Generalized Aggregation and Coordination of Residential Loads in a Smart Community

He Hao, Abhishek Somani, Jianming Lian, and Thomas E. Carroll

Abstract— **Flexibility from residential loads presents an enormous potential to provide various services to the smart grid. In this paper, we propose a unified hierarchical framework for aggregation and coordination of various residential loads in a smart community, such as Thermostatically Controlled Loads (TCLs), Distributed Energy Storages (DESs), residential Pool Pumps (PPs), and Electric Vehicles (EVs). A central idea of this framework is a virtual battery model, which provides a simple and intuitive tool to aggregate the flexibility of distributed residential loads. Moreover, a multi-stage Nash-bargaining-based coordination strategy is proposed to coordinate different aggregations of flexible loads for demand response. Case studies are provided to demonstrate the efficacy of our proposed framework and coordination strategy in managing peak power demand in a smart residential community.**

I. Introduction

Distributed residential loads contribute to a substantial portion of electricity consumption in the United States. For example, TCLs such as air conditioners, heat pumps, water heaters, and refrigerators, consume about 20% of the total electricity [1]. Additionally, there are about 8 million residential pools in the United States [2], and each pool pump consumes approximately 1 kW when it is operating. Moreover, electric vehicles and home energy storage are becoming widespread popular [3], [4]. It is projected that 50% of the U.S. vehicles (more than 150 million) will be electrified by 2050; this would amount to nearly $2,900$ GWh of mobile electric storage capacity [5]. If these resources are intelligently controlled, the demand response capability they bring to the smart grid will be phenomenal.

In this paper, we consider coordination of distributed residential loads in a smart community for demand response. In particular, we consider four types of flexible loads: TCLs, DESs, PPs, and EVs. We cluster them into two categories: *adjustable loads* and *deferrable loads*. An *adjustable load* is an electric load whose energy requirement is flexible during its service time. For example, for an air-conditioner (TCL), its temperature difference from the setpoint represents the SoC (State of Charge) of the thermal storage of a building. As long as the temperature difference is in the user-specified temperature band, it is acceptable; for an ideal battery, its SoC can be anywhere between zero and its energy capacity. A *deferrable load* is an electric load whose energy requirement is fixed at the end of its service time. For example, a pool pump must be run for a certain amount of time at the end of a day, and the SoC of an EV must be charged to a certain level at the end of its service time.

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Aggregation and control of distributed flexible loads for various grid services have been extensively studied in literature, and various modeling techniques have been applied. For example, Partial Differential Equations and Bin State Transition models were used to aggregate residential TCLs in [6]–[8]. Meyn *et. al.* proposed a Markov Decision Process model to aggregate a large collection of residential pool pumps [9]. Mean-field-game-based and hysteresis-based charging strategies of electric vehicles have been studied in [10], [11]. However, it is very challenging to integrate various modeling methodologies into the same framework to manage. Therefore, it is essential to develop a unified modeling framework to coordinate various heterogeneous classes of flexible loads.

To this end, we propose a unified hierarchical coordination framework for aggregation, characterization, and scheduling of residential loads. The core of such a coordinated aggregation framework is a virtual battery model, motivated by previous work [12]–[15]. This model is simple; it is only specified by a few parameters such as power limits, energy limits, and dissipation rate. Additionally, it is scalable; the number of parameters is independent of the size of the load population. This offers the resource coordinators great convenience when dispatching flexible loads, since they only have to deal with a few "batteries", and all the complexity and local constraints are absorbed in the aggregator level.

We show that the aggregate flexibility of both adjustable loads and deferrable loads can be concisely modeled by virtual battery models. We then propose a Nash-bargaining based coordination strategy (convex programming) for a collection of aggregated resources, and present local scheduling policies to control individual loads to follow a dispatched power profile. In contrast to centralized (integer programming) coordination approach (when considering on-off loads), our hierarchical approach requires much less communication and computation. Case studies are also provided to demonstrate how coordination of residential flexible loads in a smart community can help cap their aggregate power to avoid peak demand charge, while respecting the flexibility and constraints of each individual load. Comparing with our prior works [12], [14], we propose a unified hierarchical coordination framework to mange distributed residential loads. Moreover, we extend the single-stage Nash bargaining method in [16] to a multi-stage setting in this paper.

