Novel Adaptive Blind-Equalizer-Order Selection Scheme for Multiple-Input Multiple-Output Channels

Kun Yan, Hsiao-Chun Wu, Shih Yu Chang and Yiyan Wu

Electrical and Computer Engineering Department, Louisiana State University
Baton Rouge, Louisiana, 70803, U. S. A.

Abstract—This paper introduces a novel adaptive finite-impulse-response (FIR) equalizer-order selection scheme for multiple-input multiple-output (MIMO) channels. The model-order selection for blind FIR equalizers is determined heuristically but hardly on any theoretical basis. However, this heuristical strategy would often lead to a suboptimal solution. Thus, we design a non-pilot-aided MIMO channel-length estimation scheme and propose a new blind-equalizer-order determination scheme built on such an adaptive channel-length estimation method. Our simulations show that the proposed scheme greatly outperforms the conventional systems using the fixed heuristical model-order selection technique.

Index Terms—MIMO, channel length estimation, blind equalizer.

I. INTRODUCTION

Multi-input multi-output (MIMO) is one of the most promising techniques for achieving high data rate and reliability over the broadband wireless medium in recent years [1], [2], [3]. Using the spatial diversity provided by multiple paths, MIMO technique significantly improve channel capacity. However, with the increasing data rate of wireless applications, fading and multipath propagation might introduce *intersymbol interference* (ISI) in addition to additive white Gaussian noise (AWGN).

To deal with the ISI distortion, many equalizers, such as *minimum mean-square error* (MMSE) blind equalizer [4], equalizer based on Hidden Markov Models (HMM) [5], blind equalizer using *high order statistics* (HOS) [6], and Turbo equalizer [7], have attract researchers' eyes. Recently, equalizers for MIMO system have also been discussed in many literatures [8], [9]. Beside of the progress in practical area, some important theoretical properties for blind equalizer, such as, convergency of blind equalization algorithm with respect to optimization condition [10], and signal-to-interference ratio (SIR) with respect to equalizer's length are discussed [11], [12]. However, the scheme for choosing equalizer's order based on theoretical analysis have never been touched. In the existing literature, equalizers' length are mostly determined by heuristical experiments. Consequently, the equalization scheme might not be optimal for accuracy and computational complexity simultaneously.

To deal with this crucial problem, we would like to introduce a novel equalizer order selection scheme based on channel estimation. A non-pilot-aided channel-length estimation algorithm [13] based on the second-order statistics of the received signals from the MIMO transceiver is employed to estimate the longest channel impulse response (CIR). Then, the resulting MIMO channel-length information is used to estimate the channel impulse response. A novel scheme based on theoretical analysis of L_2 approximation error [12] is adopted to estimate length of blind equalizer consequently. With this novel equalizer choosing scheme, we can construct equalizer with optimal computational complexity and accuracy. Simulation results justify that our proposed order selection method greatly improve the system efficiency without sacrificing the equalizer's estimating accuracy.

The paper is organized as follows: System model is introduced in Section II, then the equalizer's order choosing scheme is briefly discussed in Section III. Simulation result will be presented in Section IV. Finally, conclusion is drawn in Section V.

Notations: $\vec{A}$ denotes a vector and $\tilde{A}$ denotes a matrix where $\tilde{A}^T$ is its transpose. The set of all integers is denoted as $\mathcal{Z}$ and the statistical expectation operation is denoted as $\mathbf{E}\{\}$.

II. MIMO-OFDM SYSTEM MODEL

We provide the transmission model for MIMO transmission system in this section. An N_t -transmitting/ N_r -receiving antenna configuration is considered. The signal at transmitting antenna i is written as $\vec{s}_i[n] = \tilde{F}_{N_s} \vec{a}_i[n]$, where $\vec{s}_i[n] \stackrel{\text{def}}{=} (s_i[n] \ s_i[n+1] \ \cdots \ s_i[n+N_s-1])^T$, $\tilde{F}_{N_s}$ is the $N_s \times N_s$ inverse discrete Fourier transform (IDFT) matrix, N_s denotes the total number of subcarriers and $\vec{a}_i[n] \stackrel{\text{def}}{=} (a_i[n] \ a_i[n+1] \ \cdots \ a_i[n+N_s-1])^T$ is the complex-valued symbol vector sent from antenna i at time instant $n+N_s-1$. Assume that we collect $N+1$ received signal samples at each receiving antenna. The received signal $x_j[n]$ at receiving antenna j is expressed as

