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## Nulling at Symmetric Pattern Location with Phase-Only Weight Control

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**Abstract**—Sidelobe nulling at symmetric locations in linear array patterns can be accomplished with phase-only weight control if no restriction is placed on the magnitude of the phase perturbations. Nonlinear programming techniques can be used to calculate the required phases. Several examples are presented.

### INTRODUCTION

Computational difficulties associated with the nonlinear problem of synthesizing nulls in linear array patterns with phase-only weight control are generally avoided by assuming that the phase perturbations are small. The small phase perturbation assumption enables the nulling equations to be linearized. A consequence of this assumption is [1], [2], [3] that the imposing of a null in the pattern of an ideal linear array is accompanied by the reinforcement of the pattern at the location symmetric with respect to the main beam, thus making it impossible to impose nulls at symmetric locations with small phase perturbations. In [1] it is also noted that if the phase perturbations are not restricted in size, or if the array elements have realistic errors, then phase-only nulling at symmetric pattern locations is indeed possible, and a measured pattern for a small experimental array is shown with symmetrically located nulls formed by an adaptive algorithm. No computational method has been given, however, for obtaining the phases required for symmetric nulling. It is the purpose of this communication to show that the phases to impose nulls at symmetric pattern locations can be calculated by nonlinear programming techniques. The resulting patterns are characterized by considerable distortion, however, as a consequence of the fact that some of the phase perturbations are large.

### ANALYSIS

We consider a linear array of  $N$  equispaced isotropic elements as shown in Fig. 1. The array pattern is given by

$$f_0(u) = \sum_{n=1}^N a_n e^{j d n u}$$

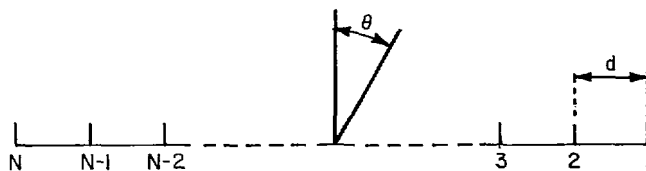


Fig. 1. Geometry of array.

where  $a_n$  is the complex excitation of the  $n$ th element,  $d_n = (N-1)/2 - (n-1)$ , and  $u = 2\pi/\lambda d \sin(\theta)$  with  $d$  the interelement spacing and  $\theta$  the angle from broadside. The pattern is assumed real so that  $a_{N-n+1} = a_n^*$ . The equations for imposing nulls at a set of  $M$  locations,  $u = u_m$ ,  $m = 1, 2, \dots, M$  with phase-only weight perturbations,  $\{\exp(j\phi_n)\}$ , are

$$\sum_{n=1}^N a_n e^{j\phi_n} e^{j d n u_m} = 0, \quad m = 1, 2, \dots, M. \quad (1)$$

This set of equations is underdetermined if  $M < N/2$ ,  $N$  even, or  $M < (N-1)/2$ ,  $N$  odd. A unique solution can be defined, however, by adding the requirement that the absolute weight perturbations,  $\{|a_n[\exp(j\phi_n) - 1]|$ , be minimized in a least squares sense,

$$\sum_{n=1}^N |a_n(e^{j\phi_n} - 1)|^2 = 4 \sum_{n=1}^N \left[ |a_n| \sin\left(\frac{\phi_n}{2}\right) \right]^2 = \text{minimum}, \quad (2)$$

a requirement that is useful to make in null synthesis to ensure that the perturbed pattern closely resembles the original pattern. The minimized weight perturbation, phase-only null synthesis problem is then to find the set of phases  $\{\phi_n\}$  satisfying (1) and (2). This problem is nonlinear in general and cannot be solved analytically. Numerical solutions can be obtained, however, by using nonlinear programming techniques [4].

If the main beam of the pattern is directed toward broadside and nulls are required at a pair of symmetric locations  $u = \pm u_1$ , the null equations are then

$$\sum_{n=1}^N a_n e^{j\phi_n} e^{\pm j d n u_1} = 0 \quad (3)$$

with the  $\{a_n\}$  now real amplitudes. It is simple to show (see Appendix) that if the assumption of small phase perturbations is made and (3) linearized via the approximation  $\exp(j\phi_n) \sim 1 + j\phi_n$ , then there is no solution to the resulting equation pair. Hence as correctly noted in [1], [2], [3], symmetric, phase-only nulling with small phase perturbations is impossible. If, however, the phase perturbations are not restricted in size, then there is an infinity of solutions to (3) and nonlinear programming methods can be used to obtain the solution that satisfies (2).

