CHARACTERISTICS OF EXTREME WAVE GROUPS

Ming-Yang Su

Naval Ocean Research and Development Activity
NSTL, MS 39529

Abstract

We discuss statistics of wave groups containing the maximal wave height over wave records of 20 minutes. These wave groups are called extreme wave groups (EWG's). The concept of the EWG combines the two previously separately considered design factors of extreme waves and wave groups into a more severe storm seas wave action. The wave data used in this investigation consists of nearly 540 records of wave staff measurements in the Gulf of Mexico. Specifically, we shall present (a) the dependence of EWG properties on the bandwidth of wave spectra, (b) the mean envelope shape of EWG's, exhibiting distinct differences from the envelope shape of regular beats.

1. Introduction

The study of wave groupiness has gained increasing attention in the past decade, because of its importance for long-period excitations to marine fixed and/or floating structures. Statistical analyses of ocean waves in storms (Rye, 1974; Goda, 1976; Cavanie and Ezraty, 1978; Su, et al., 1982) have firmly established the dependence of wave groupiness, such as its distributions and mean length, on the associated spectral bandwidth. Theoretical analyses (Longuet-Higgins, 1983; Kimura, 1980; Goda, 1970, 1983; Nolte and Hsu, 1972) have further related these properties in terms of either the linear narrow-band Gaussian process, or Markovian process. On the other hand, the most important classical design factor for marine structures is the estimation of extreme waves which are usually assumed to occur singly in nature (see Ochi, 1982 for a review on this subject). It can be convincingly argued, however, that a wave group containing an extremely large wave, if exists, would be a even more appropriate environmental design factor than either the single extreme wave or the "regular" wave group. For the sake of easy reference in this study, we shall call this special type of wave groups, each containing an extremely large wave (in a sense to be defined more precisely later) by the name of "extreme wave groups" (EWG) in contrast to the "regular wave group" (RWG) which is normally defined to be a sequence of successive waves whose heights all

exceed some prescribed critical value (H_c).

Goda (1976) seems to be the first investigator who published a statistical comparison of these two different types of wave groups. He noted that the mean length of runs for EWG is 2.4 when the critical value $H_c > H_{1/3}$, while the corresponding mean length for RWG is only 1.4. In other words, the highest wave does not appear singly, but is frequently accompanied by two to three high waves. Furthermore, the probability for the highest wave occurring in a group is even higher than regular high waves.

During the past decade, we have also witnessed a very intense research on nonlinear dynamical properties of finite-amplitude gravity waves (for a fairly complete review, see Yuen and Lake, 1982). One aspect of these researches, which is more pertinent to study of wave groupiness, is the side-band instability of two-dimensional wave trains/packets, and the formation of envelope solitons (Su, 1982; Zakharov and Shabat, 1972). There are strong statistical and dynamical evidences to support the assertion that wave groupiness in actual ocean waves, particularly in rapid growth stages in storm conditions, may be more likely produced by the side-band instability, rather than by the classical concept of linear beat phenomenon, which is conceptional basis for the narrow-band Gaussian process used in the majority of wave group modeling and analyses.

The purpose of this paper is two-fold: first, based on a large data set of ocean storm wave records, statistical characteristics of EWG will be described. Second, a discussion based on nonlinear wave dynamics will be presented to show that EWG are more likely caused by wave instability than simple beat phenomenon.

2. Analysis

Wave Data. The set of wave records used in this investigation was obtained by wave staffs from an offshore platform about 60 miles offshore in the Gulf of Mexico (Ocean Data Gathering Program (ODGP), Station #1) during 1969-1971 under the sponsorship of a consortium of eight oil companies (Ward, 1974). The station is located in South Pass Block 62A, at a depth of about 340 feet.

Each wave record is 19.85 minutes long and has been digitized into 4096 data points at the rate of 3.41 samples per second. For this analysis, we

selected a total of 533 wave records, each with the significant wave height ($H_s = H_{1/3}$) ≥ 2 meters in order to emphasize storm waves. These wave records were measured mostly in winter months, with the exception of during three hurricanes occurring in summer months. The eye of one of the hurricanes, Camille, happened to pass within 10 miles of the measuring platform.

