Multilook SAR From Measurements Partitioned Based On Synthetic Sensor Parameters

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Abstract—The balance between spatial resolution and the speckle in multilook SAR imaging may be improved by intelligently partitioning measurements of a sparsely populated, nonuniform, three-dimensional receiver array. This paper presents an approach to partitioning measurements from a single transmitter and multiple coherent receivers using previously developed two-dimensional synthetic sensor locations based on eigen analysis of first-order Taylor expansions of the sensor measurement parameters. The approach is general and is demonstrated using simulated data for a nonuniformly-spaced two-dimensional sidelooking array and low time-bandwidth product.

I. INTRODUCTION

Multilook synthetic aperture radar (SAR) images may be calculated from sparsely populated, nonuniform, three-dimensional arrays. In general, space-time measurements from these arrays are not uniformly sampled in space and/or time. The phase of the space-time radar response at a receiver for a given scatterer varies with time, frequency, and relative location of the receiver and target. Thus, the data will be sampled over five dimensions; range, along-track, elevation, slow time, and fast frequency, where the notation fast frequency refers to the Fourier transform of the phase history data with respect to fast-time. For uniformly-spaced arrays with element separations on the order of wavelengths, the subapertures reduce to fast-time, slow-time partitions. However, intelligently partitioning measurements collected over the five measurement dimensions into subapertures is not simple for nonuniformly-spaced two-dimensional arrays.

In antenna array theory, the width of the main beam of an array depends on the spatial separation between the two antenna elements that are the farthest apart. The characteristics of the array's sidelobes depend on the distance between adjacent elements. Subaperturing space-time data has the same relationships; therefore, the spatial resolution and ambiguity function of the multilook SAR image will be affected by how the measurements are partitioned.

DeGraaf [1] notes that independent look averaging reduces spatial resolution because the subapertures are smaller than the full aperture. For that reason, he says the periodogram (multilook SAR) is of little practical value. For some applications, such as estimating the clutter covariance matrix in GMTI or remote sensing applications, spatial resolution

may not be as important as accurately estimating the expected clutter spectrum. Additionally, multilook SAR may prove to be a quicker method of reducing speckle than classification techniques.

Curlander and McDonough [2] discuss subaperturing data in the Doppler (slow-time) domain for multilook SAR. However, they only consider SAR measurements collected from a single aperture. On the other hand, in Section 2.2 of Goodman's dissertation [3] and a subsequent paper [4], synthetic sensors are developed as a way to represent five-dimensional data in two dimensions. He rigorously develops the synthetic sensor locations as a method to determine a radar system's resolution and ambiguity function. The general radar model used to develop the synthetic sensors allows for sparsely populated, nonuniform, three-dimensional antenna array and a wide range of look geometries.

This paper applies the synthetic sensor locations developed in [4] to partitioning a measurement vector for multilook SAR imaging. Ambiguity functions of measurement subapertures based on these synthetic sensor locations are compared to ambiguity functions based on slow-time subapertures and fast-frequency subapertures.

II. RADAR MODEL

Following the notation of [4], the received signal due to a single scatterer on the Earth's surface is

$$d(\mathbf{x}, \mathbf{r}, \omega, t) = \frac{\gamma(\mathbf{x}) g(\mathbf{x})}{R(\mathbf{x})^2} w(\mathbf{r}, \omega, t) \exp[-j\omega\tau(\mathbf{x}, \mathbf{r}, t)]. \quad (1)$$

Using the center of the receiver array as the origin, the scatterer's reflection coefficient at a spatial location on the Earth $\mathbf{x} = [x, y, -h]^\dagger$ is $\gamma(\mathbf{x})$, where $(\cdot)^\dagger$ is the transpose operator. The transmit field intensity pattern is $g(\mathbf{x})$, and $w(\mathbf{r}, \omega, t)$ is a weight function describing the receiver response across the measurement parameters of space, fast frequency, and slow time. The range from the transmitter to the scatterer and the range from the scatterer to the receiver is $R(\mathbf{x})$, which leads to a round trip delay from the transmitter to the scatterer and back to the receiver $\tau(\mathbf{x}, \mathbf{r}, t)$. The vector $\mathbf{r}$ is the location in three-dimensional space of a receiver at $t = 0$ and is defined as $\mathbf{r} = [r_x \ r_y \ r_z]^\dagger$. Again, the notation fast frequency refers to

the Fourier transform of the phase history data with respect to fast-time.

