Communications

Electromagnetic Wave Propagation in an Asymmetrical Coal Seam

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Abstract—Electromagnetic wave propagation in a coal seam is analyzed for the case where the surrounding floor and roof rocks have different electrical properties. Numerical results are presented for the attenuation rate and field distribution of the dominant mode. Even when the roof and floor conductivities are different, the vertical electric field and the horizontal magnetic field are the dominant components, and they are nearly constant within the coal seam. The results have application to mine communication and remote sensing of coal seams.

I. INTRODUCTION

During the 1970's, communication between loop antennas using medium frequencies (MF) (300 kHz–3 MHz) was demonstrated in coal seams for horizontal ranges of several hundred meters [1], [2]. The dominant mode of propagation in coal seams at MF is transverse magnetic (TM) with the magnetic field horizontally polarized. Because the electric field is primarily vertically polarized with only a small longitudinal component, the mode is nearly transverse electromagnetic (TEM) and is commonly called the quasi-TEM or coal seam mode. This mode also has potential for use in remote sensing of coal seams [3] because its propagation constant depends on the coal seam parameters (thickness and electrical properties).

Previous models for coal seams have assumed a uniform coal layer bounded above and below by rock of higher conductivity [1], [3], [4]. For simplicity, it was assumed that the floor and the roof rock had the same electrical properties. Recent experimental work in coal mines has revealed that the roof rock is often not the same type of rock as the floor and hence has different electrical properties [5]. The purpose of this communication is to analyze an asymmetrical coal seam where the floor and roof have different electrical properties.

II. FORMULATION

The geometry of an asymmetrical coal seam is shown in Fig. 1. The coal layer has thickness $2h$, permittivity ϵ_c , and conductivity σ_c . The roof ($z > h$) has permittivity ϵ_r and conductivity σ_r , and the floor ($z < -h$) has permittivity ϵ_f and conductivity σ_f . Free space permeability μ_0 is assumed everywhere.

The time dependence is $\exp(j\omega t)$, and it is suppressed throughout. The wavenumbers for the coal (k_c), roof (k_r), and floor (k_f) are given by

$$k_c = \omega \sqrt{\mu_0 \epsilon_{cc}}, \quad k_r = \omega \sqrt{\mu_0 \epsilon_{rc}}$$

and

$$k_f = \omega \sqrt{\mu_0 \epsilon_{fc}}, \quad (1)$$

Fig. 1. Coal seam geometry.

where

$$\epsilon_{cc} = \epsilon_c - j\sigma_c/\omega, \quad \epsilon_{rc} = \epsilon_r - j\sigma_r/\omega$$

and

$$\epsilon_{fc} = \epsilon_f - j\sigma_f/\omega.$$

In our two-dimensional analysis, we seek a solution which is propagating in x and independent of y . The lowest order mode is TM and has only an H_y component of the magnetic field and E_x and E_z components of the electric field. H_y must satisfy the Helmholtz equation in each region:

$$\begin{aligned} (\nabla^2 + k_c^2)H_y &= 0, & |z| < h, \\ (\nabla^2 + k_r^2)H_y &= 0, & z > h, \\ (\nabla^2 + k_f^2)H_y &= 0, & z < -h. \end{aligned} \quad (2)$$

Within the coal seam ($|z| < h$), H_y can be written in the following general form:

$$H_y = [H_e \cos(k_c Cz) + H_0 \sin(k_c Cz)] \exp(-jk_c Sx),$$

where

$$S^2 + C^2 = 1. \quad (3)$$

H_e and H_0 are unknown constants, and S is a normalized propagation constant to be determined from the mode equation. The electric field components are determined from Maxwell's curl equation:

$$E_x = \frac{-1}{j\omega\epsilon_{cc}} \frac{\partial H_y}{\partial z} \quad \text{and} \quad E_z = \frac{1}{j\omega\epsilon_{cc}} \frac{\partial H_y}{\partial x}. \quad (4)$$

In the roof ($z > h$), we require a solution that decays for large z . H_y can be written

$$H_y = A \exp(-j\alpha_r z) \exp(-jk_c Sx), \quad (5)$$

where

$$\alpha_r = \sqrt{k_r^2 - k_c^2 S^2}, \quad \text{Im}(\alpha_r) < 0.$$

A is an unknown constant, and Im denotes imaginary part. The

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electric field components are given by

$$E_x = \frac{-1}{j\omega\epsilon_{rc}} \frac{\partial H_y}{\partial z} \quad \text{and} \quad E_z = \frac{1}{j\omega\epsilon_{rc}} \frac{\partial H_y}{\partial x}. \quad (6)$$

