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A Note on Pulse Distortion by Ionospheric Reflection

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Abstract—An expression, suitable for a small desk-top computer, is presented to calculate the time domain envelope of a rectangular pulse reflected from a sech^2 model ionosphere and observed at the output of a simple receiver bandpass filter. Ionospheric, source pulse, and filter parameters can be varied to estimate distortion characteristics of the received pulse.

I. INTRODUCTION

Investigations concerning the distortion of ionospherically reflected pulses continue to be of interest in such fields as digital communication and navigation systems and pulse signature detection [1]. Elaborate computer programs can be written to evaluate numerically various system and propagation configurations, but it is also helpful to have a simpler model that can estimate characteristics of the received pulse. This note suggests an expression derived from a sech^2 model ionosphere, a rectangular source pulse, and a simple *RLC* series circuit receiver filter. The equation can be evaluated with a small desk-top computer and is applicable to a variety of ionospheric layer conditions.

The time domain waveform $E_r(t)$ of a pulse that is reflected from the ionosphere and observed at the output terminals of a receiver bandpass filter can be expressed as

$$E_r(t) = (E_0/2\pi) \int_{-\infty}^{+\infty} S(\omega)R(\omega)B(\omega)e^{i\omega t} d\omega, \quad (1)$$

where E_0 is an amplitude constant determined from the source, $\omega = 2\pi f$ denotes the electromagnetic (angular) frequency, $S(\omega)$ and $B(\omega)$ are the frequency spectra of the source pulse and bandpass filter, respectively, and $R(\omega)$ represents the ionospheric reflection coefficient.

A recent paper [2] has investigated the impulse response for an ionospheric model characterized by a sech^2 electron density height profile and constant values of electron collision frequency and vertical component (only) of the earth's magnetic field. With ω_p denoting the penetration frequency, and z_m and σ the maximum density height and width parameter of the ionospheric layer, the

refractive index n for a sech^2 model is given by

$$n_\epsilon^2 = 1 - \{(\omega_p/\omega)^2/b_\epsilon\} \text{sech}^2 \{(z-z_m)/2\sigma\}, \quad (2a)$$

$$(\epsilon = \pm 1)$$

$$b_\epsilon = 1 + (\epsilon\omega_H - i\nu)/\omega, \quad (2b)$$

where ν and ω_H are the collision frequency and gyro frequency, respectively. For oblique incidence we also define $C \equiv \cos \theta$ with θ being the angle of incidence that the wave-normal in the x - z plane makes with the vertical z axis in a Cartesian coordinate system.

Solutions for the field components were derived (exact for vertical incidence—approximate for oblique incidence), and two reflection coefficients were defined in terms of the linearly polarized incident and reflected E -fields:

- 1) R_v : the ratio of the reflected x to the incident x component;
- 2) R_h : the ratio of the reflected y to the incident x component.

The reflection coefficients involve gamma functions of complex argument, and numerical methods can be used to evaluate (1) and show the detailed behavior of the received pulse. However, the envelope of the pulse often provides a more practical means of presenting significant pulse shape characteristics.

In many cases an expressions for the absolute value of the envelope of (1) can be obtained by applying the method of stationary phase. This results in

$$E_r/E_0 \sim \sum_{\omega_t} |f(\omega_t)/\{2\pi\phi''(\omega_t)\}^{1/2}| e^{i\{\omega_t t + \phi(\omega_t)\}}, \quad (3)$$

where

$$f(\omega) = S(\omega)R(\omega)B(\omega), \quad (4a)$$

$$\phi(\omega) = \arg S(\omega) + \arg R(\omega) + \arg B(\omega), \quad (4b)$$

and ω_t represents the value (or values) for which

$$(d/d\omega)[\omega t + \phi(\omega)] = t + \phi'(\omega) = 0. \quad (5)$$

In general the summation in (3) should be over all values of ω_t . However, if the only significant roots of (5) are the pair, $\pm\omega_t$, and if the Hermitian relationship holds, i.e., $f(-\omega) = f^*(\omega)$, then (3) becomes

$$E_r/E_0 \sim (2/\pi)^{1/2} |S(\omega_t)R(\omega_t)B(\omega_t)/\{\phi''(\omega_t)\}^{1/2}| \cos \{\omega_t t + \phi(\omega_t)\}, \quad (6)$$

where

$$\phi''(\omega_t) = (d^2/d\omega^2)[\arg S(\omega) + \arg R(\omega) + \arg B(\omega)]|_{\omega=\omega_t} \neq 0. \quad (7)$$