### RESULTS

In this section we present some examples of minimized weight perturbation, phase-only nulling at pattern locations that are symmetric with respect to the main beam. The phases were calculated using the nonlinear programming computer code LPNLP [5] in double precision on a CDC 6600 computer. All computations were performed for an array of 41 elements with half-wavelength spacing and with the main beam directed toward

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broadside. Imposed null depths  $< -90$  dB were achieved in all the examples.

In Figs. 2(a)-2(c) we show the original, perturbed, and cancellation (perturbed minus original) field patterns for nulls imposed at  $\pm 9.74^\circ$ , the locations of the peaks of the third sidelobes, in the pattern corresponding to a uniform amplitude distribution of the array. The interferometric shape of the cancellation pattern is a consequence of the fact that only one pair of elements has significant phase perturbations:  $\pm 154^\circ$  for the fifteenth and twenty seventh elements of the array. All other elements are shifted less than  $1^\circ$  in phase. The distortion of the pattern is considerable, exceeding 20 dB in many regions.

It can be clearly seen from Figs. 2(a), 2(b) that the perturbed pattern is not symmetric with respect to the main beam. This is a consequence of the odd-symmetry of the phase perturbations with respect to the phase reference at the array center which, coupled with the even symmetry of the element amplitudes, results in a pattern that is not symmetric about the main beam peak at  $0^\circ$ , even though the nulling problem itself is completely symmetric; i.e., nulls imposed at symmetric locations in a symmetric pattern.

For comparison with Figs. 2(a)-2(c), in Figs. 3(a)-(c) we show the original, perturbed, and cancellation patterns for the same array but with only one null imposed at  $+9.74^\circ$ . All phase shifts were less than  $10.5^\circ$  in magnitude with 26 of the phase perturbations between  $5.8^\circ$  and  $10.5^\circ$ . Note the reinforcement of the pattern ( $\sim 6$  dB) at the location  $-9.74^\circ$  corresponding to the null imposed at  $+9.74^\circ$ . The cancellation pattern, Fig. 3(c), can be represented as the sum of two beams [1], [3], one directed toward the null location, and the other of opposite sign directed toward the symmetric location adding in phase to the original pattern there.

In Figs. 4(a)-4(c) we show the patterns for a pair of nulls imposed at the symmetric locations  $\pm 10.48^\circ$ , the outer 3 dB points of the third sidelobes of the 41 element, uniform amplitude array pattern. In this example four pairs of elements undergo significant phase shifts:  $\pm 103^\circ$  for the fifth and thirty seventh elements,  $+26^\circ$  for the sixteenth and twenty sixth elements,  $\pm 15^\circ$  for the fifteenth and twenty seventh elements, and  $\pm 10^\circ$  for the fourth and thirty eighth elements. The remainder of the phase shifts are less than  $0.2^\circ$ . The overall form of the cancellation pattern is strongly interferometric, dominated by the pattern corresponding to the phase shifts of the fifth and thirty seventh elements.

As the final example, in Figs. 5(a)-5(c) we show patterns for nulls imposed in the pattern of a 41 element array with a 20 dB Chebyshev taper, at the symmetric locations  $\pm 14.7^\circ$ , the peaks of the fifth sidelobes. The cancellation pattern, shown in Fig. 5(c), is the superposition of two interferometric patterns, one from phase shifts of  $\pm 179^\circ$  in the second and fortieth elements, and the other from phase shifts of  $\pm 112^\circ$  in the ninth and thirty third elements. All other phase shifts are less than  $1.4^\circ$  in magnitude.

## CONCLUSION

Nulls can be imposed with phase-only weight control at sidelobe locations in a linear array pattern that are symmetric with respect to the main beam provided the phase perturbations are not restricted to be small. Such symmetric nulling is not possible with small phase perturbations. The phase perturbations required to impose symmetric nulls subject to minimizing the