III. EIGEN ANALYSIS OF MEASUREMENT PARAMETERS

Using Goodman's [3], [4] synthetic sensor concept, subapertures of the data necessary for multilook SAR may be based on two-dimensional synthetic sensor parameters. The goal is multilook SAR images with well-balanced estimation accuracy (less speckle in homogeneous scattering regions) and spatial resolution. Therefore, it is important to partition the measurement subspace the subapertures to result ambiguity functions that are well-behaved in along track and cross track. As a reminder, subaperturing is just a way to partition the measurements for multilook SAR. Subaperturing itself performs no operations on the data.

The following discussion follows Goodman's [3], [4] development of synthetic sensors. The actual five-dimensional sensor parameters are three-dimensional space, fast frequency, and slow time. The vector $\mathbf{s}$ is then the five-by-one vector of sensor parameters $\mathbf{s} = [\mathbf{r}^\dagger \ \omega \ t]^\dagger$, where $(\cdot)^\dagger$ is the transpose operator. Goodman rationalizes that since only two independent variables are necessary to define the location of a stationary scatterer on a two-dimensional plane, (location in along track and cross track for airborne radar), it is reasonable to assume two spatial sensor dimensions are sufficient to represent radar measurements. A two-by-five transformation matrix $\mathbf{\Lambda}_s$ is then developed based on first-order Taylor series expansions around the sensor parameters, $\mathbf{s}$. In terms of partial derivatives, $\mathbf{\Lambda}_s$ is given by

$$\mathbf{\Lambda}_s = \begin{bmatrix} \frac{\partial^2}{\partial x \partial r_x} & \frac{\partial^2}{\partial x \partial r_y} & \frac{\partial^2}{\partial x \partial r_z} & \frac{\partial^2}{\partial x \partial \omega} & \frac{\partial^2}{\partial x \partial t} \\ \frac{\partial^2}{\partial y \partial r_x} & \frac{\partial^2}{\partial y \partial r_y} & \frac{\partial^2}{\partial y \partial r_z} & \frac{\partial^2}{\partial y \partial \omega} & \frac{\partial^2}{\partial y \partial t} \end{bmatrix} \Psi \Big|_{\mathbf{x}_0, \mathbf{s}_0} \quad (2)$$

where Ψ is the phase of the space-time radar response vector for a given receiver location and along-track, cross-track resolution cell. The vector $\mathbf{x}_0$, defines the center of imaged scene, and $\mathbf{s}_0$ is the set of initial sensor parameters around which the expansion is performed.

For the sidelooking case, the sensor transformation and its singular value decomposition (SVD) may be analytically determined. For the sidelooking case, $\mathbf{x}_0 = [x_0 \ y_0 \ -h]^\dagger$, and $\mathbf{s}_0 = [0 \ 0 \ 0 \ \omega_0 \ 0]^\dagger$. Evaluating $\mathbf{\Lambda}_s$ at $\mathbf{x}_0$ and $\mathbf{s}_0$ and suppressing the $\frac{\omega_0}{c}$ term, the sensor transformation matrix becomes

$$\mathbf{\Lambda}_s = \begin{bmatrix} \frac{-(h^2 + y_0^2)}{R_0^3} & \frac{x_0 y_0}{R_0^3} & \frac{-x_0 h}{R_0^3} & \frac{2x_0}{\omega_0 R_0} & \frac{-2v(h^2 + y_0^2)}{R_0^3} \\ \frac{x_0 y_0}{R_0^3} & \frac{-(h^2 + x_0^2)}{R_0^3} & \frac{-y_0 h}{R_0^3} & \frac{2y_0}{\omega_0 R_0} & \frac{2vx_0 y_0}{R_0^3} \end{bmatrix} \quad (3)$$

where $R_0 = \sqrt{h^2 + x_0^2 + y_0^2}$ is the distance from the center of the radar aperture to the center of the imaged scene $\mathbf{x}_0$. The along-track velocity of the radar platform is v , its height above the earth is h , the along-track dimension is x , the range dimension is y , the fast-frequency sampling interval of the transmitted and received signals is ω_0 , and c is the velocity of the transmitted signal in freespace.

