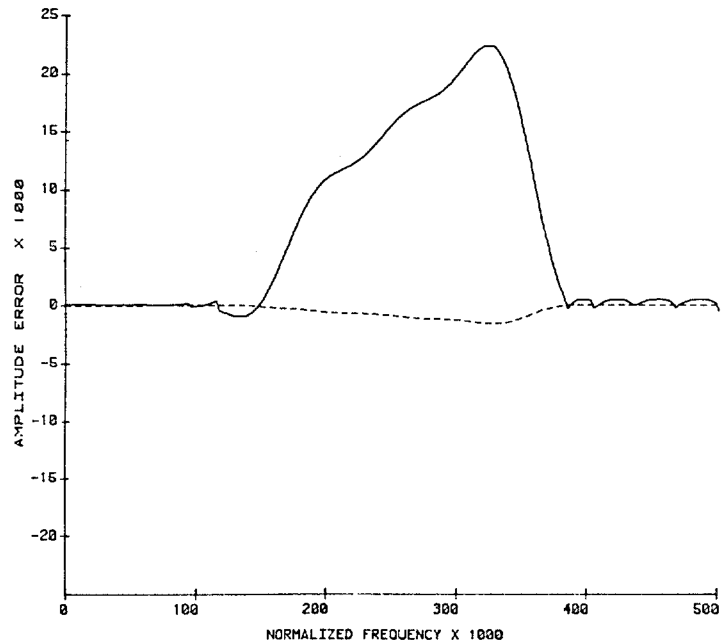

(c)

Fig. 2. (c) As in (b) for the phase error in the passband of interest.

bandwidth close to the boundaries of the filter passband. This behavior can easily be fully justified from (5). Fig. 2 also shows that the coefficient compensation (9) reduces the amplitude and phase errors by more than an order of magnitude. The residual errors are to be attributed to the terms neglected in the approximation (4) and to the fact that the charge-transfer loss actually transforms an FIR filter into an IIR one, as can be seen from the model of Fig. 1 and from relation (3).

IV. Conclusions

In many applications of CCD transversal filtering, the CTI ϵ and the filter length N are such that $N\epsilon << 1$. In this case, the general expressions (5) for the transfer function deviation caused by the device CTI have been obtained. From these relations, the amplitude and phase errors of the filter frequency response can be derived in a straightforward way. The accordance between the computed errors and those obtained from computer simulations and/or those already known in the literature was verified for many filters.

Furthermore, the simple modification algorithm (9) of the filter coefficients derived from the preceding analysis has proved to be quite effective in reducing the amplitude and phase deviations by more than an order of magnitude. The simplicity of the compensation algorithm (9) is, in principle, particularly attractive when the coefficients of a CCD transversal filter are externally evaluated and/or modified by means, for example, of a microprocessor and supplied to a multiplying digital-to-analog converter to perform the filtering operation [2].

References

[1] *IEEE J. Solid-State Circuits, Joint Special Issue on Charge-Transfer Devices*, vol. SC-11, Feb. 1976.
[2] A. Gersho, "Charge-transfer filtering," *Proc. IEEE*, vol. 67, pp. 196–218, Feb. 1979.
[3] K. K. Thornber, "Optimum linear filtering for charge-transfer devices," *IEEE J. Solid-State Circuits*, vol. SC-9, pp. 285–291, Oct. 1974.
[4] D. D. Buss, D. R. Collins, W. H. Bailey, and C. R. Reeves, "Transversal filtering using charge-transfer devices," *IEEE J. Solid-State Circuits*, vol. SC-8, pp. 138–146, Apr. 1973.
[5] A. Gersho and B. Gopinath, "Filtering with charge-transfer devices," in *Proc. Int. Symp. Circuits Syst.*, 1975, pp. 183–185.

Confidence Bounds for Signal-to-Noise Ratios from Magnitude-Squared Coherence Estimates

JOHN W. FAY

Abstract—**Coherence is used frequently to determine the degree to which one observed voltage is related to another observed voltage. Typically, in practice, these observables are degraded by system noise that is often independent, white, and Gaussian. Often, in measuring coherence, the interest is to determine the fraction of the observed power that is due to coherent signals and the fraction that is due to the uncorrelated noise floor. The term "signal" as used here describes a component of voltage of interest to an observer. With accurate coherence estimates, uncorrelated noise power can be separated from coherent signal power. Therefore, the concern in this article is with the accuracy of signal-to-noise ratio (SNR) calculations made from magnitude-squared coherence (MSC) estimates. Use is made of work by Carter and Scannel [1] in which they determine confidence bounds of MSC estimates for stationary Gaussian processes. Their results are used in this article to derive corresponding confidence bounds for SNR calculations without recourse to the complicated details of the underlying SNR statistics.**