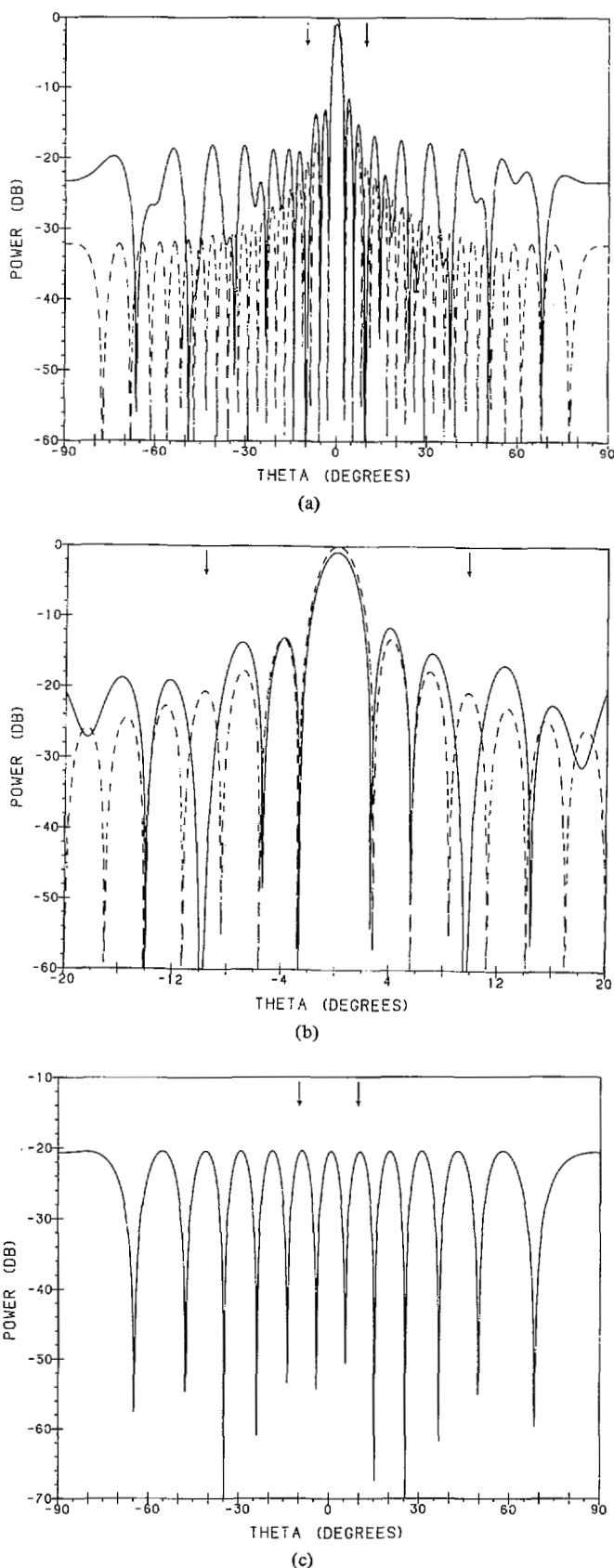
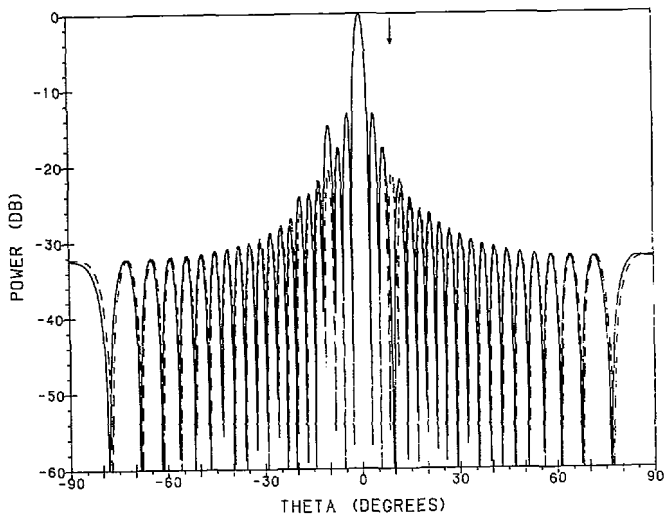
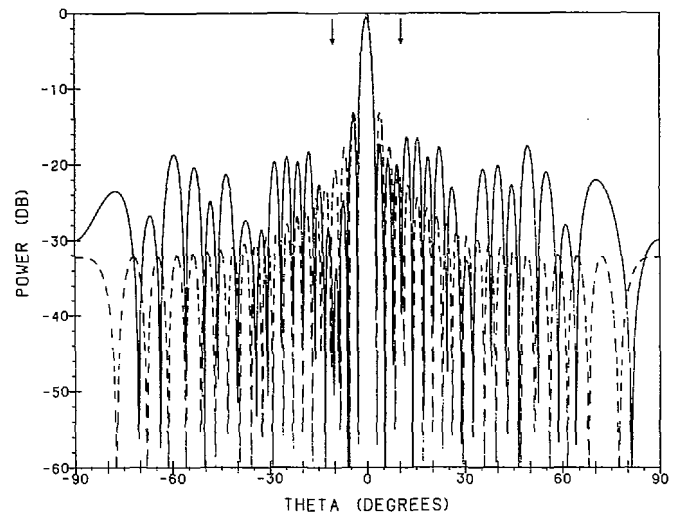