Methods. The individual waves with their heights (H_j) and periods (T_j) for $j=1,2,\dots,N$ in each wave record are determined by the standard up zero-crossing method. In the set of wave records we used, the value of N lies between 100 and 240 with the average close to 200. The power spectrum $E(f)$ for each wave record is also computed by the Fast Fourier Transform, and is used in turn for computing two parameters indicative of the average bandwidth of the power spectrum. The first parameter was proposed originally by Longuet-Higgins (1983) and called the spectral bandwidth parameter, ν , and defined by

$$\nu = (m_2 m_0 / m_1^2 - 1)^{1/2}$$

where m_0 , m_1 and m_2 are spectral moments as defined by

$$m_i = \int_0^\infty f^i E(f) df$$

When $\nu^2 \ll 1$, the power spectrum is said to be narrow. The second parameter was proposed by Goda (1976) and called the spectral peakedness parameter, Q_p , and defined by

$$Q_p = \int_0^\infty 2f E^2(f) df / m_0^2$$

The value of Q_p is always greater than unity, and increases with decreasing spectral bandwidth. We have used certain specified ranges of ν and Q_p , independently, to classify the set of wave records into several subsets in order to see dependence of the statistical characteristics of wave groupiness on the spectral bandwidth.

Definition of Extreme Wave Group (EWG). For the purpose of our analysis which is constrained by the availability of each wave record being about 20 minutes long, the extreme wave group (EWG) is defined as the wave group containing the largest wave height in the wave record. For convenience of data analyses, we first assume that the EWG may cover up to seven waves and can be expressed as

$$W_{-3}, W_{-2}, W_{-1}, W_0, W_1, W_2, W_3$$

where W_0 is the wave with the largest wave height, the three waves with negative subscripts are waves preceeding W_0 , and those with positive subscripts are waves following W_0 . The corresponding wave heights, periods, wavelengths, and steepness are denoted by H_i , T_i , L_i and S_i ;

$$S_i = a_i k_i = (H_i/2) \cdot (2\pi/L_i) = \pi H_i/L_i$$

where a_i is the wave amplitude and k_i the wave number.

In the next section, we shall present the computed results which show the mean values (denoted with an overbar) of H_i , T_i and S_i for

subsets of wave records with prescribed ranges of Q_p or ν .

3. Results

The scatter diagram between the two spectral bandwidth parameters Q_p and ν for the 533 wave records is shown in Figure 1. The majority of the wave records has the values of Q_p between 1.3 and 4.0, or has the values of ν between 0.40 and 1.0. Based on this distribution, we have classified the wave records into the following subsets: (1) for Q_p

- (a) $1.0 \leq Q_p \leq 2.0$
- (b) $2.0 \leq Q_p \leq 3.0$
- (c) $3.0 \leq Q_p \leq 4.0$
- (d) $1.0 \leq Q_p \leq 5.0$ (the entire data set)

and (2) for ν ;

- (a) $0.40 \leq \nu \leq 0.50$
- (b) $0.50 \leq \nu \leq 0.60$
- (c) $0.60 \leq \nu \leq 0.70$
- (d) $0.60 \leq \nu \leq 0.80$.

Table 1 gives the mean values of the wave heights, periods and steepness of the extreme wave groups for each of the three subsets according to the spectral classification of Q_p , plus the entire

Figure 1. Scatter diagram of two spectral bandwidth parameters Q_p and ν .

Table 1. Statistics of the Extreme Wave Groups based on the Classification of Spectral Peakedness Parameter, Q_p

	$j = -3$	-2	-1	0	$+1$	$+2$	$+3$
(a) $\overline{H}_j$	2.335	2.510	3.016	6.284	3.389	2.238	2.322
$\overline{T}_j$	1.093	1.041	1.217	1.369	1.315	1.035	1.003
$\overline{S}_j$	0.152	0.142	0.144	0.186	0.122	0.150	0.143
(b) $\overline{H}_j$	2.491	2.735	3.943	6.335	3.931	2.677	2.361
$\overline{T}_j$	1.024	1.083	1.369	1.289	1.364	1.105	1.014
$\overline{S}_j$	0.154	0.137	0.125	0.195	0.138	0.136	0.146
(c) $\overline{H}_j$	2.667	2.997	4.325	6.352	4.435	3.156	2.704
$\overline{T}_j$	1.085	1.177	1.314	1.233	1.318	1.163	1.059
$\overline{S}_j$	0.122	0.117	0.133	0.208	0.128	0.142	0.136
(d) $\overline{H}_j$	2.506	2.753	3.864	6.325	3.929	2.699	2.418
$\overline{T}_j$	1.047	1.093	1.334	1.291	1.348	1.104	1.022
$\overline{S}_j$	0.153	0.134	0.129	0.196	0.134	0.139	0.143

