VI. CONCLUDING REMARKS

In conclusion, we have developed a cavity-model theory which appear to predict successfully the performance of the circular-disk microstrip antenna with an air gap.

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A Comparative Study of Linear Array Synthesis Technique Using a Personal Computer

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Abstract—Five computer programs for synthesizing low-sidelobe sum patterns from linear arrays are evaluated in terms of run time and precision. Three of the programs are based on the Dolph-Chebyshev synthesis procedure, in which all sidelobes are set at the same level. The other two programs are based on a discretized version of the Taylor synthesis procedure, in which far-out sidelobes are allowed to decay. The programs were written for use on small 8- and 16-bit personal computers. It was found that the fastest running programs are also the most precise. The only Chebyshev program that gave satisfactory precision for arrays as large as 100 elements is based on Bresler's nested product algorithm, and the only similarly acceptable Taylor program is based on Shelton's discretized synthesis.

INTRODUCTION

Procedures for synthesizing the radiation patterns of linear arrays based on the specification of the sidelobe structure are well established. One of these is the technique originally proposed by Dolph [1] and it provides for a uniform sidelobe level. Another technique, although developed for line sources, is due to Taylor [2] and can be adapted for linear arrays. The latter is popular due to its synthesized aperture distributions which are more readily realizable. A discrete version of the Taylor synthesis procedure is discussed by Shelton [3].

The problem of computation of the aperture distribution for the Dolph-Chebyshev synthesis technique has been addressed

by several authors [4]–[9] over the past few decades. The Taylor synthesis procedure uses a discrete Fourier transform technique (called Woodward [10] synthesis) to obtain the aperture distribution. Although both of these synthesis techniques, fundamentally, rely on the manipulations of the zeros of the linear array pattern function, the procedures for computation of aperture distribution are vastly different. Even for the Dolph-Chebyshev synthesis, the expressions for a broadside array and an endfire array do not have any commonality. In contrast, the convolution synthesis procedure proposed for planar arrays [11] can be used to develop a uniform procedure for the synthesis of both Dolph-Chebyshev and Taylor syntheses. From the programming standpoint such a procedure is attractive.

The possibility of utilizing several alternative procedures for these syntheses motivated the authors to conduct this investigation. This study carries out computation on personal computers because of their increasing popularity with engineers. We present the results of a comparative study of various linear array synthesis techniques. In the next section, after a brief discussion of the three basic techniques for evaluation of Chebyshev coefficients, we discuss the accuracy and computation times associated with these techniques. The following section presents the results of the study involving two different techniques (one due to Shelton [3] and the other using the convolution procedure [11]) for Taylor synthesis. In the last section, some general observations about the investigation and on the results are offered.

DOLPH-CHEBYSHEV SYNTHESIS

Following Dolph's paper on Chebyshev synthesis, Barbieri [4], Van Der Maas [5], Salzer [6], and Brown [7], [8] reported on alternative means of evaluating aperture distribution for Chebyshev arrays. Although they are not the same, the expressions by Barbieri, Salzer, and Brown are similar in that they express the current in an element in terms of a finite series of terms involving ratios of factorial functions and with alternating sign. The expression by Elliott [12] is representative of this group and is the one used in this work and is reproduced below. We shall call this the classical expression. Also, although our results are valid for odd or even number of elements, for simplicity we will present examples of odd number of elements. Thus, all linear arrays discussed in the following have $(2N + 1)$ elements; the element numbering scheme is shown in Fig. 1 where the elements are assumed to have a symmetric excitation leading to the broadside radiation.

Classical technique:

$$I_n = \sum_{p=n}^N (-1)^{N-p} \frac{N}{N+p} \frac{\Gamma(N+p+1)}{\Gamma(N-p+1)\Gamma(p+n+1)\Gamma(p-n+1)} \cdot (u_0)^{2p}, \quad n = 0, 1, 2, \dots, N \quad (1)$$

where $T_{2N}(u_0) = R$ and $SLL = 20 \log R$. Here, SLL is the desired sidelobe level in dB, $T_{2N}(x)$ is the Chebyshev polynomial of degree $2N$ and $\Gamma(x)$ is the gamma function.

In contrast, the expression given by Van Der Maas involves terms of the same sign inside the summation. Bresler [9] reformulated the expression into a recursive form using nested products. This, we feel is a distinctly different form of representa-

Manuscript received January 5, 1984; revised March 15, 1984. This work is based on a paper presented at the 1983 Antenna Applications Symposium, Monticello, IL, September 21-23, 1983.