Discussion

The magnitude-squared coherence (MSC), or the coherence function as it is sometimes called, between two stationary pro-

Manuscript received October 29, 1979; revised March 6, 1980.
The author is with New London Laboratory, Naval Underwater Systems Center, New London, CT 06320.

cesses $x(t)$ and $y(t)$ is defined by

$$\gamma_{xy}^2 = \frac{|\overline{G}_{xy}|^2}{\overline{G}_{xx}\overline{G}_{yy}}, \quad 0 \leqslant \gamma_{xy}^2 \leqslant 1. \tag{1}$$

The quantity $x(t)$ could be the input to a system, with the $y(t)$ the corresponding output.

Frequency dependence is implicit in (1). The hyperbars denote ensemble averaging. The quantity in the numerator of the fraction in (1) is the magnitude squared cross spectrum which, for the most part, shows spectral components that are correlated between voltages $x(t)$ and $y(t)$. Typically, the cross spectrum has deterministic errors due to such things as gain differences. Quantities in the denominator of the fraction are autospectra of observed processes $x(t)$ and $y(t)$. The coherence in (1) measures the fraction of power in $y(t)$ that is related in a linear way to $x(t)$. Note that the MSC is independent of gain or level difference between $x(t)$ and $y(t)$.

As pointed out by Carter and Scannel [1], the MSC is usually estimated in practice by means of a dual-channel fast Fourier transform (FFT) algorithm, as follows:

$$\hat{\gamma}_{xy}^2 = \frac{\left|\sum_{n=1}^{N} x_n y_n^*\right|^2}{\sum_{n=1}^{N} |x_n|^2 \sum_{n=1}^{N} |y_n|^2}. \tag{2}$$

The asterisk denotes complex conjugation and N is the number of data points averaged. The factors x_n and y_n are FFT outputs of the nth data segments of periodically sampled $x(t)$ and $y(t)$, respectively. Typically, we have N independent data segments, each of length $M\Delta T$, where ΔT is the sampling interval. In general, this partitioning is done to gain statistical stability at the expense of increased bias and decreased frequency resolution. Bias and frequency resolution are not a concern here.

If we have uncorrelated noise and coherent signals between $x(t)$ and $y(t)$, it is possible to correct the spectrum of $y(t)$ to determine the amount of power due only to uncorrelated noise and the amount due only to coherent signal. If the uncorrelated noise power in $x(t)$ is assumed nonexistent, the power in $y(t)$ as a result of a coherent signal between $x(t)$ and $y(t)$ at frequency f is

$$\overline{G}_{yy}\hat{\gamma}_{xy}^2. \tag{3}$$

Similarly, the power in $y(t)$ due to uncorrelated noise is

$$\overline{G}_{yy}(1 - \hat{\gamma}_{xy}^2). \tag{4}$$

The ratio of (3) and (4) yields the coherent signal-to-uncorrelated-noise ratio (SNR) [2] for $y(t)$ at a single frequency in terms of measured coherence,

$$\text{SNR} = \frac{\overline{G}_{yy}\hat{\gamma}_{xy}^2}{\overline{G}_{yy}(1 - \hat{\gamma}_{xy}^2)} = \frac{\hat{\gamma}_{xy}^2}{1 - \hat{\gamma}_{xy}^2}. \tag{5}$$

Note from (5) that the SNR is a function of frequency and is a random variable because of the randomness in the estimate $\hat{\gamma}_{xy}^2$. Thus, we are limited in the use of (5) until we can put some confidence bounds on its estimate.

Using the results in Carter and Scannel [1], we can find corresponding confidence intervals for SNR. A computer program in [1] permits calculating arbitrary confidence limits for MSC estimates of stationary Gaussian processes. Table I is a sample output from this program.