For the sidelooking case assumed in this paper, $x_0 = 0$, and $R_0 = \sqrt{h^2 + y_0^2}$. The sensor transformation matrix becomes

$$\mathbf{\Lambda}_s = \frac{\omega_0}{c} \begin{bmatrix} \frac{-1}{R_0} & 0 & 0 & 0 & \frac{-2v}{R_0} \\ 0 & \frac{-h^2}{R_0^3} & \frac{-y_0 h}{R_0^3} & \frac{2y_0}{\omega_0 R_0} & 0 \end{bmatrix}. \quad (4)$$

The SVD of the sidelooking sensor transformation matrix can be taken to yield:

$$\mathbf{\Lambda}_s = \mathbf{U} \mathbf{S} \mathbf{V}^\dagger, \quad (5)$$

where the SVD matrices are:

$$\mathbf{U} = [\mathbf{u}_1 \ \mathbf{u}_2], \quad (6)$$

$$\mathbf{S} = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

and

$$\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4 \ \mathbf{v}_5]. \quad (8)$$

Goodman [3] shows that only the first two column vectors of $\mathbf{V}$, $\mathbf{v}_1$ and $\mathbf{v}_2$, project the five sensor parameters into the two independent eigensensor dimensions. Calculating the SVD of $\mathbf{\Lambda}_s$, the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are

$$\mathbf{v}_1 = \begin{bmatrix} \frac{-1}{\sqrt{1+4v^2}} & 0 & 0 & 0 & \frac{-2v}{\sqrt{1+4v^2}} \end{bmatrix}^\dagger \quad (9)$$

and

$$\mathbf{v}_2 = \frac{1}{\sqrt{\frac{h^4 + h^2 y_0^2}{R_0^4} + \frac{4y_0^2}{\omega_0^2}}} \begin{bmatrix} 0 & \frac{-h^2}{R_0^2} & \frac{-hy_0}{R_0^2} & \frac{2y_0}{\omega_0} & 0 \end{bmatrix}^\dagger. \quad (10)$$

The location of the first synthetic sensor dimension is obtained by taking the inner product of $\mathbf{v}_1$ with $\mathbf{\Lambda}_s$, which is the change in the sensor parameters from $\mathbf{s}_0$. Similarly, the second synthetic sensor dimension is a result of the inner product of $\mathbf{v}_2$ with $\mathbf{\Lambda}_s$. Consequently, the coordinates in the synthetic sensor dimensions are given by $\alpha = \mathbf{v}_1^\dagger \mathbf{\Lambda}_s$ and $\beta = \mathbf{v}_2^\dagger \mathbf{\Lambda}_s$. Completing the inner products,

$$\alpha = \frac{-1}{\sqrt{1+4v^2}} (r_x + vt) \quad (11)$$

and

$$\beta = \frac{1}{\sqrt{\frac{h^4 + h^2 y_0^2}{R_0^4} + \frac{4y_0^2}{\omega_0^2}}} \left(-\frac{h^2 r_y}{R_0^2} - \frac{hy_0 r_z}{R_0^2} + \frac{2y_0 \Delta \omega}{\omega_0} \right). \quad (12)$$

The parameter α depends on the along-track position of the receiver, the along-track velocity, and slow time, and is therefore called the effective along-track position of the synthetic sensor. The effective cross-track position of the sensor is given by the value of β because of its dependence on fast time and the receiver's position in height and cross track. Using these parameters, each measurement can be mapped into the two-dimensional coordinate system of effective along track and effective cross track for the synthetic array.