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$$U = kd \sin \theta$$

Fig. 1. $(2N + 1)$ element linear array.

tion of the coefficients. Thus, we use this representation (called nested product technique) in our comparison. This expression (in our notation) is shown below.

Nested product technique:

$$I_{N-n} = 2N\alpha \text{ NP}(n, f_m, \alpha), \quad n = 0, 1, 2, \quad (2)$$

where

$$\text{NP}(n, f_m, \alpha) = \sum_{m=1}^n \alpha^{n-m} \prod_{j=m}^n f_j; \quad f_n \equiv 1.$$

and

$$f_m = \frac{m(2N - 2n + m)}{(n - m)(n + 1 - m)};$$

also

$$\alpha = 1 - \frac{1}{u_0^2}.$$

The third technique is based on the convolution of three element canonical arrays [11]. These canonical arrays have outer element excitations of unity, whereas the center element excitation C_j ; for $j = 1, 2, \dots, N$ is chosen such that the j th canonical array has a pattern null at the location of the j th symmetric zero pair of the Chebyshev polynomial. These arrays are then convolved to generate the large array.

Convolution technique:

$$w_{0j} = \cos \frac{(2j-1)}{2N} \pi; \quad j = 1, 2, \dots, N.$$

where w_{0j} are the zeros of the Chebyshev polynomial $T_{2N}(w)$.

$$C_j = -2 \cos u_{0j};$$

$$u_{0j} = 2 \arccos(w_{0j}/u_0)$$

and the aperture distribution

$$I(x) = \sum_{n=-N}^N I_n \delta(x - nd) = f_1 * f_2 * \dots * f_N \quad (3)$$

where $f_j = \delta(x - d) + C_j \delta(x) + \delta(x + d)$.

Using these three expressions ((1), (2), and (3)), computer programs NESTED, CHEB, and CONCHEB, respectively, were written to implement the Chebyshev synthesis. Different versions of the program suitable for implementation on different machines were written. There were two personal computers used in the numerical phase; one is an 8-bit Radio Shack TRS-80 Model II which has available an interpretive Basic language. The other computer is a 16-bit NEC Advanced Personal Computer with Basic and Fortran IV compilers. Also, in order to ascertain the numerical accuracy, some of the programs were run on a 32-bit mainframe computer (Texas Instrument's Advanced Scientific Computer at the Naval Research Laboratory) using double precision (REAL*8) arithmetic.

Fig. 2. Run times for Chebyshev programs—Fortran IV on NEC-APC.

The computation was carried out for several different array sizes ranging from 15 to 99 elements. All designs specified a sidelobe level of 30 dB.

Fig. 2 shows the run time, under Fortran, for the three aforementioned Chebyshev synthesis programs versus number of elements. The CHEB program was the slowest; but more importantly, the program failed to converge to the correct element excitations beyond 31 elements. Over 21 elements the accuracy of the excitation was only to two digits. When the program was run using double precision arithmetic it still failed to converge above 31 elements. This indicates that the classical technique inherently has a limitation as to the largest size of array that may be synthesized.

The convolution synthesis program CONCHEB, although much faster than CHEB, certainly cannot compete with NESTED program in speed. Also, beyond 61 elements, the CONCHEB program failed to converge. The current version of the program convolves three-element arrays using zeros of the Chebyshev polynomial in an alternating sequence; i.e., the sequence in j is 1, N , 2, $N - 1$, 3, $\dots$. No attempt was made to modify this convolution process to improve the accuracy; past experience with the convolution process indicates that some improvement may be possible. However, with reference to Fig. 2, it is obvious that the NESTED program is the most efficient one.

The results of the element excitations indicate that it is an extremely stable algorithm; provides a very good accuracy in single precision (six digit accuracy); and of course, it is very fast. This program, NESTED, was translated into Basic and run on the TRS-80, Model II computer. The execution time ranged from 2 s for 21 element array to 36 s for a 99 element array. Although, the execution times in Basic are about 15 to 20 times longer than that in Fortran, they are not significantly long to be of any major consequence. The NESTED program was also run using double precision (16 significant digits) on the mainframe computer. The total execution time for all 10 different arrays was less than 0.3 s!

The most important consideration on small computers is not the speed of execution but the accuracy of the final result. In this sense as well, the nested product algorithm proposed by Bresler [9] is the winner.