The factor multiplying the cosine term in (6) now provides an approximation to the positive portion of the envelope of (1).

In presenting results for the sech^2 model, it is convenient to introduce the quantities

$$\zeta = \alpha\omega, \quad T = \{t - 2(C/c)(z_m - z_1)\}/\alpha, \quad (8a)$$

$$\alpha = 2(\sigma/c)C, \quad \delta = 2(\sigma/c)\omega_p, \quad (8b)$$

where c is the speed of light and z_1 is the height to which the reflection coefficient is referred. Then, as shown in [2], the envelope of the impulse response obtained by setting $S(\omega)$ and $B(\omega)$ in (1) to

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TABLE I
VALUES OF IONOSPHERIC AND FILTER PARAMETERS FOR FIGS. 1-3

	ω_p	σ (km)	θ^0	ν	ω_H	ω_0	Q
Figure 1.	50	60	0	0	0	28.87	100
Figure 2.	30	1	15	0.01	1	11.30	0.1, 0.7
Figure 3.	25	3	60	0	0	25.33	6

$\omega_p, \omega_H, \omega_0$ in MHz ($\omega = 2\pi f$); ν in collisions/ μ s

unity is

$$\text{env } (\alpha E_r/E_0) \sim [\zeta_T/\pi(1+e^T)]^{1/2} e^{-A_T} |P(A\omega_H)|, \quad (9)$$

$$\zeta_T = p_T \delta, \quad p_T = \{e^T/(1+e^T)\}^{1/2}, \quad (10a)$$

$$A = (\alpha/2p_T) \ln [(1+p_T)/(1-p_T)]. \quad (10b)$$

The factor $P(A\omega_H)$ depends on whether R_v or R_h is assumed in (1) and is defined by

$$P(A\omega_H) = \begin{cases} \cos(A\omega_H), & R = R_v \\ \sin(A\omega_H), & R = R_h. \end{cases} \quad (10c)$$

In deriving (9), the following asymptotic approximation has been used in (5) to determine ζ_T :

$$(d/d\zeta) \arg R(\zeta) \sim \ln [(\delta^2 - \zeta^2)/(1 + \zeta^2)]. \quad (11)$$

Except for small δ and near the leading edge of the pulse, (9) agrees well with the envelope as evaluated by numerical quadrature. In the leading edge region, details of the pulse shape can be obtained by applying a fast Fourier transform (FFT) to (1).

To assess the effects of a source pulse and receiver filter on the propagation channel, we will assume a simple rectangular source with frequency spectrum

$$\hat{S}(\zeta) = (1/2)[S(\zeta) + S(-\zeta)], \quad (12a)$$

$$S(\zeta) = A_p [\sin \{\Delta_p(\zeta - \zeta_c)\} / \{\Delta_p(\zeta - \zeta_c)\}], \quad (12b)$$

where $\Delta = \alpha\Delta_p$ is the half-width of the pulse centered at $T = 0$, A_p is a constant representing the area of the pulse, and $\zeta_c = \alpha\omega_c$ denotes an effective carrier frequency. The time domain form of the source pulse is given by

$$\alpha s(T) = (1/2\pi) \int_{-\infty}^{+\infty} \hat{S}(\zeta) e^{i\zeta T} d\zeta = (A_p/2\Delta_p) U(T) \cos(\zeta_c T), \quad (13a)$$

where

$$U(T) = \begin{cases} 1, & |T| < \Delta_p \\ 1/2, & T = \pm \Delta_p \\ 0, & \text{otherwise.} \end{cases} \quad (13b)$$

