SEQUENTIAL CLASSIFICATION OF ACTIVE SONAR RETURNS

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ABSTRACT

An automated active sonar classifier is designed based on the principle of deferred decision. Given the loglikelihoods of all classes up to the current return a classification decision is deferred until the next return if certain performance criteria are not satisfied. This active sonar sequential classification procedure has the advantage of reducing errors and minimizing the number of returns required to reach a correct decision. The performance of the active sonar sequential classifier is demonstrated by simulation. Four typical sonar classes representing rigid and elastic spheres of different materials are simulated. The results which demonstrate the advantage of a deferred decision strategy are displayed for two performance criteria in a confusion matrix extended to include a decision pending category.

1. INTRODUCTION

In active sonar, a return from a target is often analyzed in a time-frequency

representation[1,2,3]. Both the ambiguity function and Wigner distribution function are examples time-frequency representations. Here the fourth-order cumulant spectrum will be used to extract information about the targets to be classified. Four classes of targets are considered. Three are spherical with the same radius. The material of the spheres are either steel or lucite. Both rigid and elastic modes of the spheres are considered. The fourth class is noise.

The classification model employed assumes a two-dimensional grid of frequency components from a slice of the fourth-order cumulant spectrum. For subsequent returns the model allows the components of the fourth-order cumulant spectrum to be correlated with previous corresponding components.

2. SONAR MODEL

The active sonar data model assumed here uses the output of the fourth-order cumulant spectrum[4],

$$C_4(\omega_1, \omega_2, \omega_3) =$$

This work was supported by the IR Program of NUWC and by ONT under the NU3B Program at NUWC.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_4(\tau_1, \tau_2, \tau_3) d\tau_1 d\tau_2 d\tau_3, \quad (2.1)$$

where, $c_4(\tau_1, \tau_2, \tau_3)$, is the fourth-order cumulant. Details of the fourth-order cumulant and spectrum can be found in[5]. Here, a special case will be used to extract information from the impulse response of a rigid sphere, a thin elastic steel sphere, and a thin elastic lucite sphere. The impulse responses of the spheres were obtained from a program developed by J. Patel from NUWC, based on Junger and Feit[6].

Classification is based on the components of the fourth-order cumulant spectrum. For each return a set of components are extracted and sent to the sequential classifier. In general all components of the fourth-order cumulant spectrum could be extracted and used for classification. But this would require a significant processing capability. Here a judicious selection of components were extracted and used in the classifier. Future work will include all components in the classifier.

The fourth-order cumulant spectra for the spheres are shown below:

- (1) rigid sphere, designated as class 1,

- (2) thin elastic steel sphere, designated as class 2,

- (3) thin elastic lucite sphere, designated as class 3,

- (4) noise, designated as class 0,

The axes for each figure are in normalized frequency. Since the patterns depicted above are all distinctive, a classifier properly encoded when presented with the data

from one class could in principle decide correctly. In practice noise and resolution will limit performance. Here, only a small number of samples from each class are used in the classification simulation to demonstrate the method. Actually, two simulations were performed with the data. In the first case the data were used as given. In the second case the data were compressed or normalized. This case proved to be difficult for the classifier because of the reduced dynamic range of the data. The symmetry in the patterns above appear to aid identification. But samples from only one octant in each fourth-order cumulant spectrum were chosen to encode the sequential classifier.

3. ACTIVE SONAR SEQUENTIAL CLASSIFICATION

Since the fourth-order cumulant spectrum is in general a complex function, its components can be represented by two multidimensional probability density functions. To simplify the analysis without losing generality only the real part of the output of the fourth-order cumulant spectrum will be considered. Therefore, the components of the fourth-order cumulant spectrum will be represented by a multidimensional Gaussian density function over all returns. This assumption could perhaps be extended to non-Gaussian densities by utilizing mixtures of Gaussians[7]. But this will be left to future work.