The rest of the paper is organized as follows. In Section II, we present the problem statement. Section III describes the aggregate modeling and coordination framework. A multi-stage Nash-bargaining-based resource coordination strategy is presented in Section IV. Section V is devoted to case studies. The paper ends with conclusions and future work in Section VI.

II. Problem Statement

There is an emerging consensus that the control intelligence that is currently centralized in the power system operator will be distributed to the periphery in the future grid (see for example GRIP in [17]). It is believed that the future power system will increasingly rely on localized grids, which aggregate diverse geographically co-located distributed energy resources, and support coordination of generation, storage, and flexible demand on the distribution system to reduce the need of building new transmission facilities to meet increasing electricity demand and integrate renewable energy resources. In this paper, we consider such a localized grid (or resource cluster [17]) - a smart residential community, in which residents cede physical controls of their flexible loads to a community manager, who is responsible for forecasting the aggregate baseline load of the community and implement coordination and scheduling algorithms to manage the power consumption of flexible loads to avoid peak demand charge while respecting resource constraints.

In this paper, time is designated as discrete with discretization step δ over a time horizon $[0,(K+1)\delta]$. We use $t\in\mathcal{K}=\{0,1,\cdots,K\}$ to index the time interval $[t\delta,(t+1)\delta)$, and denote U_t as the power supplied to a collection of loads over that time interval. The sequence $U=(U_0,\cdots,U_t,\cdots,U_K)$ is regarded as a power profile. We consider four types of distributed flexible loads in the residential community: TCLs, DESs, PPs and EVs. The dynamics for each type of loads are described in the following:

TCLs: The thermal dynamics of a TCL are given by

$$\theta^i_{t+1}=\alpha^i\theta^i_t+(1-\alpha^i)(\theta_a-m^i_tb^ip^i_m)+w^i, \tag{1}$$

where θ^i_t is the TCL temperature, and $\alpha^i=e^{-\delta/(R^iC^i)}, b^i=R^i\eta^i$ can be expressed in terms of the thermal resistance R^i, thermal capacitance C^i and coefficient of performance η^i. Additionally, θ_a is the ambient temperature, p^i_m is the TCL's rated power, and w^i is the external disturbance. Each TCL has a temperature setpoint θ^i_r with a user-specified temperature band $[\theta^i_r-\Delta^i,\theta^i_r+\Delta^i]$, and its binary operating state m^i_t (1 when ON and 0 when OFF) switches when the TCL temperature crosses the temperature bounds.

DESs: For each DES such as a battery, its SoC can be described by the following simplified model

$$x^i_{t+1}=x^i_t+u^i\delta, \tag{2}$$

with initial condition x^i_0. Additionally, its charging/discharging rate and SoC are bounded as below

$$-n^i_-\le u^i_t\le n^i_+,\quad 0\le x^i_t\le c^i_+,$$

where n^i_-/n^i_+ is the discharge/charge rate limit, and c^i_+ is the energy capacity limit.

EVs: The SoC of an EV can be also modeled by (2). However, to respect its battery life, we assume each EV is in the charging mode when plugged in. Additionally, its SoC must be charged to a certain level before its service time ends. Therefore, we have

$$0\le u^i_t\le n^i_+,\quad x^i_{d^i}=E^i,$$

where d^i is its departure time and E^i is its energy requirement.

PPs: Each PP has ON-OFF switching behavior, and its power consumption at time step t is given by

$$u^i_t=m^i_tp^i_m, \tag{3}$$

where m^i_t is its binary operating state, and p^i_m is its rated power. For each PP, it is required to run h^i hours a day to filter the water in the pool. Therefore, we have

$$x^i_K=\sum_{t=0}^{K}u^i_t\delta=E^i=p^i_mh^i.$$

We observe from the above presentation that different loads have different dynamics and constraints, and it will be very challenging to incorporate various heterogeneous individual loads into the same coordination and/or optimization framework for the community manager to manage, especially when the number of loads is very large. Therefore, it is essential to develop a unified aggregation and coordination framework to manage various heterogeneous classes of flexible loads. To this end, we propose a generalized aggregate modeling and coordination framework for aggregation, characterization, and scheduling of distributed residential loads.

III. A Unified Aggregation and Coordination Framework

A. Virtual Battery Model

The core of such a coordinated aggregation framework is a virtual (generalized) battery model.