TAYLOR SYNTHESIS

Synthesis procedure proposed by Taylor [2] applies to a continuous aperture. In practice, this procedure is used for discrete aperture (arrays) by properly discretizing the continuous distribution. Shelton [3] presented a synthesis procedure for discrete aperture distribution for Taylor type sidelobe structure. He expressed the pattern function in the form of a product function of zeros and then carried out the synthesis exactly analogous to that by Taylor; that is, to use the Woodward synthesis technique. In particular, for a $2N + 1$ element array, all $2N$ zeros are explicitly specified in the pattern function. Thus, analogous to the Chebyshev synthesis, this synthesis is amenable to the convolution procedure. In view of this, in the case of Taylor synthesis, we compare the two techniques; one proposed by Shelton and the other being the convolution synthesis. Before presenting and discussing the results of the investigation, the pertinent expressions for the two syntheses are given below. Once again, we will limit our discussion to arrays with odd $(2N + 1)$ number of elements.

Discrete Taylor (Shelton [3]) technique:

$$u_{0n} = \frac{2\pi\bar{n}}{(2N+1)} \frac{\sqrt{A^2 + (\bar{n} - 1/2)^2}}{\sqrt{A^2 + (\bar{n} - 1/2)^2}}, \quad n = 1, 2, \dots, \bar{n} - 1$$

$$= \frac{2\pi n}{(2N+1)}, \quad n = \bar{n}, \dots, N. \quad (4)$$

where $A = (1/\pi) \cosh^{-1}(R)$; $\bar{n}$ is equal to the number of near-in zeros that are moved in order to achieve the desired sidelobe ratio R (or equivalently the number of near-in sidelobes that are required at the specified level). The element excitations are

$$I_p = 1 + 2 \sum_{m=1}^{\bar{n}-1} a_m \cos \frac{2mp\pi}{2N+1}, \quad p = 0, \dots, N. \quad (5)$$

where

$$a_m = E \left(\frac{2\pi m}{2N+1} \right);$$

$$E(u) = \prod_{n=1}^N \frac{(\cos u - \cos u_{0n})}{(1 - \cos u_{0n})}.$$

For the case of the convolution synthesis procedure, once the symmetric zero pairs are established, the excitation of the center element of a three element canonical array is readily determined. The procedure and expressions are analogous to the case of Chebyshev convolution synthesis. They are zeros are $\pm u_{0j}$; $j = 1, 2, \dots, N$ where u_{0j} are defined through (4), and the excitation $C_j = -2 \cos u_{0j}$.

The synthesis of the large array is carried out using the convolution of three element arrays, chosen in the same alternating zero sequence as indicated for the Chebyshev array.

Based on these two procedures, computer codes STAYL and CONTAYL, respectively, were developed in Fortran using single precision arithmetic. Run time associated with these codes for $\bar{n} = 6$ and the sidelobe level of 30 dB for various number of elements from 15 to 99 were recorded and are shown in Fig. 3.

The program CONTAYL failed to converge, once again, for arrays with more than 71 elements and provided only two to three digit accuracy between 31 and 61 elements. These results are similar to the Chebyshev convolution synthesis. Even the run time data is very close.

Fig. 3. Run times for Taylor programs—Fortran IV on NEC-APC.

The computation time associated with STAYL has an interesting behavior with increasing number of elements; it is almost linear. This is to be expected, since the number of computations to be carried out for each element is determined by $\bar{n}$ and not $(2N + 1)$. The corresponding growth for CONTAYL is exponential. Thus, for small number of elements CONTAYL may save some computation time but will suffer in accuracy as the number of elements increases. A check of STAYL program using double precision arithmetic on the mainframe computer indicates that it has five to six digit accuracy in single precision on a small computer.

It should be noted that the STAYL program code was developed by Shelton for the HP-41C, a pocket calculator. On this calculator, one has 10 significant digit capacity and thus the results obtained are more accurate than with a single precision Fortran code. But, as one would expect, the HP-41C is very slow; it took approximately 5 min to synthesize a 31 element array.

STAYL code was also run on NEC-APC using CBasic, a compiler Basic. In CBasic, the computation times were significantly higher, ranging from 30 s for a 15 element array to 217 s for a 99 element array. However, the computation was carried out to 14 significant figures.

Once again, as with Chebyshev synthesis, we find the overriding consideration in Taylor synthesis is not the computation time, but the accuracy of the results. In this sense, Shelton's procedure is more efficient.