IV. SUBAPERTURE COMPARISON

Figure 1 is a plot of the synthetic array sensor locations for a measurement vector of a radar model with nine receive elements randomly distributed over a two-dimensional space. The receive vector for each receiver in this example consists of 39 pulses and 37 fast-frequency samples per pulse. These samples appear as blocks in Figure 1, while the locations of the blocks depend directly on the location of the receiver in along track and cross track. The receiver whose block of measurements is in the lower left of Figure 1 happened to be the farthest from the array center in both along track and cross track. Its spatial separation from the center of the receiver array was 94.8 wavelengths in along track and 54.2 wavelengths in cross track. As a comparison, the receiver closest to array center was separated by 7.9 wavelengths in along track and 0.86 wavelengths in cross track.

To illustrate the effect of the proposed subaperturing technique compared to traditional techniques, measurements from a single receiver are presented in the follow subsections.

Figure 1. Synthetic array sensor locations for an array of 9 receivers randomly distributed in a 2D array (planar array in along track and cross track). The measurement samples from the 9 distinct receiver locations are denoted by the different colors

A. Ambiguity and Resolution

Using the two synthetic sensor location coordinates in place of the five-dimensional measurement parameters, the radar model may be reorganized into the two synthetic dimensions. This reorganization enables easier subaperturing of SAR data for multilook processing with the goal improving the ambiguity function as compared to other subaperturing techniques.

a) *Azimuth Subapertures:* SAR systems typically have better resolution in azimuth than range, so looks are most often taken from the azimuth spectrum using fixed bandwidth bandpass filters. The bandwidth of the bandpass filters depends on the desired azimuth resolution of the resulting multilook image. Cumming and Wong detail azimuth look extraction,

Figure 2. Synthetic sensor array locations for 4 azimuth subapertures

detection, and summation in Section 6.5 of their book [5]. The azimuth subaperturing technique used in this research is accomplished differently than in Cumming and Wong [5] with the same desired effect. The desired effect is better estimation accuracy, in the form of reduced speckle, at the expense of reduced azimuth resolution.

Azimuth subaperturing is accomplished by selecting all the measurements from sequential slow-time increments for each subaperture. Partitioning the measurements in slow-time increments shortens the timewidth of each subaperture, when compared to the full measurement vector. The decreased timewidth effectively increases the Doppler beamwidth reducing the Doppler resolution.

Each azimuth subaperture is then SAR processed, and all the resulting SAR images are incoherently averaged. For a single receive element, Figure 2 shows where the synthetic array sensor locations are for four azimuth subapertures. Figure 3 shows the ambiguity plot for 64 azimuth subapertures for a resolution cell near the center of a 32x32 scene. As expected, the near-center resolution cell is clearly ambiguous with the other resolution cells at the same azimuth, or along-track location.

Figure 3. Ambiguity plot for 64 azimuth subapertures

b) *Range Subapertures*: Similarly, the radar model may be subapertured in range by selecting the measurements for each subaperture based on fast-frequency sampling. Partitioning the measurements according to fast-frequency reduces the fast-frequency bandwidth of each subaperture, as compared to the full measurement vector. The reduced fast-frequency bandwidth results in an increased range beamwidth and corresponding reduction in range resolution.

Figure 4. Synthetic sensor array locations for 4 range subapertures

Figure 4 shows where the synthetic array sensor locations are for four range subapertures. Figure 5 shows the resulting ambiguity plot for the near-center resolution cell of the same 32x32 scene. As expected, the center resolution cell is ambiguous with the other resolution cells at the same cross-track location.