$$x_j[n] = \sum_{i=1}^{N_t} \sum_{m=0}^{L_{ji}} h_{ji}[m] s_i[n-m] + \zeta_j[n],$$

$$n = 0, 1, \dots, N, \quad j = 1, 2, \dots, N_r, \quad (1)$$

where $h_{ji}[n]$ is a linear-time-invariant impulse response modeling the propagation path between the i th transmitting and the j th receiving antenna and $N \geq \max_{i,j} (L_{ji})$ such that

$$h_{ji}[n] = 0, \text{ for } n < 0 \text{ and } n > L_{ji}. \quad (2)$$

We further define

$$L_{j,\max} \stackrel{\text{def}}{=} \max(L_{j1}, L_{j2}, \dots, L_{jN_t}). \quad (3)$$

$\zeta_j[n]$ is the additive noise incurred at the j th receiving antenna. We can write the matrix form for Eq. (1) as

$$x_j[n] = \text{trace}\{\tilde{H}_j \tilde{S}^T[n]\} + \zeta_j[n], \quad (4)$$

where

$$\tilde{H}_j \stackrel{\text{def}}{=} 2 \sum_{k=1}^K \left\{ \begin{array}{cccc} h_{j1}[0] & h_{j1}[1] & \cdots & \\ \vdots & \vdots & \vdots & \\ h_{j\chi}[0] & h_{j\chi}[1] & \cdots & \\ \vdots & \vdots & \vdots & \\ h_{jN_t}[0] & h_{jN_t}[1] & \cdots & \\ h_{j1}[L_{j1}] & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & h_{j\chi}[L_{j,\max}] \\ \vdots & \vdots & \vdots & \vdots \\ h_{jN_t}[L_{jN_t}] & 0 & \cdots & 0 \end{array} \right\}. \quad (5)$$

Note that $h_{j\chi}[n] \in \{h_{ji}[n] \mid L_{ji} = L_{j,\max}\}$.

$$\tilde{S}[n] \stackrel{\text{def}}{=} 2 \sum_{k=1}^K \left\{ \begin{array}{cc} s_1[n] & s_1[n-1] \\ \vdots & \vdots \\ s_{N_t}[n] & s_{N_t}[n-1] \\ \cdots & s_1[n-L_{j,\max}] \\ \vdots & \vdots \\ \cdots & s_{N_t}[n-L_{j,\max}] \end{array} \right\}_{N_t \times (L_{j,\max}+1)}. \quad (6)$$

and $\text{trace}\{\cdot\}$ denotes the trace operation of matrices. $\zeta_j[n]$ is assumed both spatially and temporally white, with zero mean and variance

$$\mathbf{E}\{|\zeta_j[n]|^2\} = \sigma_{\zeta_j}^2. \quad (7)$$

Unlike the assumption of the identical delay and fading property in [14], [15], we assume that the delays L_{ji} are not necessarily the same for all i .

III. NOVEL EQUALIZER ORDER SELECTION SCHEMA

To determine the order for equalizer in MIMO system, we estimate the channel impulse response at first, then the equalizer's order is calculated according to a predetermined truncation error upper bound. Consequently this order selection scheme provide an effective way for finding

the minimum equalizer length with given truncation error, a tradeoff between computational complexity and error caused by equalizer's truncation is obtained thereby.

A. Non-Pilot-Aided Channel Estimation in MIMO system

In this section, a blind channel impulse response estimation based on channel length estimation will be discussed. A novel channel length estimation algorithm has been studied in [13]. The Second order statistics for the received signal is used to estimated the channel length. We employ the estimated channel length to the iterative CIR estimation algorithm discussed in [16]. Without loss generality, we assume the MIMO channel to be linear-time-invariant(LTI) FIR filter with distinct length in the following.

B. Novel Equalizer Order Selection Schema

Based on the estimation in Section III-A, we introduce a novel scheme for choosing equalizer's order here. The theoretical analysis of the relationship between equalizer's order and equalizer's truncation error is presented in [12]. We briefly derive the truncation error for FIR equalizer based on the analysis in [12] here.