For each class the information from a set of components of the fourth-order cumulant spectrum are encoded into a covariance matrix of a multidimensional Gaussian density function. This is done by letting the square of the sample values represent the variances

in the covariance matrices. For the data given above the samples were assumed independent and to have zero mean. However, the classifier can encode correlated and non-zero mean samples in more general cases. Given a sequence of returns the sequential classifier decides which class is present based on the covariance matrices representing the classes.

Since each class is modeled as a multidimensional Gaussian process, the loglikelihood ratio of two classes is,

$$\Lambda_{npq}(\underline{X}) = \left\{ \frac{1}{2} \ln \left[\frac{|M_q|}{|M_p|} \right] - \frac{1}{2} \underline{X}^T (M_p^{-1} - M_q^{-1}) \underline{X} \right\}, \quad (3.1)$$

where,

$$\underline{X} =$$

$$[X_{11}(1), X_{12}(1), \dots, X_{MD}(1), \dots, X_{MD}(n)],$$

is a vector representing the components of the fourth-order cumulant spectrum for each return, $p, q = 0, 1, 2, 3$, $p \neq q$, represent the four classes, and, n is a random variable representing the number of returns required to make a decision. The matrices of (3.1) are defined as,

$$M_m = E[\underline{X}_{kj} \underline{X}_{kj}^T],$$

for, $m = 0, 1, 2, 3$, over all returns. These results can easily be extended to higher dimensions.

The performance of the sequential classifier for each loglikelihood ratio is based on the operating characteristic function (OCF) and the average sample number (ASN) as

defined by Wald[8,9,10,11]. The OCF is the probability, $L(h)$, of accepting H_q , defined as,

$$L(h) = (A^h - 1)/(A^h - B^h), \quad (3.2)$$

where, $(H_p : h = -1, H_q : h = 1)$, A and B are upper and lower thresholds, respectively. For the loglikelihood ratios the upper and lower thresholds are, respectively,

$$a = \ln(A) = \ln\left(\frac{1 - \beta}{\alpha}\right), \quad (3.3)$$

$$b = \ln(B) = \ln\left(\frac{\beta}{1 - \alpha}\right). \quad (3.4)$$

The parameters, α , and, β , represent the false alarm and false dismissal probabilities, respectively. If, $\Lambda_{npq} \geq a$, H_p is decided, if, $\Lambda_{npq} \leq b$, H_q is decided. Whereas, if, $b < \Lambda_{npq} < a$, it is not used in the decision at this return. In the simulations, $\alpha = \beta$, therefore, $b = -a$. Two sets of thresholds will be considered in the simulations.

5. RESULTS

In the simulations four hypotheses or classes representing the data discussed in section 2 will be considered. In the figures, 0 will represent, H_0 , 1 will represent, H_1 , 2 will represent, H_2 , and 3 will represent, H_3 . The classifier considers simultaneously all independent pairs of classes for a total of 6. The performance is summarized in a matrix called the performance matrix. The performance matrix compares the classification performance for each class as a function of the number of returns. Since the classifier is sequential a classification may not be made at each return. To account for this a decision pending (DP) category is included

in the performance matrix. For both cases five independent trials were run. The performance matrix for each return gives the average of the five trials. To demonstrate the effect of deferred decision two thresholds settings, $(-.5, .5)$ and $(-2.1972, 2.1972)$, representing, $(\alpha = \beta = .3774)$ and $(\alpha = \beta = .1)$, respectively, were compared for each return over the five trials. The thresholds were determined from equations (3.3) and (3.4).

CASE 1 UNNORMALIZED DATA:

The following four figures represent the first four returns. The figure on the left is for thresholds representing higher error probabilities.

FIRST RETURN (CASE 1)

n= 1 sphere						n= 1 sphere					
a= .5 b= -.5						a= 2.1972 b= -2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	4/5	0	0	0	1/5	0	2/5	0	0	0	3/5
1	1/5	2/5	0	0	2/5	1	0	0	0	0	1
2	0	0	1	0	0	2	0	0	0	0	1
3	0	0	0	1	0	3	0	0	0	1	0

There's one missclassification in the left figure, none in the right figure but classification is deferred more frequently.