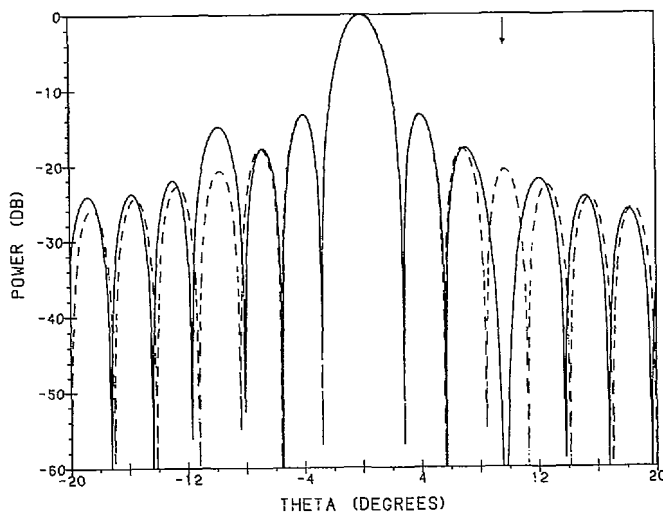
Fig. 2. (a) Original uniform amplitude array pattern (---) and perturbed pattern (—) with symmetric nulls imposed at  $\pm 9.74^\circ$ .  $\theta = -90^\circ$  to  $+90^\circ$ . (b) Original uniform amplitude array pattern (---) and perturbed pattern (—) with symmetric nulls imposed at  $\pm 9.74^\circ$ .  $\theta = -20^\circ$  to  $+20^\circ$ . (c) Cancellation pattern to impose symmetric nulls at  $\pm 9.74^\circ$  in uniform amplitude array pattern.  $\theta = -90^\circ$  to  $+90^\circ$ .



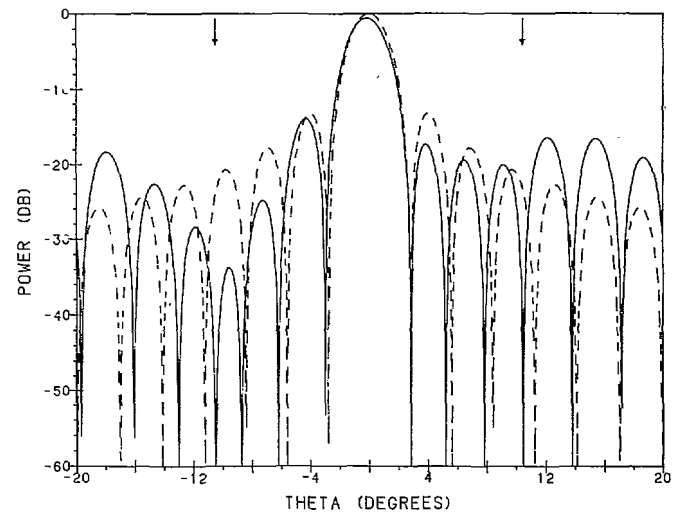
(a)



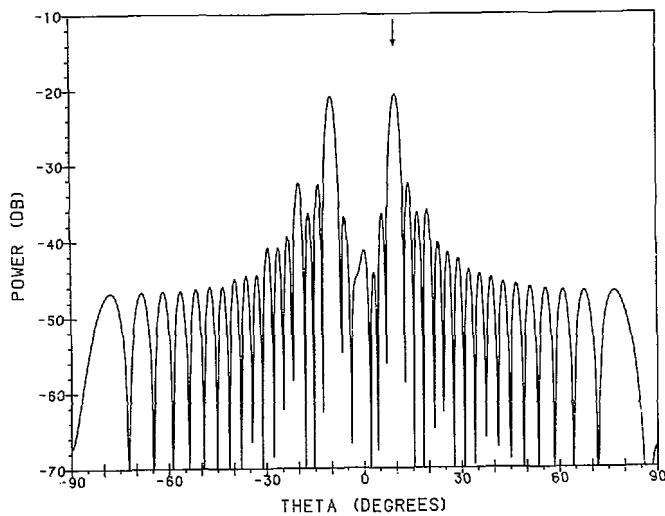
(a)