Given LTI channel impulse response as Eq. (2), we can easily factorize its z-transform $H_{ji}(z)$ as

$$H_{ji}(z) = \frac{D(z)}{Q(z)}, \quad (8)$$

where $D(z)$ and $Q(z)$ are polynomial with variable z^{-1} . We can always factorize $D(z)$ as

$$D(z) = \prod_{l_1=1}^{n_{>}} (z_{-1} - P_{l_1})^{m_{l_1}} \prod_{l_2=1}^{n_{<}} (z_{-1} - p_{l_2})^{m_{l_2}}, \quad (9)$$

where $|P_{l_1}| > 1$ and $|p_{l_2}| < 1$, m_{l_1} , m_{l_2} denote the number of distinct roots P_{l_1} and p_{l_2} . To simplify the derivation, we only consider the case when $m_{l_1} = m_{l_2} = 1$. The other cases will be discussed in our following work. Considering these assumption, the z-transform for corresponding equalizer is

$$\begin{aligned} E(z) &= \frac{Q(z)}{\prod_{l_1=1}^{n_{>}} (z_{-1} - P_{l_1}) \prod_{l_2=1}^{n_{<}} (z_{-1} - p_{l_2})} \\ &= \sum_{l_1=1}^{n_{>}} \frac{\rho_{l_1}}{(z^{-1} - P_{l_1})} + \sum_{l_2=1}^{n_{<}} \frac{\eta_{l_2}}{(z^{-1} - p_{l_2})} \\ &\quad + r_0 + r_1 z + r_2 z^2 + \dots + r_{\Gamma} z^{\Gamma}, \end{aligned} \quad (10)$$

where $\deg(r_0 + r_1 z + r_2 z^2 + \dots + r_{\Gamma} z^{\Gamma}) \stackrel{\text{def}}{=} \Gamma$, and $\Gamma \stackrel{\text{def}}{=} \deg(Q(z)) - \deg(D(z))$ then the error caused by truncation is given by

$$\begin{aligned} \epsilon_{M,N} &= \sum_{k=M}^{M+\deg(D(z))-1} \left| \sum_{l_1=1}^{n_{>}} F_{l_1,k} \right|^2 \\ &\quad + \sum_{k=N+\deg(D(z))-1}^N \left| \sum_{l_2=1}^{n_{<}} f_{l_2,k} \right|^2. \end{aligned} \quad (11)$$

Note that

$$F_{l_1,k} = \frac{\rho_{l_1} \left(\frac{d^{\deg(D(z))-1} \left[\frac{D(z)}{(z^{-1}-P_{l_1})} \right]}{d(z^{-1})^{\deg(D(z))-1}} \right)}{(\deg(D(z)) - 1)!} \Big|_{z^{-1}=0} \left(-\frac{1}{P_{l_1}^M} \right),$$

$$f_{l_2,k} = \frac{\eta_{l_2} \left(\frac{d^{1-\deg(D(z))} \left[\frac{D(z)}{(z^{-1}-P_{l_2})} \right]}{d(z^{-1})^{1-\deg(D(z))}} \right)}{(1 - \deg(D(z)))!} \Big|_{z^{-1}=0} (-P_{l_2}^N).$$
(12)

where $M \in \mathcal{Z}$ and $N \in \mathcal{Z}$ denote that $-N^{\text{th}}$ to M^{th} indexed element in the corresponding time domain series for $E(z)$ in Eq (10). We can define

$$\Phi_{l_1,k} = \frac{\rho_{l_1} \left(\frac{d^{\deg(D(z))-1} \left[\frac{D(z)}{(z^{-1}-P_{l_1})} \right]}{d(z^{-1})^{\deg(D(z))-1}} \right)}{(\deg(D(z)) - 1)!} \Big|_{z^{-1}=0},$$

$$\Psi_{l_2,k} = \frac{\eta_{l_2} \left(\frac{d^{1-\deg(D(z))} \left[\frac{D(z)}{(z^{-1}-P_{l_2})} \right]}{d(z^{-1})^{1-\deg(D(z))}} \right)}{(1 - \deg(D(z)))!} \Big|_{z^{-1}=0}.$$
(13)

Clearly $\Phi_{l_1,k}$ and $\Psi_{l_2,k}$ are functions with respect to roots defined after Eq. (9). Then Eq. (11) can be simplified as

$$\epsilon_{M,N} = \sum_{k=M}^{M+\deg(D(z))-1} \left| \sum_{i_1=1}^{n>} -\frac{\Phi_{l_1,k}}{P_{l_1}^M} \right|^2 + \sum_{k=N+\deg(D(z))-1}^N \left| \sum_{i_2=1}^{n<} -\Psi_{l_2,k} P_{l_1}^N \right|^2. \quad (14)$$