We will further assume an RLC circuit filter model described by Price [3] with frequency spectrum

$$B(\zeta) = [1 + i(Q/\zeta_0)(\zeta^2 - \zeta_0^2)/\zeta]^{-1}, \quad (Q \geq 0) \quad (14)$$

where $\zeta_0 = \alpha\omega_0$ is the effective filter characteristic frequency and Q is a bandwidth parameter.

If the differentiations indicated in (5) and (7) are now performed (but substituting the dimensionless variables ζ and T for ω and t), it is

found that the terms involving $\arg S(\zeta)$ and $\arg B(\zeta)$ are negligible compared with the reflection coefficient terms as long as $\delta \gg 1$ and $(Q/\zeta_0) \ll 1$. Thus from (6) and (9), the envelope of (1) can be approximated by

$$\text{env } (\alpha E_r/E_0) \sim |\hat{S}(\zeta_T) B(\zeta_T)| [\zeta_T/\pi(1+e^T)]^{1/2} e^{-A_T} |P(A\omega_H)|, \quad (15)$$

where

$$|B(\zeta_T)| = [1 + (Q/\zeta_0)^2 \{(\zeta_T^2 - \zeta_0^2)/\zeta_T\}^2]^{-1/2}, \quad (16)$$

$$|\hat{S}(\zeta_T)| = (1/2) |S(-\zeta_T) + S(\zeta_T)|, \quad (17)$$

and the other quantities are defined in (10) and (12). Note that the sinusoidal variation arising from the effects of the magnetic field component ω_H has been included in the envelope expression.

Comparisons of (15) with FFT evaluations of (1) indicate that (15) is good to within 0.5 for $\delta \geq 10$ and $(Q/\zeta_0) \leq 0.01$. As the latter ratio becomes larger, it is necessary to include the effect of the filter term in determining the roots of (5). Consequently, the envelope would be computed using (3) rather than (15). The present note is restricted to the condition $(Q/\zeta_0) \ll 1$.

Another condition of the stationary phase method is that the magnitude terms should be slowly varying, at least in the region where they are numerically significant. Thus, it is necessary to restrict the source function $S(\zeta_T)$, which in turn requires a restriction on Δ_p . If we specify that $\Delta_p(\zeta_T - \zeta_c) = \pi$, then FFT comparisons indicate a suitable relationship for Δ_p is $\Delta_p \leq 10/\zeta_c$.

An example of the use of (15) is shown in Fig. 1 for the values given in Table I. In this case the ionospheric layer is characterized by the dimensionless quantity $\delta = 2 \times 10^4$ and $\alpha = 400 \mu$ s. The effective filter frequency, ζ_0 , that is assumed is that value for which the maximum peak amplitude of the reflected pulse can be observed. When ν and ω_H are both zero, the peak occurs at $T = \ln(1/2)$ and $\zeta_0 = \sqrt{1/3}\delta$. Choosing other values of ζ_0 will, among other things, diminish the magnitude of the pulse observed at the output of the receiver filter.

Fig. 1 indicates how the shape of the observed reflected pulse will vary (for a constant $Q = 100$) as the effective width of the rectangular source pulse, $2\Delta_p$, is increased. For a narrow enough pulse corresponding to a large source bandwidth, the source is essentially an impulse. For the system depicted in Fig. 1, the impulse response is well approximated by the curve labeled $\Delta_p = 4 \times 10^{-4}$ and, in fact this curve corresponds to that shown in [3, fig. 11 (the dashed curve)]. Smaller values of Δ_p used in (15) (after appropriate normalization by A_p) will result in this same impulse response curve. Larger Δ_p with more restricted source bandwidths give received pulse shapes exhibiting a distorted $(\sin x)/x$ behavior. An example is shown by the curve labeled $\Delta_p = 4 \times 10^{-3}$. The pulse shows the effect of the bandwidth limitation in the source.