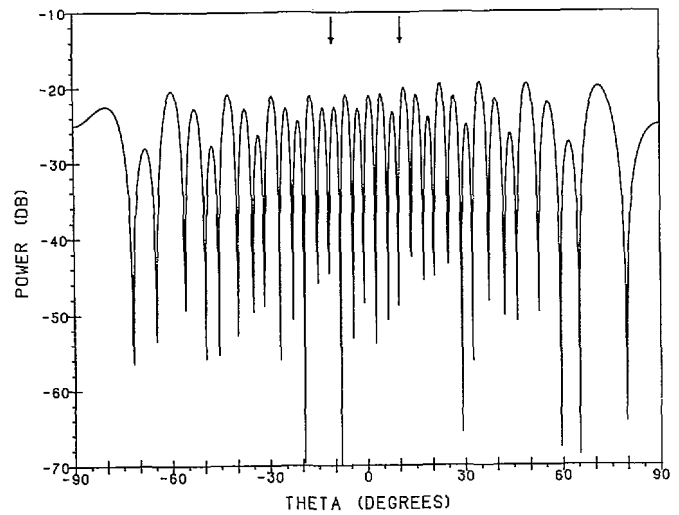
(b)



(b)



(c)



(c)

Fig. 3. (a) Original uniform amplitude array pattern (—) and perturbed pattern (---) with one null imposed at  $+9.74^\circ$ .  $\theta = -90^\circ$  to  $+90^\circ$ . (b) Original uniform amplitude array pattern (—) and perturbed pattern (---) with one null imposed at  $+9.74^\circ$ .  $\theta = -20^\circ$  to  $+20^\circ$ . (c) Cancellation pattern to impose one null at  $+9.74^\circ$  in uniform amplitude array pattern.  $\theta = -90^\circ$  to  $+90^\circ$ .

Fig. 4. (a) Original uniform amplitude array pattern (—) and perturbed pattern (---) with symmetric nulls imposed at  $\pm 10.48^\circ$ .  $\theta = -90^\circ$  to  $+90^\circ$ . (b) Original uniform amplitude array pattern (—) and perturbed pattern (---) with symmetric nulls imposed at  $\pm 10.48^\circ$ .  $\theta = -20^\circ$  to  $+20^\circ$ . (c) Cancellation pattern to impose symmetric nulls at  $\pm 10.48^\circ$  in uniform amplitude array pattern.  $\theta = -90^\circ$  to  $+90^\circ$ .