Magnitude squared coherence values from Table I have been used to compute confidence bounds for SNR in (5) (see the Appendix). These bounds are given in Figs. 1 and 2 for 80 and 95 percent confidence, respectively. The outermost curve in

TABLE I
CONFIDENCE BOUNDS FOR MSC ESTIMATES[1]

N	Estimated magnitude squared coherence	80% lower bounds	80% upper bounds	95% lower bounds	95% upper bounds
8	0.000	0.015	0.280	0.004	0.410
8	0.167	0.055	0.487	0.015	0.611
8	0.333	0.154	0.626	0.061	0.727
8	0.500	0.305	0.741	0.175	0.815
8	0.667	0.500	0.836	0.365	0.886
8	0.833	0.732	0.924	0.637	0.947
8	1.000	1.000	1.000	1.000	1.000
16	0.000	0.007	0.142	0.002	0.218
16	0.167	0.065	0.372	0.023	0.466
16	0.333	0.190	0.533	0.111	0.614
16	0.500	0.354	0.669	0.259	0.732
16	0.667	0.546	0.789	0.458	0.832
16	0.833	0.762	0.899	0.705	0.921
16	1.000	1.000	1.000	1.000	1.000
32	0.000	0.003	0.072	0.001	0.112
32	0.167	0.084	0.307	0.044	0.369
32	0.333	0.225	0.470	0.164	0.532
32	0.500	0.393	0.618	0.327	0.668
32	0.667	0.580	0.754	0.523	0.789
32	0.633	0.783	0.882	0.747	0.899
32	1.000	1.000	1.000	1.000	1.000
64	0.000	0.002	0.036	0.000	0.057
64	0.167	0.102	0.258	0.071	0.308
64	0.333	0.253	0.428	0.209	0.478
64	0.500	0.423	0.586	0.377	0.625
64	0.667	0.805	0.729	0.567	0.758
64	0.833	0.793	0.869	0.775	0.889
64	1.000	1.000	1.000	1.000	1.000
128	0.000	0.001	0.018	0.000	0.029
128	0.167	0.119	0.230	0.094	0.269
128	0.333	0.275	0.407	0.243	0.447
128	0.500	0.445	0.565	0.413	0.599
128	0.667	0.623	0.718	0.597	0.740
128	0.833	0.808	0.860	0.793	0.879
128	1.000	1.000	1.000	1.000	1.000

[1]Sample output from program in [1].

each figure corresponds to $N = 8$, whereas the innermost curves are for $N = 128$. In order to use these curves, it is necessary to obtain an estimate of MSC from (2) for some value of N, e.g., $N = 8$ and 95 percent confidence of MSC. This specifies a point on the abscissa in Fig. 2 at a value of 0.5, for example. The line normal to the abscissa at 0.5 intersects the $N = 8$ curves at -6.73 and 6.44 dB. These SNR bounds correspond to the lower and upper MSC bounds, respectively. We can, therefore, make the statement that the true SNR is within this interval 95 percent of the time. Expressing this another way, true SNR is no less than the estimated SNR minus 6.73 dB, or not greater than the estimated SNR plus 6.44 dB, 95 percent of the time.

Overall, there is a tightening of the confidence interval as N increases, and a leveling off of the confidence curves with increases in estimated MSC. Confidence is poor for small MSC estimates. An obvious rule of thumb is to choose as large a value of N as is practically possible.

SUMMARY

We have presented confidence bounds for SNR calculations from MSC estimates for Gaussian processes. Confidence

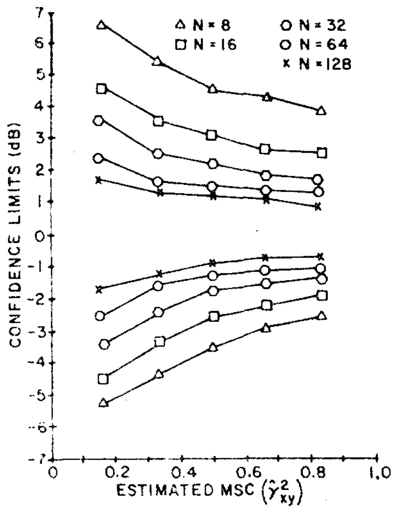

Fig. 1. 80 percent signal-to-noise confidence limits for $N = 8$, 16, 32, 64, and 128.

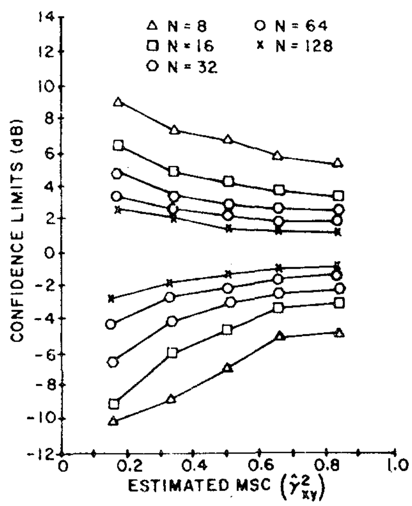

Fig. 2. 95 percent signal-to-noise confidence limits for $N = 8$, 16, 32, 64, and 128.

bounds for MSC from Carter and Scannel [1] have been mapped into corresponding confidence bounds for SNR. The user interested in separating coherent signals from incoherent noise can do so with a quantitative measure of confidence from simple calculations.