In the floor ($z < -h$), we require a solution that decays for the large negative z . H_y can be written

$$H_y = B \exp(j\alpha_f z) \exp(-jk_c S x), \quad (7)$$

where

$$\alpha_f = \sqrt{k_f^2 - k_c^2 S^2}, \quad \text{Im}(\alpha_f) < 0,$$

and B is an unknown constant. The electric field components are given by

$$E_x = \frac{-1}{j\omega\epsilon_{fc}} \frac{\partial H_y}{\partial z} \quad \text{and} \quad E_z = \frac{1}{j\omega\epsilon_{fc}} \frac{\partial H_y}{\partial x}. \quad (8)$$

At $z = \pm h$, the tangential components H_y and F_x must be continuous; this leads to the following equations:

$$H_e \cos(k_c Ch) + H_0 \sin(k_c Ch) = A \exp(-j\alpha_r h),$$

$$H_e \cos(k_c Ch) - H_0 \sin(k_c Ch) = B \exp(-j\alpha_f h),$$

$$\frac{k_c C}{j\omega\epsilon_c} [H_e \sin(k_c Ch) - H_0 \cos(k_c Ch)] = \frac{\alpha_r A}{\omega\epsilon_{rc}} \exp(-j\alpha_r h),$$

$$\frac{-k_c C}{j\omega\epsilon_{cc}} [H_e \sin(k_c Ch) + H_0 \cos(k_c Ch)] = \frac{-\alpha_f B}{\omega\epsilon_{fc}} \exp(-j\alpha_f h).$$

(9)

Equation (9) is a linear, homogeneous system of four equations in four unknowns (H_e , H_0 , A , and B), and it has a nontrivial solution if and only if the determinant is zero. This determinant condition yields the following mode equation for C :

$$f_e + f_0 = 0, \quad (10)$$

where

$$f_e = jk_c C \tanh(jk_c Ch) + \frac{j\epsilon_{cc}}{2} \left(\frac{\alpha_r}{\epsilon_{rc}} + \frac{\alpha_f}{\epsilon_{fc}} \right)$$

and

$$f_0 = \frac{-j \tanh(jk_c Ch) \left[\epsilon_{cc} \left(\frac{\alpha_r}{\epsilon_{rc}} - \frac{\alpha_f}{\epsilon_{fc}} \right) \right]^2}{\tanh(jk_c Ch) \frac{\epsilon_{cc}}{2} \left(\frac{\alpha_r}{\epsilon_{rc}} + \frac{\alpha_f}{\epsilon_{fc}} \right) + k_c C}.$$

For the special case where the roof and floor parameters are equal ($\epsilon_r = \epsilon_f$ and $\sigma_r = \sigma_f$), f_0 is zero and (10) reduces to the previous mode equation for the symmetrical coal seam [3], [4], [6]. Once C is determined, the propagation constant Γ is given by

$$\Gamma = jk_c S = jk_c \sqrt{1 - C^2}, \quad \text{Re}(\Gamma) < 0, \quad (11)$$

where Re indicates the real part. The attenuation rate is a limiting factor in communication and remote sensing applications, and it is given by $8.686 \text{ Re}(\Gamma)$ in dB/m.

When the wavenumber in the roof and floor are much greater than the coal wavenumber ($|k_r| \gg |k_c|$ and $|k_f| \gg |k_c|$), an approximate

solution for C can be obtained

$$C \approx \sqrt{\frac{j}{2}} \sqrt{\frac{1}{k_r h} + \frac{1}{k_f h}}. \quad (12)$$

If C in (12) is small compared to unity, the propagation constant Γ in (11) is given approximately by

$$\Gamma \approx jk_c + \frac{k_c}{4k_r h} + \frac{k_c}{4k_f h}. \quad (13)$$

The first term in (13) represents plane wave propagation in a infinite medium, and the remaining two terms in (13) are due to the finite conductivity of the roof and the floor. When the roof and floor wavenumbers are equal ($k_r = k_f$), the approximations in (12) and (13) reduce to the earlier approximation for a symmetrical seam [7].

III. NUMERICAL RESULTS

The mode equation (10) has been solved numerically by Newton's method [8]. For a starting value of C , we use the approximation in (12). The computer code has been checked against previous results for the special case of a symmetrical seam [3], [6].