Definition 1: A *Virtual (Generalized) Battery Model* $\mathbb{B}$ is a set of power profiles U that satisfy

$$-\underline{U}_t\le U_t\le\overline{U}_t,\quad\forall\, t\ge0,$$
$$X_{t+1}=\alpha X_t+U_t\delta,\ X_0=0,\ \Rightarrow\ -\underline{X}_t\le X_t\le\overline{X}_t,\ \forall\, t>0.$$

The model is specified by non-negative parameters $\phi_t=(\underline{X}_t,\overline{X}_t,\underline{U}_t,\overline{U}_t,\alpha_t)$, and we write it compactly as $\mathbb{B}(\phi_t)$. □

We can regard U_t as the power draw of the battery, X_t as its SoC, α_t as its dissipation rate, $\underline{U}_t/\overline{U}_t$ as its discharge/charge power limits, and $\underline{X}_t/\overline{X}_t$ as its lower/upper energy capacity limits.

The virtual battery model is used to aggregate the flexibility of each category of residential loads.

B. Aggregate Flexibility of Residential Loads

We first consider aggregating flexibility of adjustable loads. It was shown in [12] the aggregate behavior of a collection of TCLs with the hybrid model (1) can be approximated by that with the following continuous model,

$$\theta^i_{t+1}=\alpha^i\theta^i_t+(1-\alpha^i)(\theta_a-b^ip^i_t)+w^i,$$

where the power input p^i_t can be continuously modulated between 0 and the rated power p^i_m. Assuming w^i in (1) is zero-mean and negligible [12], the nominal power required

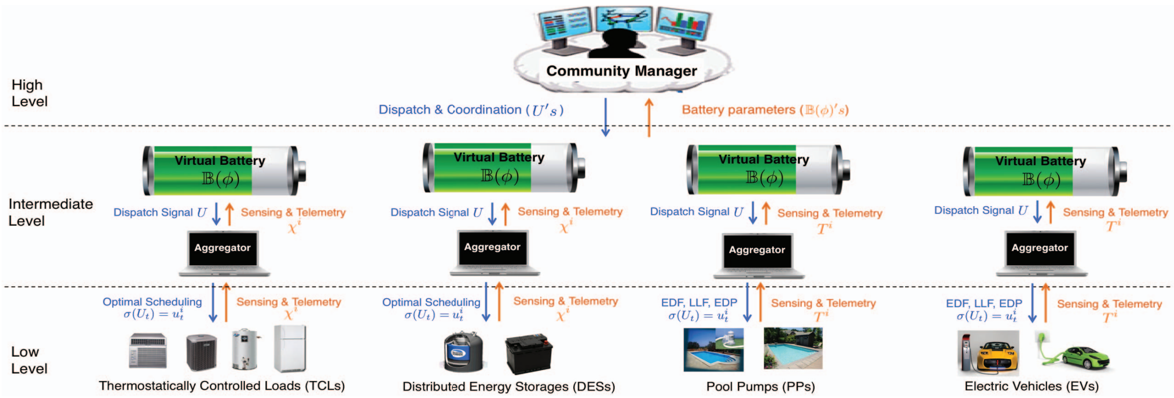


Fig. 1. Schematic representation of the proposed hierarchical coordination framework.

to keep a TCL at its set-point is given by $p_0^i = \frac{\theta_a - \theta_r^i}{R^i \eta^i}$. After a change of variable $x^i = \frac{C^i(\theta_r^i - \theta^i)}{\eta^i}$, we obtain that

$$x_{t+1}^i = \alpha^i x_t^i + u_t^i \delta, \tag{4}$$

where $u_t^i = p_t^i - p_o^i$. Additionally, it is straightforward to see that $u^i(t) \in [-p_0^i, p_m^i - p_0^i]$, and $x^i(t) \in [-\frac{C^i \Delta^i}{\eta^i}, \frac{C^i \Delta^i}{\eta^i}]$, with initial condition $x_0^i = C^i(\theta_r^i - \theta_0^i)/\eta^i$. We observe that for each adjustable load (either TCL or DES), it can be uniformly represented by (4) subject to constraints

$$-n_-^i \leq u_t^i \leq n_+^i, \quad -c_-^i \leq x_t^i \leq c_+^i, \tag{5}$$

and it is parameterized by nonnegative parameters $\chi^i = (c_-^i, c_+^i, n_-^i, n_+^i, \alpha^i)$.