Case (a) $1.0 < Q_p < 2.0$, Number of wave records = 87
 (b) $2.0 < Q_p < 3.0$, Number of wave records = 348
 (c) $3.0 < Q_p < 4.0$, Number of wave records = 91
 (d) $1.0 < Q_p < 5.0$, Number of wave records = 533

data set. We shall now study the content of this table in detail to bring out the important silent features which are rather remarkable. Note that the wave heights given here have been normalized by the root-mean square of wave surface displacement (m_0)^{1/2} of each corresponding wave record. The wave periods are normalized by the mean wave period of the corresponding wave record.

We shall start with case (d) in Table 1, which includes the entire set of wave records in order to see the overall mean characteristics first, before discussing their dependence on spectral bandwidth. From the mean values of wave heights, $\overline{H}_j$, one can see the envelope of the EWG is rather symmetric with respect to the highest wave height, H_0 . The mean wave heights for the three central waves are $\overline{H}_0 = 6.3$ and $\overline{H}_{\pm 1} = 3.9$, while the mean wave heights for outlying waves; $\overline{H}_{\pm 2} = 2.7$ and $\overline{H}_{\pm 3} = 2.5$, are much smaller. Recalling that the mean wave height for a linear narrow band Gaussian process is $H = \sqrt{2\pi} = 2.506$ (Arhan et al., 1976), we can infer, based on the height distribution alone, that (W_0, W_{+1}) is definitely distinctive from (W_{+2}, W_{+3}).

The mean wave periods for the three central waves are ($\overline{T}_0$ and $\overline{T}_{\pm 1}$) = 1.3, while the outlying waves are ($\overline{T}_{\pm 2}, \overline{T}_{\pm 3}$) < 1.1. Since the expected value of the wave periods for a linear random process would be unity, by definition, we find again that the three central waves (W_0, W_{+1}) are qualitatively different from the other waves. We also note that $\overline{T}_0 = 1.29$ is appreciably smaller than $\overline{T}_{\pm 1} = 1.34$.

The mean wave steepness for the highest wave is $\overline{S}_0 = \overline{a_0 k_0} = 0.2$, while the other waves are about 0.14. (The wave steepness for the highest Stokes waves is 0.443.) So, the highest wave turns out to be also the steepest, in the mean, a very significant feature in its own right.

Taking together the above observed properties of both wave heights and wave periods, we may assert with confidence that each extreme wave group, in the mean, contains three waves. Furthermore, if we use a threshold wave height $H_c = H_{1/3} = 4m_0^{1/2}$ for determining a wave group, then the mean (run) group length of the extreme wave group would be slightly less than 2.5. (Since $\overline{H}_{-1} = 3.86$ and

$\bar{H}_{+1} = 3.93$ are only slightly smaller than the threshold value of 4.0. W_{-1} and W_{+1} will have slightly less than 50% of chance exceeding the prescribed threshold.) This estimated value is thus in good agreement with the value of 2.36 found by Goda (1976). The mean wave group length for EWG is considerably larger than that of regular wave groups (≈ 1.5) (Goda, 1976; Su and Bergin, 1983).

Next, we shall examine the effects of spectral bandwidth on the above group characteristics by comparing cases (a), (b) and (c) against (d) in Table 1. As the spectral bandwidth becomes narrower, i.e., Q_p increasing from 1.0 to 4.0, or from case (a) to case (c), $\bar{H}_{+1}$ is found to increase from about 3.2 to 4.4, and $\bar{H}_0$ increases from 6.28 to 6.35, and $\bar{S}_0$ increases from 0.186 to 0.208, and $\bar{T}_0$ decreases from 1.37 to 1.23. Therefore, one may conclude that the specific features noted for the extreme wave group from the entire data set (case d) becomes even more prominent as the wave power spectra becomes narrower as parametrized by Q_p . In other words, for narrower spectral bandwidth, the mean group length will become longer, and the highest waves become steeper, and, increasingly, the

wave period of the central waves become shorter than those of the surrounding waves.