In Fig. 2 we show the attenuation rate of the quasi-TEM mode as a function of roof conductivity σ_r for the following parameters: $\sigma_c = 10^{-4}$ S/m, $\epsilon_c/\epsilon_0 = 6$, $\epsilon_r/\epsilon_0 = \epsilon_f/\epsilon_0 = 15$, $2h = 2$ m, and frequency = 500 kHz. The permittivities are normalized to the free space value ϵ_0 . The frequency of 500 kHz was chosen because it has been found to provide a long range for communications [2] and remote sensing [5]. When the floor and roof conductivities are equal ($\sigma_f = \sigma_r$), the results in Fig. 2 agree with previous results for the symmetrical seam [3]. When σ_f is constant at 10^{-1} S/m and σ_r varies (a common case in real coal mines [5]), the variation with σ_r is less pronounced. This behavior is in agreement with the approximate result in (12). The decrease in attenuation rate for $\sigma_r < 2 \times 10^{-3}$ S/m is not predicted by the approximate result in (12). The decrease occurs because for small σ_r , more energy propagates in the roof, and a smaller value of σ_r yields a lower roof loss.

At MF, all of the higher order TM modes and all of the transverse electric (TE) modes are cut off and have very high attenuation rates. We have not considered TE modes here because they are not important at MF. This conclusion is consistent with the ray optical analysis of transmission in coal seams by Holmes and Balanis [9]. The first higher order propagating mode occurs at the frequency of 30.6 MHz where the coal height is half the wavelength in coal.

In Figs. 3-5, the electric and magnetic field distributions are shown for three different values of roof conductivity σ_r . The normalization field E_e is the value of E_z at $z = 0$ and is given by

$$E_e = \eta_c S H_e,$$

where

$$\eta_c = \sqrt{\mu_0/\epsilon_{cc}}. \quad (14)$$

In Fig. 3, the seam is symmetrical, and E_z and H_y are even in z . E_x is odd and is thus zero at the center of the seam. In Figs. 4 and 5, the roof and floor conductivities are not equal and the field distributions are not symmetrical. The asymmetry is most evident in the field decay outside the coal seam ($|z| > h$). However, in all three cases the dominant field components inside the coal seam are E_z and H_y , and they are nearly constant for $|z| < h$. Thus the quasi-TEM mode can be efficiently excited (or received) by either a vertical electric dipole or a horizontal magnetic dipole (vertical loop) located anywhere within the seam. In addition to providing information on excitation of

Fig. 2. Attenuation rate for the quasi-TEM mode as a function of σ_r . Parameters: $\sigma_c = 10^{-4}$ S/m, $\epsilon_c/\epsilon_0 = 6$, $\epsilon_r/\epsilon_0 = \epsilon_f/\epsilon_0 = 15$, $2h = 2$ m, and frequency = 500 kHz.

Fig. 3. Field distributions for the quasi-TEM mode for $\sigma_r = \sigma_f$. Parameters: $\sigma_c = 10^{-4}$ S/m, $\epsilon_c/\epsilon_0 = 6$, $\epsilon_r/\epsilon_0 = \epsilon_f/\epsilon_0 = 15$, $\sigma_f = 1$ S/m, $2h = 2$ m, and frequency = 500 kHz.

the quasi-TEM mode, the field distributions in Figs. 3–5 also indicate how anomalies in or near the seam will be illuminated in remote sensing applications.

IV. CONCLUSION

Propagation of the dominant TM mode in a coal seam has been studied for the case where the bounding floor and roof rocks have different electrical properties. Numerical results have been presented for the attenuation rate and the field distribution at MF. Even when the roof and floor conductivities are different, the vertical electric field and the horizontal magnetic field are the dominant components,

Fig. 4. Field distributions for the quasi-TEM mode for a higher value of roof conductivity. Parameters: $\sigma_c = 10^{-4}$ S/m, $\epsilon_c/\epsilon_0 = 6$, $\epsilon_r/\epsilon_0 = \epsilon_f/\epsilon_0 = 15$, $\sigma_f = 10^{-1}$ S/m, $2h = 2$ m, and frequency = 500 kHz.

Fig. 5. Field distributions for the quasi-TEM mode for a lower value of roof conductivity. Parameters: $\sigma_c = 10^{-4}$ S/m, $\epsilon_c/\epsilon_0 = 6$, $\epsilon_r/\epsilon_0 = \epsilon_f/\epsilon_0 = 15$, $\sigma_f = 10^{-1}$ S/m, $2h = 2$ m, and frequency = 500 kHz.

and they are nearly constant within the coal seam. The asymmetry is more evident in the field decay in the floor and roof.