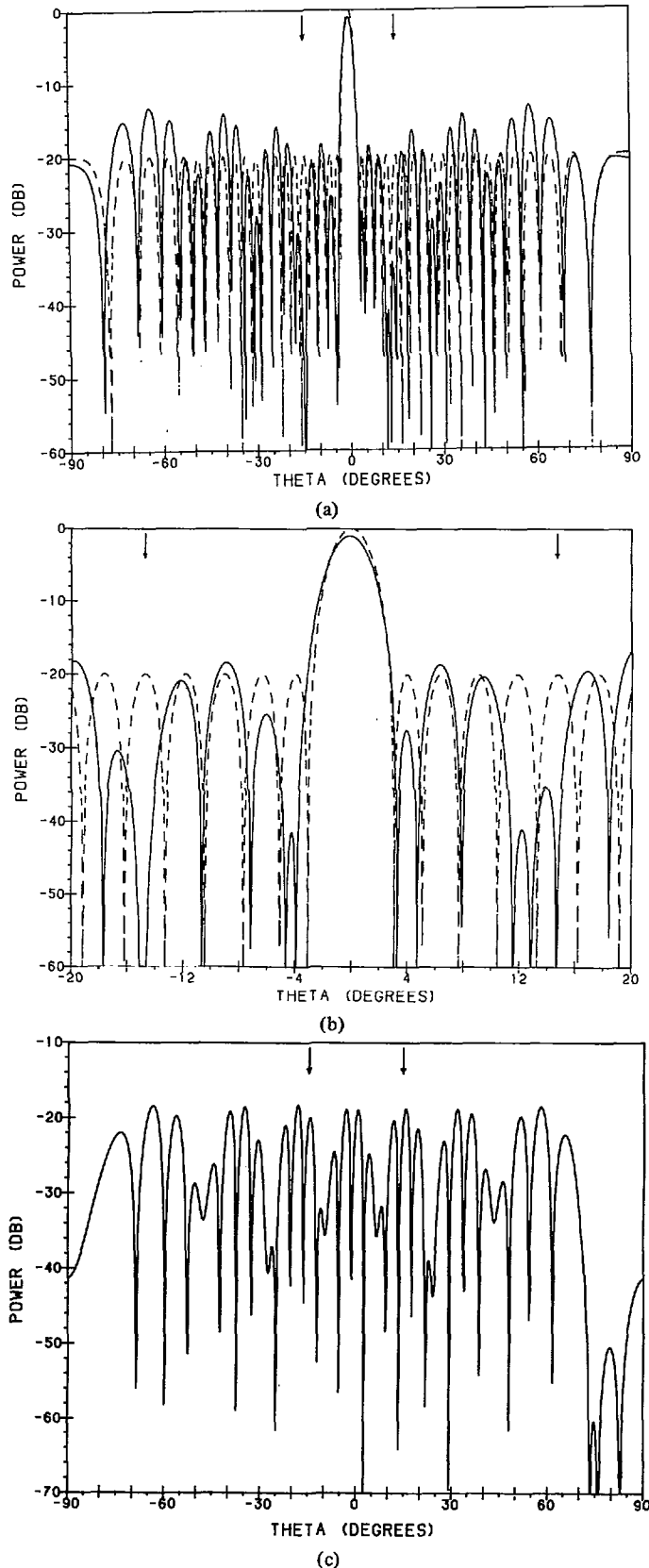


Fig. 5. (a) Original 20 dB Chebyshev array pattern (---) and perturbed pattern (—) with symmetric nulls imposed at  $\pm 14.70^\circ$ ,  $\theta = -90^\circ$  to  $+90^\circ$ . (b) Original 20 dB Chebyshev array pattern (---) and perturbed pattern (—) with symmetric nulls imposed at  $\pm 14.70^\circ$ ,  $\theta = -20^\circ$  to  $+20^\circ$ . (c) Cancellation pattern to impose symmetric nulls at  $\pm 14.70^\circ$  in 20 dB Chebyshev array pattern,  $\theta = -90^\circ$  to  $+90^\circ$ .

absolute weight perturbations must be calculated numerically using techniques such as nonlinear programming. The cancellation patterns for phase-only nulls at symmetric pattern locations tend to be interferometric in shape and result in significant distortion over much of the pattern. In contrast, the paired-beam shape that characterizes the cancellation patterns for minimized weight perturbation phase-only nulling at nonsymmetric sidelobe locations, results in relatively small pattern distortion apart from some pattern reinforcement at the locations symmetric to the imposed nulls.

## APPENDIX

Substituting the small phase linearization  $\exp(j\phi_n) \sim 1 + j\phi_n$  in (3) and rearranging leads to

$$\sum_{n=1}^N a_n \phi_n e^{jd_n u_1} = -j \sum_{n=1}^N a_n e^{jd_n u_1} = -j f_0(u_1) \quad (4a)$$

$$\sum_{n=1}^N a_n \phi_n e^{-jd_n u_1} = -j \sum_{n=1}^N a_n e^{-jd_n u_1} = -j f_0(u_1) \quad (4b)$$

where  $f_0(u_1)$  is the (real) value of the unperturbed pattern at  $\pm u_1$ . But the left hand sides of 4(a), 4(b) are complex conjugates and so cannot both be equal to the same imaginary quantity. Hence the assumptions of symmetric imposed null locations and small phase perturbations are incompatible.

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## Spectral-Domain Analysis of Circular Patch Frequency Selective Surfaces

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**Abstract**—A computationally efficient method is presented for analyzing the scattering from frequency selective surfaces (FSS) comprised of circular metal patches. The formulation is carried out in the spectral domain where the convolution form of the integral equation for the induced current reduces to an algebraic one and the spectral-Galerkin technique is used to solve the resulting equation. Entire-domain basis functions that satisfy the edge

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