The case of a weak magnetic field on a thin reflecting layer for

Fig. 1. Envelope of the received pulse ($R = R_v$) for the values in Table I. Actual source pulse width $= 2\alpha\Delta_p$. Assumed carrier frequencies: f_c (MHz) $= 3/4\alpha\Delta_p$.

Fig. 2. Envelope of the received pulse ($R = R_v$) for the values in Table I and $A_p = 3\pi/2$. Actual source pulse width $= 0.0644 \mu s$. Assumed carrier frequency $f_c = 23.292$ MHz.

propagation at a slightly oblique angle from the vertical is shown in Fig. 2 with the parameter values again given in Table I. A constant electron collision frequency $\nu = 0.01$ has been assumed, and only curves for the reflection coefficient $R = R_v$ are presented. The original carrier-modulated rectangular pulse has a width of ~ 64 ns ($= 2\alpha\Delta_p$), and calculations were made for two values of the filter bandwidth parameter, $Q = 0.1$ and 0.7 . The vertical magnetic field component introduces oscillations into the envelope of the received signal which would tend to oscillate more rapidly with a stronger field. The effect of increasing Q is to narrow the pulse width in the first oscillation and to reduce the magnitude of the envelope once this main portion of the pulse has passed. For Q large enough, the main portion widens and decreases in amplitude, although this cannot be shown by (15) because of the restriction on (Q/ξ_0) .

Fig. 3 shows the effect of varying the receiver filter characteristic frequency. The ionospheric parameters are given in Table I, and the source pulse is assumed to be an impulse ($\Delta_p = 0$). For this case $\delta = 500$ and $\alpha = 10 \mu s$; thus the characteristic frequency in MHz is given by $f_0 = \xi_0/2\pi\alpha = 0.016\xi_0$. Because the reflected pulse contains frequencies up to ω_p that arrive sequentially, the peak of the pulse after passing through the filter will change with time depending on ξ_0 . With the receiver tuned at the higher frequency, the pulse peak is delayed relative to that which would be received at a lower frequency. Estimates of this time delay can be useful in studies of various radio systems. Furthermore, because the 3 dB filter bandwidth is proportional to f_0 , there is greater dispersion in the received pulse at the higher characteristic frequency.

It is obvious, of course, that (15) will characterize observed measurements only insofar as the model and approximations upon which it is based are representative of actual conditions. In considering the limitations of (15), the following assumptions and approximations are relevant.

1) In the ionosphere the electron density profile is assumed to have

Fig. 3. Envelope of the received pulse ($R = R_v$) for the values in Table I and $A_p = 1$, $\Delta_p = 0$, $\omega_0 = \xi_0/\alpha(\mu s) = 0.1\xi_0$.

a sech^2 height variation and the electron collision frequency is constant with height.

2) The model assumes plane wave propagation and cannot account for the effects of Pedersen rays [4] on the reflected pulse.

3) Because of approximations made in the derivation of the electromagnetic field components, the full reflection coefficient as obtained in [2] is only approximate for oblique incidence propagation, but becomes exact in the limit as $\Theta \rightarrow 0$ (normal incidence).

4) General conditions under which the method of stationary phase may be applied to the present model are: δ large, ω_H small, (Q/ζ_0) small, and $\hat{S}(\zeta_T)$ slowly varying near ζ_c .

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The Measurement of the Electric Field on the Surface Above Conducting Objects Immersed in a Dissipative Half-Space

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Abstract—The experimental set-up and techniques for the measurement of the electromagnetic fields on the surface above metal targets submerged in salt water are described. The variation of the electric field with respect to the depth of the scattering target is discussed. An attempt is made to relate the measured field components to the size and the shape of the scattering targets.