It is well known that power spectra have narrower bandwidth in the fetch-limited growth stages than in either the fully saturated stages or in decaying stages. (Here we have excluded the consideration of swells which could have extremely sharp narrow spectra for which the beat phenomenon can be suitably applied.) One thus expects to find most clearly the unique feature of the extreme wave group in the rapidly growing sea.

Table 2 gives the similar statistics of the extreme wave groups as given in Table 1 with the exception that the wave records are grouped by the spectral bandwidth parameter, ν . We found much the same group characteristics as noted in Table 1 in all three cases, (a), (b) and (c), for $\nu = 0.4-0.5$, $0.5-0.6$, and $0.6-0.8$, respectively, but the effects of spectral bandwidth based on ν is not as clear as based on Q_p . One possible reason for this relative sensitivity between the classifications of Q_p and ν is as follows: by its definition, Q_p emphasizes the contribution from the peak of the spectrum due to the square of $E(f)$, while ν , by its definition, considers equally the contribution from the higher frequency range due to the factor of f^2 in the m_2

Table 2. Statistics of the Extreme Wave Groups based on the Classification of Spectral Bandwidth Parameter, ν .

	$j = -3$	-2	-1	0	+1	+2	+3
(a) $\bar{H}_j$	2.586	2.823	3.971	6.320	3.993	2.739	2.404
$\bar{T}_j$	1.046	1.085	1.324	1.228	1.311	1.088	1.029
$\bar{S}_j$	0.131	0.132	0.122	0.213	0.125	0.140	0.129
(b) $\bar{H}_j$	2.537	2.701	3.703	6.233	3.780	2.532	2.429
$\bar{T}_j$	1.068	1.101	1.335	1.316	1.340	1.086	1.012
$\bar{S}_j$	0.148	0.120	0.109	0.169	0.120	0.128	0.140
(c) $\bar{H}_j$	2.310	2.653	3.895	6.347	3.976	2.874	2.478
$\bar{T}_j$	1.018	1.106	1.327	1.389	1.418	1.190	0.972
$\bar{S}_j$	0.140	0.136	0.125	0.177	0.119	0.122	0.159
(d) $\bar{H}_j$	2.557	2.653	3.982	6.420	4.098	2.890	2.429
$\bar{T}_j$	0.965	1.073	1.320	1.359	1.420	1.143	1.000
$\bar{S}_j$	0.139	0.145	0.133	0.193	0.122	0.139	0.156

Case (a) $0.40 < \nu < 0.50$, Number of wave records = 246
 (b) $0.50 < \nu < 0.60$, Number of wave records = 182
 (c) $0.60 < \nu < 0.70$, Number of wave records = 47
 (d) $0.60 < \nu < 0.80$, Number of wave records = 65

computation. In our determination of individual waves by the zero-crossing method which acts roughly as a low-pass filter, these individual waves thus obtained are primarily energy-containing lower-frequency waves centering around the spectral peak. Hence, Q_p will be a more sensitive parameter than ν in our analysis.

4. Discussion

Several unique features of the extreme wave group, particularly for the case with narrower spectral bandwidth normally expected for growing seas, prompt us to consider a hypothesis that these extreme wave groups may be caused by nonlinear wave instabilities, rather than simply due to the linear beat phenomenon. In this section we shall present some reasoning in support of this hypothesis.

The side-band instability is characterized by two perturbations with the frequencies $f_1 = f_0 - \Delta f$ and $f_2 = f_0 + \Delta f$, where f_0 is the original primary (unperturbed) frequency of a wave train and/or wave packet under consideration. Furthermore, the Δf is related to the wave steepness, $a_0 k_0$, of the original primary waves by

$$\Delta f = a_0 k_0 f_0.$$

Experimental observation (Su, 1982) showed that at the maximum modulation due to the side-band instability the amplitudes for the two side-band components, a_1 and a_2 , are approximately equal to each other, and further related to the amplitude, a_0 , of the primary wave at the particular moment by $a_1^2 = a_2^2 = 1/2 a_0^2$.