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Determination of the Green's Function in the Spectral Domain Using a Matrix Method: Application to Radiators or Resonators Immersed in a Complex Anisotropic Layered Medium

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Abstract—A planar structure having arbitrarily located conductor patches immersed in complex anisotropic layered media presents a very general field problem. This problem is solved here by a rigorous formulation technique characterizing each layer by a 6×6 tensor and finding the appropriate Fourier transformed Green's function matrix G of $2n \times 2n$ size. The technique finds a set of field eigenvectors for each layer. Using G , a method of moments numerical solution for radiation characteristics of probe fed patch(es) can be had in the spectral domain employing, for example, a zero reaction method. Variation of real frequencies of the driving probe fed signal is allowed by that approach. Those workers desirous of radiator or resonator fields and frequency behavior at only selected resonant frequencies can use G to derive a matrix S_x given here. Setting the determinant of S_x equal to zero yields complex resonant frequency solutions, and the field solutions as a consequence to the nonprobe fed or free standing patch structure. The method is very versatile and can handle a large class of microwave or millimeter wave integrated circuit or monolithic circuit problems, no matter how simple or complex as long as they possess planar layers.

I. INTRODUCTION

Advance in materials technology is allowing the contiguous growth of substances of considerably different properties. Present integrated circuit processing techniques allow various combinations of metals, dielectrics, and semiconductors to be layered together where these materials may or may not be crystalline. We may expect to see in the future the use of magnetic films [1], [2] (metallic or nonmetallic), uniaxial and biaxial dielectric films [3], [4], ferrite films, magnetically induced semiconductor gyroelectric films, and widely varying compositions of compound films such as binary, ternary, and quaternary compounds. More creative use of materials, especially for

monolithic integrated circuits, will probably occur in the future. Besides some of the more familiar classes of anisotropic materials mentioned above, materials with optical activity may be employed in integrated circuit applications. Finally, very complex materials with a combination of birefringent gyroelectric, gyromagnetic, optical rotation, or other anisotropic properties may be utilized. Furthermore the use of semiconductors, dielectrics, or magnetomaterials rotated off principal axes or convenient axis coordinates can be envisioned.

Conventional methods either in direct or Fourier transformed space using planar symmetry are not general enough to enable the interested worker in the microwave or millimeter wave area to readily solve such complex problems outlined above. The theoretical formulation presented in this communication shows how to solve for an interface Green's function leading to determination of the electromagnetic fields in a multilayered planar structure having complex anisotropic layers. The only restrictions to the formulation below are that the conductors are assumed to be lossless and infinitesimally thick.

Expedient ways of solving field problems based on Maxwell's equations, but avoiding gauge methods, are possible by using field matrix techniques. For two field components, two second-order partial differential equations (PDE's) must be employed in two field components to find the field solution. Matrix techniques using two field components have been often used in the optics [5], [6] and microwave/electromagnetics [7] areas. Nevertheless, as the medium becomes more complex with less symmetry, the two-component methods become increasingly difficult to implement. Lack of conductor line symmetry also complicates the two-component solution methods.

Use of four components has been shown in reflection and transmission light problems to lead to simple PDE's [8], [9]. A four-component method has the great advantage of enabling the use of only first-order PDE's. The four-component technique also has the ability to allow direct field matching at layer boundaries or interfaces [10]. In [10] it is pointed out that such field matching avoids the need to employ auxiliary equations in the two suppressed field components if using the two-component method, thus providing some economy in problem solving.

Here a new formulation technique for solving the field problem is developed for layered media possessing complex anisotropic properties. A four-component method is utilized by adapting the 4×4 matrix approach in [10] to the spectral domain or Fourier transform domain (FTD). Significant advantage is gained by working in the FTD because Green's function convolution integrals for determination of field quantities due to current sources are converted into algebraic products. Section II develops the normal mode field solution formulation in the FTD. One should be alerted that the chosen column field vector used here differs from [10] in the component selection and arrangement. Section III gives the solution to the open or uncovered multilayered patch problem. The Green's function G which is derived pertains to a perfectly conducting ground plane. From G one can obtain the electromagnetic fields using standard method of moments numerical techniques assisted by identical expansion and test basis functions (Galerkin approach). Involved in such determinations is the application of the remaining set of BC's requiring the electric field $\vec{E} = 0$ on the interface conductors. For probe fed radiators, an approach [11] similar to the zero reaction method [12], [13] leads to currents on conductors, fields in the layered structure and radiated fields, and mutual- and self-impedance Z_{ij} circuit multiport parameters all functions of a real frequency, and resonant frequencies determined by $\text{Im}(Z_{ii}) = 0$. For free standing

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