CONCLUSION

As is often the case with engineering investigations, the most significant results presented in this communication are not what we were looking for when we began the project. We were originally interested in evaluating computer run times for the various programs. However, two points soon became apparent—first, most of the programs run fast enough, even on small machines, so that run time is not a major concern, and second, only two of the programs given adequate precision for the range of array size that was investigated. It is concluded that Bresler's nested procedure algorithms gives excellent results in terms of speed and precision, and also that Shelton's discretized procedure allows precise Taylor synthesis for all sizes of arrays. Finally, it is noted that the programs are very brief; the Fortran computer codes for all five programs and the codes in Basic are available from the authors.

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Pattern Distortion of Aperture Antennas Radiating in the Presence of Conducting Platforms

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Abstract—An analytically simple and numerically efficient technique for calculating the pattern distortion of aperture antennas radiating in the presence of conducting platforms located in the near or far field of the antenna is presented. The technique presented, based on uniform geometrical theory of diffraction (UGTD), is also applicable for large aperture antennas (aperture area $> 15 \lambda^2$). An excellent agreement between the calculated and measured results obtained for a typical aperture antenna mounted on a conducting platform confirms the validity of the analytical technique developed.

I. INTRODUCTION

A study of distortion in the near or far-field radiation patterns of an aperture antenna radiating in the presence of a conducting platform located in the near field of the aperture antenna is a practical problem of great interest. In the past geometrical theory of diffraction (GTD)-based studies have been conducted to obtain the diffracted fields in the principal planes, when a point source is located above and on the axis of symmetry of a conducting rectangular box [1]. No GTD-based technique seems

to be available in open literature to study accurately the pattern distortion of such antennas when the aperture is large in terms of wavelength (aperture area $> 15 \lambda^2$) and the conducting platform is also large in terms of wavelength and the interaction of the aperture with the obstacle is in the radiating near field [2].

A systematic approach to this problem based on uniform geometrical theory of diffraction (UGTD) [3] is developed in this communication. An experimental study conducted to verify experimentally the analytical study is also described.

II. DESCRIPTION OF THE PROBLEM

The aperture antenna is supposed to be mounted on a perfectly conducting platform whose cross section in the $x-y$ plane is shown in Fig. 1.

Without loss of generality, the platform is modeled by wedges with included angles of 90° and 270° . Further, the size of the platform along the y -axis is assumed to be of finite length L_0 . The length of each face of the wedge (AB , CD etc. in Fig. 1) is assumed to be large in terms of the wavelength ($\geq 5 \lambda$) so that first order diffracted rays and diffracted reflected rays alone need be included. Further, L_0 is supposed to be much larger than the length of each face of the wedge in the $x-z$ plane. The analysis will be carried out to obtain the composite radiation pattern of the antenna mounted on the platform, as if the antenna together with the platform radiates in free space.

III. ANALYSIS

Initially a GTD analysis for the problem described above is carried out when a linearly polarized spherical point source illuminates the platform and this solution is extended subsequently to a large aperture antenna which has a caustic over the main beam.

A. Spherical Point Source of Illumination

The linearly polarized spherical point source located at (h, g, f) above the platform illuminates a series of conducting rectangular parallelopipeds simultaneously as illustrated in Fig. 1(a). Initially one attempts to obtain an expression for the field diffracted by any one of the faces of the conducting platform in explicit closed form. Toward this end the parallelopiped directly beneath the point source is considered (Fig. 1(b)). For this source and obstacle configuration, four edge diffracted rays and 12 corner diffracted rays will contribute to the scattered field in any plane of observation $\phi = \phi_0$. Corner diffracted rays will be much weaker than the geometrical optics (GO) and edge diffracted rays and hence these are ignored. Moreover, no corner diffraction coefficients are available for the corners that are encountered in the problem described above. The total field will be the sum of the incident or direct field, reflected or GO field and the diffracted field.

With the coordinate system shown in Fig. 1, consider the plane P given by

$$x \tan \phi - y + (g - h \tan \phi) = 0. \quad (1)$$

The direction of any ray which lies in the plane P is defined by α , where α is measured from the line of intersection of P with the $x-y$ plane and $0 \leq \alpha \leq \pi$.

The field incident on the wedge from the spherical point source

Manuscript received July 1, 1983; revised January 3, 1984. This work was supported by the Defense Research and Development Organization, Government of India.

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