I. INTRODUCTION

Electromagnetic techniques are particularly useful in geophysical exploration when the subsurface objects or regions of interest have significantly different electrical parameters from those of the surrounding earth. Earlier theoretical studies in this area treated the scattering of a plane wave from simple conducting targets immersed in a dissipative half-space [1], [2]. More recently King [3], [4] studied scattering from a buried insulated conductor illuminated by an incident lateral wave. The lateral-wave field was maintained by a horizontal half-wave dipole in the air. On the experimental side, Chan *et al.* [5] reported a baseband pulse radar for subsurface target identification. Experiments using an FMCW radar have also been reported [6]. In contrast to the above experimental studies, the investigation reported in this communication utilized an unmodulated CW radar in a bistatic configuration [7], [8]. The experiment was

designed so that it was an approximate scale model of the actual geophysical problems of interest to the Sandia National Laboratories [7]. Such problems arise, for example, in the mapping of enhanced-oil-recovery (EOR) processes by controlled source audio magnetotelluric surveys (CSAMT). In terms of the complex wave number in a dissipative medium, $k = \beta + i\alpha$, the scattering properties of an object with a typical dimension l are determined, insofar as they depend on its size, by the quantity $\beta l = 2\pi l/\lambda$ (where λ is the wavelength in the medium) which was, therefore, of the same order of magnitude in the model as the corresponding quantity in the full-scale situation. The amplitude of the field that excites the subsurface object located at a depth d depends on the quantity $\alpha d = d/\delta$ (where δ is known as the skin depth) in the form $\exp(-\alpha d)$. This quantity also was scaled in accordance with the full-scale parameters.

II. EXPERIMENTAL SET-UP

A functional layout of the experimental set-up at the Gordon McKay Laboratory of Harvard University is shown in Fig. 1. The set-up included an above-ground Coleco® pool which was 18 ft in diameter and was filled with salt water ($\epsilon_r = 79.213$, $\sigma = 0.057$ S/m) to a nominal height of 4 ft (the exact water level was measured using a vertical scale attached to the side of the pool). The transmitting antenna was a half-wave dipole at the operating frequency of 300 MHz. It was supported in a horizontal position 2.5 cm above the surface of the water by a styrofoam block secured by two strings. At the center of the pool, the scattering object (a metal disk with or without a cut-out wedge) was mounted on a nonconducting (plastic) positioning apparatus which was actuated by a chain drive from outside the pool. The scattering object could be lowered to a maximum depth of 18 in from the surface of the water. The distance between the transmitting antenna and the center of the pool is 1 m as measured along the surface of the water. An electrically small (length $= 0.05 \lambda_{\text{air}}$) dipole antenna connected to a Hewlett-Packard 8405A vector voltmeter was used to map the horizontal components of the electric field. The probe was at a height of 2.5 cm from the surface of the water and could be positioned in steps of 1 cm within an area of 50 cm $\times$ 50 cm at the center of the pool. Microwave absorbers were used on the sides of the pool as well as over parts of the ceiling to isolate the experimental set-up from undesirable reflections from the room. The absorber in the most sensitive parts of the set-up was AAP-24™ (rated at 30-dB max. reflection at normal incidence at 300 MHz [9]). Other areas of the set-up were surrounded by Emerson and Cummings FR-350™ blocks.

III. MEASUREMENTS

A. Constitutive Parameters of Salt Water

Two different techniques, both employing conventional microwave circuits, were used to measure the electric parameters of salt water at 300 MHz at 20°C. The first technique consisted simply of measuring the complex propagation constant $k = \beta + i\alpha$ for a traveling wave in a coaxial line filled with salt water [10]. Once β and α were known, ϵ_r and σ were computed from the following relationships:

$$\epsilon_r = \frac{1}{w^2 \mu_0 \epsilon_0} (\beta^2 - \alpha^2)$$

$$\sigma = 2\alpha\beta/w\mu_0.$$

With this technique, ϵ_r was measured to be 79.2, but the conductivity of the solution was too small to be measured. The measured value of ϵ_r compares favorably with the data reported in [10].

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