We define the *aggregate flexibility* $\mathbb{U}$ of a collection of adjustable loads described by (4) and (5) as the set of all power profiles U's that could satisfy the power and energy requirements of each load. Formally,

$$\mathbb{U} = \left\{ U \middle| \begin{array}{c} U_t = \sum_{i=1}^N u_t^i \\ -n_-^i \leq u_t^i \leq n_+^i, \; -c_-^i \leq x_t^i \leq c_+^i \end{array} \right\},$$

for all $t \geq 0$ and all $i \in \{1, 2, \cdots, N\}$. In this paper, we show that the aggregate flexibility of adjustable loads can be aggregated as a virtual battery model. The proof follows a similar line of attack as [12], in which we demonstrated that the aggregate flexibility of TCLs could be approximated by virtual battery models. For ease of illustration, we consider in this paper a collection of *homogeneous* adjustable loads. Results of aggregating flexibility of *heterogeneous* adjustable loads can be found in [12], [18]. The proof of the following theorem is given in [18].

Theorem 1: Consider a collection of N *homogeneous* adjustable loads parameterized by $\chi = (c_-, c_+, n_-, n_+, \alpha)$, and with zero SoC. The *aggregate flexibility* $\mathbb{U}$ of the collection satisfies

$$\mathbb{U} = \mathbb{B}(\phi_t),$$

where the battery parameters ϕ_t are given by

$$\underline{X}_t = Nc_-, \; \overline{X}_t = Nc_+, \; \underline{U}_t = Nn_-, \; \overline{U}_t = Nn_+,$$

and the dissipation rate of the battery, α_t, is equal to the model parameter of the adjustable load, α. □

We next consider aggregating flexibility of deferrable loads. We model each deferrable load as a *task* [13], [19]. A *task* is parameterized by its total energy need E^i, arrival time a^i, departure time d^i, and rated power p_m^i. It is denoted by a quadruple $T^i = (E^i, a^i, d^i, p_m^i)$. We define the *aggregate flexibility* $\mathbb{U}$ of a collection of deferrable loads as the set of all power profiles that could complete all tasks without surplus or deficit. Formally,

$$\mathbb{U} = \left\{ U \;\middle|\; \begin{array}{c} U_t = \sum_{i=1}^N u_t^i, \; \forall \, t \\ \sum_{t=0}^K U_t \delta = \sum_{i=1}^N E^i \\ x_{a^i}^i = 0, \; x_{d^i}^i = E^i, \; \forall \, i \end{array} \right\}.$$

Again, for ease of presentation, we consider in this paper a collection of *homogeneous* deferrable loads. The general results considering *heterogeneous* deferrable loads can be found in [13], [14], [18]. The proof of the following theorem is given in [18].

Theorem 2: Consider a collection of N homogeneous deferrable loads parameterized by $T = (E, a, d, p_m)$. The *aggregate flexibility* $\mathbb{U}$ of the collection satisfies

$$\mathbb{U} = \mathbb{B}(\phi_t),$$

where the battery parameters ϕ_t are given by

$$\underline{X}_t = \sum_{i:d^i<t} E^i + \sum_{i:a^i \leq t, d^i > t} \max\{E^i - (d^i - t)p_m^i \delta, 0\},$$

$$\overline{X}_t = \sum_{i:d^i<t} E^i + \sum_{i:a^i \leq t, d^i > t} \min\{E^i, (t - a^i)p_m^i \delta\},$$

$$\underline{U}_t = 0, \qquad \overline{U}_t = \sum_{i:a^i \leq t, d^i > t} p_m^i, \qquad \alpha_t = 1. \qquad \square$$

C. Description of the Coordinated Aggregation Framework

The coordination framework we propose contains three hierarchies: a load level (low level), an aggregator level (intermediate level), and a community manager level (high level). The schematic representation of the coordination framework is depicted in Fig. 1. The information flow and decision making process are as follows. First of all, each flexible load at the load level sends its load parameters (T^i's, χ^i's) to the aggregator. At the aggregator level, there is an aggregator associated with them for each class of flexible loads. Based on the received information, the aggregator constructs a virtual battery model $\mathbb{B}(\phi)$ for each class of flexible loads. The aggregator then reports the battery parameters ϕ to the community manager. To provide demand response to the grid, the community manager will allocate power profiles U's to different virtual batteries while respecting their flexibility and preferences. At the aggregator level, each aggregator is obliged to follow this dispatched power profile. In order to do this, the aggregator implements appropriate scheduling algorithm σ to directly control the loads to follow the dispatched signal.