From Eq. (14), we build the explicit truncation error function $\epsilon_{M,N}$ with respect to equalizer's order $M + N$, then, given an upper-bound for truncation error, the minimum equalizer length can be easily determined. For example, given an estimated z-transform for a channel impulse response $h_{ij}(n)$ as follows

$$H_{ij}(z) = (z^{-1} - 0.5)(z^{-1} - 2)(z^{-1} + 1.5). \quad (15)$$

The truncation error can be derived as

$$\epsilon_{M,N} = 2.43 \times 0.25^N + \frac{9.98 \times 1.5^{2M} + 31 \times 2^{2M} + 36.2 \times 3^M}{3^{2M}}. \quad (16)$$

Given an upper bound of truncation error 10^{-2} , we can compute the minimum equalizer length as $M = 11, N = 5$, Since $M \in \mathcal{Z}$ and $N \in \mathcal{Z}$, the result can be computed by discrete approximation.

We briefly state the workflow of our proposed order selection scheme as follows:

1. Estimate the maximum channel length according to the received signal;
2. Then Estimate the channel impulse response $h_{ij}(n)$ using the estimated channel length;
3. Decompose the corresponding Z-transform of $h_{ij}(n)$ and compute $\Phi_{l_1,k}$ and $\Psi_{l_2,k}$;
4. Given an upper bound of truncation error, calculate the minimum length of equalizer.

Since we assume the channel is time invariant in this paper, the equalizer order selection scheme can be carried out at the begin of equalization loop. On the other hand, when considering time variant channel, the selection loop might be perform occasionally to tracking the change of channel.

IV. SIMULATION

A 2×2 MIMO-OFDM system is simulated ($N_t = N_r = 2$). Figure 1 illustrates the block diagram of the MIMO-OFDM system. The channel estimation and order selection block is added in the receiver. In simulation we choose the channel parameters as follows: (1) the FFT/IFFT size is 256; (2) 16QAM is employed as the mapping constellation; (3) the channel coefficients are randomly generated while ITU pedestrian B model is used to characterize the power delay profile; (4) turbo code is used in order to achieve the maximum information transfer; (5) the maximum channel length of the 4 paths is 10; (6) the parameters in the channel-length estimator are set as: $\theta_j = 0.01$ and $\Psi_j = 0.0001, \forall j$. (7) channel estimation algorithm in [16] is employed in this simulation to estimate the channel. Minimum mean-square error (MMSE) equalizer is employed for the ultimate symbol detection according to [17]. The truncation error used in our simulation is chosen as 0.05. Six iterations of turbo decoding scheme is adopted. We carry out 100 Monte Carlo trials to test our algorithm. We define a tradeoff for computational complexity and accuracy of the equalizer as

$$\Upsilon = \frac{\varphi}{Q}. \quad (17)$$

where $Q = \frac{1}{N_r} \sum_{\beta=1}^{N_r} W_{\beta}$ in MIMO system, W_{β} is the length of each equalizer in MIMO receiver. φ is given by

$$\varphi = \mathbf{E} \left\{ \frac{\max_k |w_{ij}(k) \otimes h_{ij}(k)|^2}{\sum_{k=0}^{Q+M} |w_{ij}(k) \otimes h_{ij}(k)|^2 - \max_k |w_{ij}(k) \otimes h_{ij}(k)|^2} \right\}, \quad (18)$$

where the expectation in Eq. (18) is computed according all the transmitted channels. Figure 2 depicted the tradeoff defined here. Equalizer's length is chosen according to our novel order selection scheme. The tradeoff of the same system with fixed equalizer's order $2L_{j,\max} - 1$ is also

shown in Figure 2. Clearly, the equalizer obtain similar SIR with shorter length when employ our novel order selection scheme.

V. CONCLUSION

In this paper, we propose a novel equalizer order selection scheme base on channel estimation and factorization. The novel scheme give us a theoretical analysis for determining the length of equalizer given a truncation error. Consequently the system could be optimal both in computational complexity and accuracy. A simulation for tradeoff in transmission system with our novel order selection algorithm is present to support our claim.

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Fig. 1. Block diagram of the transmission system

Fig. 2. Tradeoff comparison between our proposed order selection algorithm and the fixed equalizer order. The equalizer length is fixed as $2L_{j,\max} - 1$ in the conventional algorithm. The truncation error used in our order selection algorithm is chosen as 0.05.