true here. Up to a point, increasing the bandwidth increases the "impulsiveness" of the received noise process, allowing greater processing gain. Also, increasing the bandwidth allows a faster sampling rate; i.e., more samples for a given detection time, which also improves performance. However, increasing the bandwidth *increases* the received noise power and also allows more narrow-band interfering

Fig. 5. Probability of detection versus SNR for a probability of false alarm of 0.001.

signals. Because of these tradeoffs there is, undoubtedly, some optimum bandwidth to use for each particular situation.

In the analysis we assumed that the noise samples $z_i, i = 1, N$, were independent. We know that for atmospheric noise this is not always the case. In our case the spectral behavior of the measured noise (Evans and Griffith [8]) shows that if we sample at the bandwidth rate, the samples will probably not be completely independent. In general, if the samples are dependent and we know and make use of this dependence, we can gain further improvement over the case of independent samples. In general, such improvement turns out to be small although it never has been investigated in detail. The Soviet author Levin has treated the problem of correlated samples [ch. 3, "Asymptotically optimal signal detection algorithms," vol. 3 of Levin's series of Communication Theory books]. Levin considers the noise to be a k th order Markov process and derives the optimum receiver structure (in a sense, at least). It turns out to be, naturally, quite complex. Performance is not determined, although it is clear performance depends on the $k \times k$ covariance matrix of the process. In any case, any performance improvement probably would be small (a few decibels at the most and in order to actually make use of the correlation information in the noise process would require very detailed information about the noise, more detailed than it is possible to know (probably) by theory or measurement.

In our spatial procedure we discarded $n - 1$ of our n samples at each sampling instant. One would suppose that this is discarding potentially useful information. This is true and if we know the time and spacial correlation details, etc. of the noise field, perhaps a better algorithm could be developed (in the spatial dimension). Again, it is clear that the much greater complexity this would involve could not give us a great deal more improvement, even if it were at all practical.

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Buffer Sizing of a Packet-Voice Receiver

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Abstract—Some analytical results have been developed for the analysis and design of a packet-voice receiver in previous work [1], namely, the delay suffered by packets because of the reassembly mechanism is described in terms of the overall delay probability density function (pdf) $w(t)$. This basic analytical result is used here in order to obtain the pdf $b(t)$ of the time spent by packets in the receiver buffer. In this way, the previous work [1] is extended to analysis of the delay distribution of the receiver buffer where arriving talkspurts are delayed to compensate for the variable network delay of all packets within a talkspurt. The tools developed here facilitate a choice of the buffer size at the prescribed level of packet-loss probability.

I. INTRODUCTION

Some new results from the analytical study of packet-voice communication stated in [1] are given. In particular, the problem of buffer sizing is studied; an analytical expression of the time spent by packets in the packet-voice-receiver (PVR) buffer is obtained.

A system outline is given in Fig. 1. The time diagram of the system behavior is shown in Fig. 2. From line (a) to line (b) of Fig. 2, the voice packets are randomly delayed by the network, and from line (b) to line (c), a new delay is introduced by the PVR. The network delay pdf is $p(t)$ see Fig. 3. The overall delay pdf from (a) to (c) $w(t)$ has been analytically calculated in [1]. In this work, the buffer delay pdf $b(t)$ is considered.

The pdf $p(t)$ completely characterizes the delay process from (a) to (b) where independence among packets is a basic assumption (see Section II). This is not the case for $w(t)$ and $b(t)$, as a matter of fact, the reordering and the receiving processes introduce a correlation between delays of packets belonging to the same talkspurt.

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Fig. 1. System block diagram.

Fig. 2. Time diagram of the system behavior.

Fig. 3. pdf involved in the analytical developments.

For easy comprehension of this paper and for the discussion of the technical motivations of this work, a reading of [1] is advisable along with the references therein.

In Section II the statistical assumptions needed for the analytical study are stated; in Section III the results given in [1] which necessary here are recalled. In Section IV the reordering delay pdf $b(t)$ is obtained.

In Section V numerical results and simulations are presented.

II. PRELIMINARY ASSUMPTIONS

The system was analyzed in a wide statistical environment in [1] by means of some hypotheses that yielded a suitable compromise between generality of description and useful mathematical modeling. The basic assumptions were as follows.

1) Delays experienced by packets belonging to a single talkspurt are independent and identically distributed according to the network delay pdf $p(t)$. This assumption is guaranteed only in the case of a highly dynamic routing policy.

2a) Talkspurt durations are exponentially distributed according to $f(t) = \lambda e^{-\lambda t}$; this implies that the probability mass function of talkspurt length (in packets) is geometrical, i.e.,

$$p_n \triangleq \text{Prob} \{ \text{talkspurt contains exactly } (n+1) \text{ packets} \} \\ = (1-\rho)\rho^n \quad (1)$$

where $\rho \triangleq e^{-\lambda t_p}$; $t_p \triangleq$ packet duration (seconds). This means that (" $\triangleq$ " means "defined equal to") $q_n \triangleq \text{Prob} \{ \text{actual packet is the } (n+1)\text{th of the talkspurt} \}$ is given by

$$q_n = \begin{cases} \frac{1-\rho}{\rho} \ln \frac{1}{1-\rho}, & n=0 \\ \frac{1-\rho}{\rho} \ln \frac{1}{1-\rho} \sum_{i=0}^{n-1} \frac{1-\rho}{i+1} \rho^i, & n=1, \dots, \infty \end{cases} \quad (2)$$

or alternatively the following.