IV. Hierarchical Coordination Strategies

In this section, we study coordination of resource aggregations and scheduling of residential loads for demand response while respecting the flexibility and preference of each resource aggregator.

A. High Level Power Allocation Strategy

We first study the high level coordination problem of resource aggregators. Consider a residential community with n aggregations of distributed flexible loads. Let $j \in \mathcal{N} = \{1, 2, \cdots, n\}$ denote the j^{th} aggregation. To provide demand response (such as peak load shaving) to the grid, the community manager is responsible for allocating power to all resource aggregators subject to a total power limit Q_t for all $t \in \mathcal{K}$, where the power limit Q_t is determined either by a dispatch signal from the grid or a peak demand limit.

The community manager first assigns a default power allocation $\{p_t^{j,d}\}_{t\in\mathcal{K}}$ for each resource aggregator such that $\sum_j p_t^{j,d} \leq Q_t$, and $p_t^{j,d} \geq \underline{p}_t^{j,d}$, where $\underline{p}_t^{j,d}$ is the minimum power requirement of the j^{th} aggregator. The information of default allocations $\{p_t^{j,d}\}_{t\in\mathcal{K}}$ and total power limit $\{Q_t\}_{t\in\mathcal{K}}$ are then broadcast to all the resource aggregators. In the presence of power surplus, i.e., $Q_t - \sum_j p_t^{j,d} > 0$, each resource aggregator is then motivated to negotiate with other peer aggregators on a new allocation profile $\{(p_t^1, p_t^2, \cdots, p_t^n)\}_{k\in\mathcal{K}}$, so that the new allocation increases its own utility or preference, $\mathcal{U}^j$, which is assumed to be concave and differentiable. Note that p_t^j is the absolute power consumption of the j^{th} aggregator at time step t, instead of its deviation from baseline. Therefore, if it is a collection of TCLs, then $p_t^j = P_{o,t}^j + U_t^j$, where $P_{o,t}^j$ is their aggregate baseline power. Otherwise, $p_t^j = U_t^j$.

In this paper, we consider a cooperative game, in which each aggregator is motivated to negotiate with others in a collaborative way to jointly allocate the power while maximizing their own utility functions. We use the *Nash Bargaining Solution* (NBS) [20] to solve the multi-stage power allocation problem. NBS is an attractive approach for solving such cooperative resource allocation problems as it balances fairness with efficiency [21]. Formally, the Nash bargaining problem is formulated as

$$\max_{U_t^j\text{'s}} \quad \prod_{j=1}^{n} \Big(\mathcal{U}^j\left(\{p_t^j\}_{t\in\mathcal{K}}\right) - \mathcal{U}^j\left(\{p_t^{j,d}\}_{t\in\mathcal{K}}\right)\Big) \tag{6a}$$

$$\text{subject to: } \sum_{j=1}^{n} p_t^j \leq Q_t, \qquad \forall\, t\in\mathcal{K}, \tag{6b}$$

$$-\underline{U}_t^j \leq U_t^j \leq \overline{U}_t^j, \qquad \forall\, j\in\mathcal{N}, \quad \forall\, t\in\mathcal{K}, \tag{6c}$$

$$X_{t+1}^j = \alpha^j X_t^j + U_t^j\delta, \quad \forall\, j\in\mathcal{N}, \quad \forall\, t\in\mathcal{K}, \tag{6d}$$

$$-\underline{X}_t^i \leq X_t^j \leq \overline{X}_t^j, \qquad \forall\, j\in\mathcal{N}, \quad \forall\, t\in\mathcal{K}, \tag{6e}$$

$$-\underline{X}_{K+1}^j \leq X_K^j \leq \overline{X}_{K+1}^i, \ \forall\, j\in\mathcal{N}, \tag{6f}$$

where the objective function is referred to as the Nash product and it is defined as the social welfare.