Figure 5. Ambiguity plot for 64 range subapertures

c) *Synthetic Sensor Location Subapertures*: The synthetic array sensor locations for four eigensensor-based subapertures are shown in Figure 6 and ambiguity functions for the near-center target for 64 eigensensor-based subapertures is plotted in Figure 7. These plots reveal that subapertures based on synthetic sensor locations are a hybrid of the azimuth and range subapertures. The synthetic sensor subapertures suppress ambiguities in azimuth that are evident in the azimuth

subapertures ambiguity plot, and the ambiguities in range are suppressed, which are evident in the range subapertures ambiguity plot. Reducing the magnitudes of the ambiguities in along track and cross track come at the expense of a wider mainlobe in both dimensions of the subaperture ambiguity function, which results in decreased spatial resolution in the SAR intensity plot.

Figure 6. Synthetic sensor array locations for 4 eigensensor subapertures

Figure 7. Ambiguity plot for 64 azimuth subapertures

Subapertures based on the eigensensor locations effectively balance the reduction in slow-time timewidth and fast-frequency bandwidth for each subaperture, compared with azimuth and range subapertures. The resulting subaperture beamwidth is increased in Doppler and range, respectively, thereby reducing the spatial resolution, with respect to using the entire measurement vector.

An important observation is that a closely-spaced uniform receiver array results in redundant synthetic sensor locations. This becomes important in SAR imaging, because only one receiver is necessary to achieve the full, unambiguous resolution possible. Each subaperture may then correspond to an individual receiver. The resulting single-look images would then have full spatial resolution, and the multilook image would have full spatial resolution and reduced speckle.

Figure 8. 4 subapertures of the synthetic array sensor locations for an array of 9 receivers randomly distributed in a 2D array. The subapertures are denoted by the different colors

V. MULTILOOK IMAGES

The synthetic sensor locations for widely-spaced nonuniform receiver arrays are also nonuniform, as shown in Figure 1. Dividing the synthetic sensor subspace into uniform subapertures may result in sparsely-populated subapertures, and possibly even unpopulated subapertures. Figure 8 illustrates partitioning the synthetic sensor subspace of Figure 1 into four subapertures, and the nonuniformity of the synthetic sensor locations within the subapertures is evident. As a result, care must be taken when calculating the multilook image from the single-look images. Additionally, because averaging the single-look images averages the magnitudes of the complex scattering coefficients, multilook SAR is a biased estimate of the actual scattering intensity.

Figure 9 shows the expected scattering intensity for the simulated scattering scene to be imaged. Figures 10-12 The following figures show multilook SAR images averaged from single-look SAR images calculated using conjugate back projection. The simulated phase history samples are from the nine-receiver array briefly described in Section IV.

As expected from multilook SAR, the speckle is reduced as the number of subapertures increases, and proportionally, the number of measurements in each subaperture decreases. Additionally, the spacial resolution of the images in Figures 10-12 becomes coarser as the number of subapertures increases.

Because of the sparsity of the measurement sample distribution in the synthetic array sensor dimensions, as seen in Figure 8, some subapertures are empty when the partitions are small. This effect reduces the number of effective looks, and the number of nonempty, or populated, subapertures are noted in the captions of Figures 10-12.

VI. CONCLUSION

Multilook SAR images from measurements partitioned using synthetic array sensor locations promise to have a better

Figure 9. Expected SAR intensity image for a simulated 32x32 scene

Figure 10. SAR image generated from the synthetic sensor subspace divided into 16 subapertures, but only 13 populated subapertures

Figure 11. SAR image generated from the synthetic sensor subspace divided into 64 subapertures, but only 41 populated subapertures

Figure 12. SAR image generated from the synthetic sensor subspace divided into 256 subapertures, but only 142 populated subapertures

balance of spatial resolution and ambiguity function than traditional approaches to multilook SAR. Ambiguity plots for the proposed approach support the claim for simulated measurements from a widely-spaced uniform linear receive array, but the approach is generally applicable to nonuniform, three-dimensional arrays. Despite the advantages of better subaperture partitioning, multilook SAR from a single measurement vector is a biased estimate of the scattering intensity and suffers from reduced spatial resolution. The subaperture approach using synthetic array sensor locations may be more applicable when the sensor locations and measurement parameters are known a priori, such as in medical imaging.

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the United States Government.

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