Hence, the time series of the surface displacement, $\eta(t)$, of the wave train under the maximum modulation of the side-band instability can be expressed as

$$\begin{aligned} \eta(t) &= a_0 \sin(2\pi f_0 t) + a_1 \sin(2\pi f_1 t) + \\ &\quad a_2 \sin(2\pi f_2 t) \\ &= a_0 \sin(2\pi f_0 t) \\ &\quad + \sqrt{1/2} a_0 \sin[2\pi(1-a_0 k_0)f_0 t] + \\ &\quad \sin[2\pi(1+a_0 k_0)f_0 t] \\ &= a_0 [1 + \sqrt{2} \cos(2\pi a_0 k_0 f_0 t)] \sin(2\pi f_0 t) \end{aligned}$$

Note that the expression in the square bracket is the slower varying envelope of the wave group, which have a repetition period equal to $(a_0 k_0)^{-1}$ number of primary wave periods. Now, the mean wave steepness for the case (c) with $3.0 < 4.0$ in Table 1 is equal to $\bar{S} = 1/3 (0.133 + 0.208 + 0.128) = 0.156$. If we take the average $S = 0.16$, then the repetition length of the wave group is nearly 6 waves. The corresponding wave heights for the seven waves centering around the maximum of the wave envelope are then given by

$$\begin{aligned} H_0 &= a_0 (1 + 2 \cos 0^\circ) = 2.41 a_0 = H_0 \\ H_{\pm 1} &= a_0 (1 + 2 \cos 60^\circ) = 1.707 a_0 = 0.7 H_0 \\ H_{\pm 2} &= a_0 [(1 + 2 \cos 120^\circ)] = 0.293 a_0 = \\ &\quad 0.121 H_0 \\ H_{\pm 3} &= a_0 [(1 + 2 \cos 180^\circ)] = 0.41 a_0 = 0.170 H_0 \end{aligned}$$

The most significant feature of the distribution of the above wave heights is the considerable smallness of $(H_{\pm 2}$ and $H_{\pm 3})$ in comparison to $(H_0$ and $H_{\pm 1})$. This feature agrees remarkably well with the observation of the extreme wave group consisting of three high waves, in the mean, as presented in the last section. The observed fact that $T_0 < T_{\pm 1}$ in Table 1 is also consistent with the experimental observation in the side-band instability.

On the other hand, let's consider a wave train consisting of two sine waves with equal amplitudes but with a difference in frequencies by an amount of $\Delta f = a_0 k_0 f_0$ the same as in the side-band instability. The time series of the corresponding surface displacement for the beat phenomenon will be

$$\zeta(t) = a_0 \sin(2\pi f_1 t) + a_0 \sin(2\pi f_2 t)$$

with $f_1 = f_0 - 1/2 \Delta f$, and $f_2 = f_0 + 1/2 \Delta f$. Hence,

$$\zeta(t) = a_0 [2 \cos(\pi a_0 k_0 f_0 t)] \sin(2\pi f_0 t)$$

The envelope for the beat phenomenon is a simple cosine shape with a repetition period twice longer than the period for the side-band modulation. With the same value of $a_0 k_0 = 0.16$, the seven waves centering around the maximum of the envelope are, approximately,

$$\begin{aligned} H_0 &= 2a_0 \cos 0^\circ = 2a_0 = H_0 \\ H_{\pm 1} &= 2a_0 \cos 30^\circ = 1.732a_0 = 0.866 H_0 \\ H_{\pm 2} &= 2a_0 \cos 60^\circ = a_0 = 0.5 H_0 \\ H_{\pm 3} &= 2a_0 \cos 90^\circ = 0 \end{aligned}$$

Note that $H_{\pm 2} = 0.5 H_0$ in this case is much larger than the corresponding values for the side-band instability. Of course, when Δf used smaller than the $a_0 k_0 f_0$, the value of $H_{\pm 2}$ will increase, and vice versa.

5. Conclusions

We have analyzed a large set of ocean wave records mainly in storm conditions for the structure of wave groups each containing the highest wave height in that wave record, called the extreme wave groups. We found that the extreme wave group predominantly consists of three high waves with heights greater than the significant wave height, and with the mean periods equal about 1.3 times the average period for the entire collection of individual waves determined by the standard up zero-crossing method, and with the mean wave steepness for the highest wave equal to about 0.20. Furthermore, as the spectral bandwidth becomes narrower, these features of the extreme wave groups become more prominent. Based on a previous knowledge of experimental studies on nonlinear side-band instability of finite-amplitude waves, the extreme wave groups are found to be explainable better as the manifestation of the maximum modulation under this type of nonlinear instability, rather than the simple linear beat phenomenon.

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