Note that when Nash formulated the bargaining problem, no bargaining protocol is provided to tell palyers how to bargain or negotiate. In [16], we developed a distributed protocol for players to reach the Nash bargaining solution. Interesting readers are referred to [16] for more details.

B. Low Level Load Scheduling Methods

The high level coordination strategy will decide the power allocation for each aggregation of residential loads. We next study how to schedule individual loads at the low level to follow the allocated power profile while respecting the local run-time constraints of each individual load.

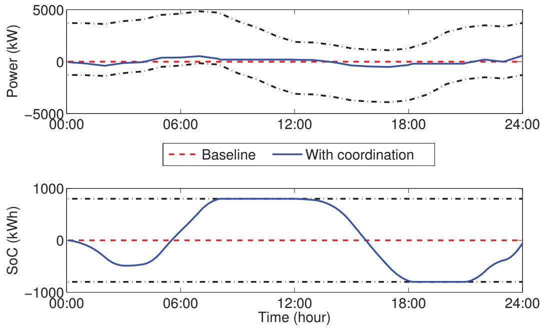
We first consider scheduling of adjustable loads. In [12], we proposed a priority-stack-based scheduling policy to turn ON/OFF TCLs to follow a dispatch signal. The priority is measured by the temperature difference from the appropriate switching boundary. For instance, for TCLs that are ON, the smaller a TCL's temperature distance from its lower temperature bound $\frac{\theta_t^i - (\theta_r^i - \Delta^i)}{\Delta^i}$ is, the higher priority it receives to be turned OFF. The motivation of this priority-stack-based scheduling policy is to minimize the ON/OFF switching for each TCL. The priority-stack-based scheduling policy works for adjustable loads with ON-OFF switching behavior. For TCLs with variable frequency drive and DESs whose power inputs can be continuously modulated, we consider a simple proportional allocation strategy $u_t^i = \beta^i U_t$, where $\beta^i \geq 0$ and $\sum_i \beta^i = 1$. We show in [18] that for a collection of *heterogeneous* loads, the parameters β^i's determine the virtual battery model parameters ϕ, and provide a guideline on how to optimize β^i's to maximize the battery parameters. For *homogeneous* loads, the optimal parameters are given by $\beta^i = 1/N$ for all i.

We next consider scheduling of deferrable loads. One of the well-known heuristic causal scheduling policies is Least Laxity First (LLF) [19], which allocates power to active tasks in the order of their *slack* time, which is defined as $s_t^i = d_i - (E^i - x_t^i)/p_m^i$. In [14], we show that given a feasible power profile U for a collection of *heterogeneous* deferrable loads, the LLF policy generally cannot finish all the tasks without surplus or deficit. A heuristic noncausal scheduling policy, Earliest Deadline Priority (EDP) policy was proposed in [14], and it was shown that the EDP policy generally could finish all tasks while LLF policy couldn't. For *homogeneous* loads, it was shown that both LLF and EDP could finish the tasks [18]. In this paper, we exploit these scheduling policies to schedule deferrable loads to follow an allocated power profile from the community manager.

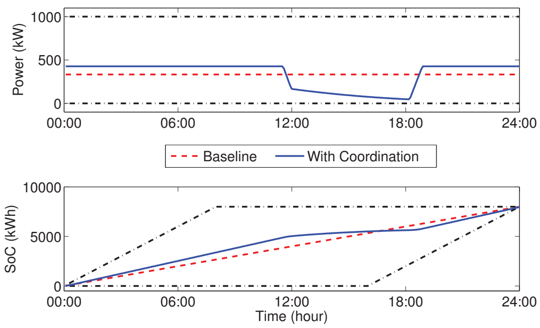
V. Case Studies

In this section, we consider a residential community with a collection of TCLs and a collection of PPs. These two types of residential loads respectively represent the adjustable load and deferrable load. The objective of the community manager is to coordinate these two types of residential loads to cap the total power consumption of the community to avoid peak demand charge.

In the simulations, we consider 1000 TCLs and 1000 PPs, where the parameters of TCLs are given by $C = 2$ kWh/^oC,



(a) Power and SoC trajectories of battery associated with TCLs



(b) Power and SoC trajectories of battery associated with PPs

Fig. 2. Trajectories of battery power and states of charge.