SECOND RETURN (CASE 1)

n= 2 sphere						n= 2 sphere					
a= .5 b= -.5						a= 2.1972 b= -2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1	0	0	0	0	0	4/5	0	0	0	1/5
1	0	3/5	0	0	2/5	1	0	3/5	0	0	2/5
2	0	0	1	0	0	2	0	0	0	0	1
3	0	0	0	1	0	3	0	0	0	1	0

There are no missclassifications but deferred decisions occur more often in the right figure.

THIRD RETURN (CASE 1)

n=3 sphere a= .5 b= - .5						n=3 sphere a= 2.1972 b= - 2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1	0	0	0	0	0	1	0	0	0	0
1	0	4/5	0	0	1/5	1	0	3/5	0	0	2/5
2	0	0	1	0	0	2	0	0	2/5	0	3/5
3	0	0	0	1	0	3	0	0	0	1	0

FIRST RETURN (CASE 2)

n=1 sphere (N) a= .5 b= - .5						n=1 sphere (N) a= 2.1972 b= - 2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1/5	0	0	0	4/5	0	0	0	0	0	1
1	2/5	0	0	1/5	2/5	1	0	0	0	0	1
2	3/5	0	0	0	2/5	2	0	0	0	0	1
3	2/5	0	0	1/5	2/5	3	0	0	0	0	1

Again, there are no missclassifications and deferred decisions occur more often in the right figure.

There are several missclassifications in the left figure but all decisions are deferred until the next return in the right.

FOURTH RETURN (CASE 1)

n=4 sphere a= .5 b= - .5						n=4 sphere a= 2.1972 b= - 2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1	0	0	0	0	0	1	0	0	0	0
1	0	4/5	0	1/5	0	1	0	3/5	0	0	2/5
2	0	0	1	0	0	2	0	0	3/5	0	2/5
3	0	0	0	1	0	3	0	0	0	1	0

FIFTEENTH RETURN (CASE 2)

n=15 sphere (N) a= .5 b= - .5						n=15 sphere (N) a= 2.1972 b= - 2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1	0	0	0	0	0	1	0	0	0	0
1	0	4/5	1/5	0	0	1	1/5	1/5	0	0	3/5
2	1/5	0	4/5	0	0	2	1/5	0	3/5	0	1/5
3	0	0	0	1	0	3	0	0	0	1	0

There's one missclassification in the left figure but none in the right. At the fifth and following returns all classes are correctly classified.

Errors in classification are in both figures.

TWENTYFIFTH RETURN (CASE 2)

n=25 sphere (N) a= .5 b= - .5						n=25 sphere (N) a= 2.1972 b= - 2.1972					
	0	1	2	3	DP		0	1	2	3	DP
0	1	0	0	0	0	0	1	0	0	0	0
1	0	4/5	1/5	0	0	1	0	3/5	1/5	0	1/5
2	1/5	0	4/5	0	0	2	1/5	0	3/5	0	1/5
3	0	0	0	1	0	3	0	0	0	1	0

CASE 2 NORMALIZED DATA:

Classification in the normalized data case required many more returns. Some classes were not decided even at the twentyfifth return. The right figures have fewer misclassifications and more deferred decisions. But errors in classification occur in both left and right figures.

Even at the twentyfifth return errors occur in both figures. However, less errors occur in the right figure because of the deferred decision strategy.

6. CONCLUSIONS

Information extracted from rigid and elastic spheres by the fourth-order cumulant spectrum were used to encode an automated active sonar sequential classifier. The sequential classifier operated under the principle of deferred decision. If at a particular return classification criteria were not satisfied a decision would be deferred. By deferring decisions, classification errors were shown to be controllable. Two simulations from the data representing rigid and elastic spheres demonstrated the deferred decision strategy. Specifically, by postponing a classification decision until the next return misclassification errors were reduced.

ACKNOWLEDGMENT

The authors acknowledge the assistance of Dr. Theresa Baus in generating the spherical test data.

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