Appendix

The MSC between two observed stationary processes, $x(t)$ and $y(t)$, is given by

$$\hat{\gamma}^2_{xy} = \frac{\left| \sum_{n=1}^{N} x_n y_n^* \right|^2}{\sum_{n=1}^{N} |x_n|^2 \sum_{n=1}^{N} |y_n|^2} \tag{A1}$$

where $\hat{\gamma}^2_{xy}$ is an estimate of the true coherence γ^2_{xy}. It is, therefore, itself a random process. The quantities in (A1) are ensemble averages of FFT's of N independent data segments. In general, the segments need not be independent. The SNR that gives a measure of causality between $x(t)$ and $y(t)$ is given by

$$\text{SNR} = \frac{\hat{\gamma}^2_{xy}}{1 - \hat{\gamma}^2_{xy}} \tag{A2}$$

where $\hat{\gamma}^2_{xy}$ is estimated from (A1).

Our interest, here, is in determining the confidence bounds for SNR, given confidence bounds for $\hat{\gamma}^2_{xy}$. We can do this by noting from (A2) that SNR is a monotone increasing function of $\hat{\gamma}^2_{xy}$. Let $u = \hat{\gamma}^2_{xy}$ and $V = 1 - \hat{\gamma}^2_{xy}$. Then $\Delta u = \hat{\gamma}^2_{xy}(C) - \hat{\gamma}^2_{xy}$ and $\Delta V = \hat{\gamma}^2_{xy} - \hat{\gamma}^2_{xy}(C)$, where $\hat{\gamma}^2_{xy}(C)$ is an upper or lower confidence bound for the coherence estimate in Table I. From (A2),

$$\Delta\text{SNR} = \frac{u + \Delta u}{V + \Delta V} - \frac{u}{V} = \frac{V\Delta u - u\Delta V}{V(V + \Delta V)}$$

and

$$\frac{\Delta\text{SNR}}{\text{SNR}} = \frac{V\Delta u - u\Delta V}{u(V + \Delta V)}.$$

Substituting for u, V, Δu, and ΔV, we obtain

$$\frac{\Delta\text{SNR}}{\text{SNR}} = \frac{\hat{\gamma}^2_{xy}(C) - \hat{\gamma}^2_{xy}}{\hat{\gamma}^2_{xy}(1 - \hat{\gamma}^2_{xy}(C))}. \tag{A3}$$

Taking $10 \log_{10} [1 + (\Delta\text{SNR}/\text{SNR})]$ provides us with the SNR confidence bounds, relative to 0 dB, presented in the main text, i.e., the interval within which the true SNR will exist a given percent of the time, which is defined by $\hat{\gamma}^2_{xy}(C_{\text{upper}})$, $\hat{\gamma}^2_{xy}$, and $\hat{\gamma}^2_{xy}(C_{\text{lower}})$ from [1].

We have presented SNR confidence intervals for 80 and 95 percent confidence bounds in the main text. Note that estimated MSC values of zero and unity have been deliberately avoided because, for MSC confidence limits in [1], $\log_{10} [1 + (\Delta\text{SNR}/\text{SNR})]$ is undefined as MSC goes to zero and ΔSNR/SNR becomes indeterminate at unity MSC. However, the MSC values we have considered should cover most practical cases. If not, the reader must invoke a metric different from the one used here.

References

[1] G. C. Carter and E. H. Scannel, Jr., "Confidence bounds for magnitude-squared coherence estimates," NUSC Tech. Doc. 5881, Naval Underwater Systems Center, New London, CT, July 13, 1978.

[2] J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedures.* New York: Wiley–Interscience, 1971.

A Continuous Recursive DFT Analyzer—The Discrete Coherent Memory Filter

B. G. GOLDBERG

***Abstract*—The discrete version of the coherent memory filter (DCMF) is introduced and it is shown that the device performs a (N point) DFT of the N input data samples. It is also shown that with a simple modification the device can be operated recursively, so that as the data samples flow in, the DFT of the last N data samples are performed continuously.**

Manuscript received November 12, 1979; revised March 28, 1980.
The author is with the Ministry of Defense, Haifa, Israel.

0096-3518/80/1200-0760$00.75