$R = 2^oC$/kW, $p_m = 5$ kW, $\eta = 2.5$, $\theta_r = 22.5^oC$, and $\Delta = 1^oC$ [12], and the parameters of PPs are $a = 0$ hour, $d = 24$ hour, $p_m = 1$ kW and $E = 8$ kWh [14]. The discretization time step is taken as $\delta = 1$ minute, and we assume the peak power limit imposed by the community manager is 3500 kW. According to Theorems 1 and 2, the battery model parameters associated with TCLs are given by $\underline{X}_t = \overline{X}_t = 24, \underline{U}_t = 200(\theta_{a,t} - 22.5), \overline{U}_t = 5600 - 200(\theta_{a,t} - 22.5), \alpha_t = e^{-1/240}$, and the battery model parameters associated with PPs are given by $\underline{X}_t = 1000 \max\{8 - (1440 - t)/60, 0\}, \overline{X}_t = 1000 \min\{8, t/60\}, \underline{U}_t = 0, \overline{U}_t = 1000, \alpha_t = 1$. The black dot dash lines in Fig. 2 (a) and (b) are respectively the power and energy capacity limits of the two virtual battery models.

The utility function of each resource aggregator is modeled by the Cobb-Douglas preference function

$$\mathcal{U}^j = \prod_{t \in \mathcal{K}} \left(\frac{p_t^j - \underline{p}_t^j}{\overline{p}_t^j - \underline{p}_t^j} \right)^{\alpha_t^j}, \tag{7}$$

where $\alpha_t^j \geq 0$ satisfying $\sum_t \alpha_t^j = 1$ represents the preference factor on the power consumption at time step t. Additionally, $(p_t^j - \underline{p}_t^j)/(\overline{p}_t^j - \underline{p}_t^j)$ is the normalized power of the j^{th} resource aggregator, where $\overline{p}_t^j$ and $\underline{p}_t^j$ are respectively its upper and lower power limits. For TCLs, $\overline{p}_t^j = P_{o,t}^j + \overline{U}_t^j$, $\underline{p}_t^j = P_{o,t}^j + \underline{U}_t^j$ and for PPs, $\overline{p}_t^j = \overline{U}_t^j$, $\underline{p}_t^j = \underline{U}_t^j$. The Cobb-Douglas utility function is widely used in economics to model the preferences of consumption on multiple goods. We assume the default allocations $\{p_t^{j,d}\}_{t \in \mathcal{K}}$ set by the community manager are the minimum powers, i.e., $p_t^{j,d} = \underline{p}_t^j$ for all j and all t. This means if a negotiation is not successful, each resource aggregator is allocated with the minimum power.

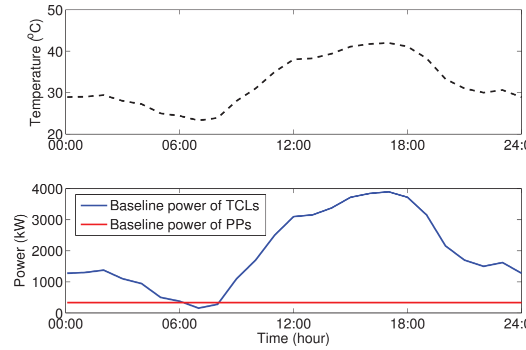


Fig. 3. Outside temperature, and baseline powers of TCLs and PPs.

For TCLs, their aggregate baseline power is dependent on the ambient temperature, which is given by

$$P_{o,t}^{\text{TCLs}} = \sum_i \frac{\theta_{a,t} - \theta_r^i}{R^i \eta^i}. \tag{8}$$

For PPs, we assume their operations are randomized, which yields the following aggregate baseline power

$$P_{o,t}^{\text{PPs}} = \sum_i \frac{E^i}{d^i - a^i}. \tag{9}$$

The outside temperature, and baseline powers of TCLs and PPs are depicted in Fig. 3. We observe that TCLs contribute more to the peak power than PPs. Therefore, we design power coefficients in the Cobb-Douglas utility functions as

$$\alpha_t^{\text{TCLs}} = \frac{\max_k \theta_{a,k} - \theta_{a,t}}{\sum_{t \in \mathcal{K}} (\max_k \theta_{a,k} - \theta_{a,t})}, \quad \alpha_t^{\text{PPs}} = \frac{1}{K+1},$$

since we want TCLs to consume more power in the off-peak hours (when the outside temperature is low) and less power in the peak hours (when the outside temperature is high). The operation of residential PPs are independent of the outside temperature, therefore we set the values of the power coefficients equally.

We next compare the community power profiles with and without coordination of residential loads. The uncoordinated power profile is obtained by simulating (1) and (3), and summing up the power trajectories of all TCLs and PPs, which can be approximated by $P_{o,t}^{\text{TCLs}} + P_{o,t}^{\text{PPs}}$. To obtain the coordinated power profile, we first conduct a high level Nash-bargaining-based power dispatch for the aggregations of TCLs and PPs. The dispatched power profiles for the two aggregations of loads are respectively depicted in Fig. 2 (a) and (b). We observe that the dispatched power profiles are strictly within the power and energy limits of the virtual battery models. We next execute low level load scheduling algorithms to actively control TCLs and PPs to follow the corresponding dispatched power profile. The coordinated community power is then obtained by summing up the power trajectories of all these actively controlled TCLs and PPs.

We observe from Fig. 4 that the uncoordinated community power exceeds the power limit in the peak hours, while the coordinated community power is always caped within the desired limit. Additionally, we see from Fig. 2 that the power consumption of TCLs and PPs under active control are shifted to the off-peak hours, and the effects of "pre-cooling" and "rebound" are also observed. Moreover, Fig. 5 shows

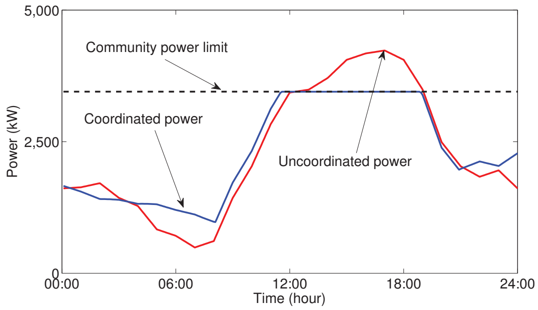


Fig. 4. Comparison of community power profiles with and without coordination.

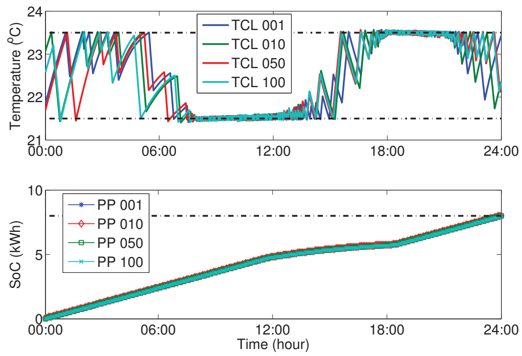


Fig. 5. Temperature trajectories of TCLs and SoCs of PPs.

that the temperatures of TCLs (scheduled by priority-stack-based policy) are well regulated within the user-specified temperature band, and the PPs (scheduled by LLF and EDP policies) are operated for the desired amount of hours at the end of a day. And also, it is interesting to see that in the periods from 08:00 to 14:00 and from 18:00 to 21:00, the TCLs are frequently switching between ON and OFF. To respect their minimum off time, reporting a smaller temperature band Δ' to the battery model will be helpful to avoid short cycling. Interesting readers are referred to [18] for details on how to design the temperature band Δ'.

VI. Conclusions and Future Work

In this paper, we proposed a hierarchical framework for aggregation and coordination of residential flexible loads in a smart community. This hierarchical coordination framework provided a unified and intuitive tool for the community manager to manage distributed demand-side resources. We exploited a virtual battery model to aggregate the flexibility of various classes of residential loads such as TCLs, DESs, PPs, and EVs. Additionally, we presented a Nash-bargaining-based power allocation strategy for resource aggregators, and various load scheduling algorithms for individual loads. We also conducted numerical experiments to demonstrate the efficacy of our proposed framework and coordination strategy to cap the peak power of a residential community. In the future, we will study coordination of residential loads for other grid services. Additionally, we will consider a more realistic residential community environment, and quantify the value of flexible loads in terms of the cost savings associated with the amount of peak demand that is shaved, reduced utility by responding to dynamic pricing schemes, and/or the revenue by providing ancillary services to the grid. Moreover, we are interested in examining the effect of communication errors on the performance of our proposed approaches.

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