TRANSFORMATION OF STORM WAVE SPECTRA IN SHALLOW WATER
OBSERVED DURING THE ARSLOE STORM

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ABSTRACT. Wave data collected along a line of gages from 36 m to 2 m depths are used to study transformation of a storm wave field. The equilibrium range form proposed by Kitaigorodskii et al. is shown to fit the data. Spectral components on the forward face of the spectrum are shown to have marked increases during onshore winds.

INTRODUCTION. Measurement of a storm wave field as the waves propagate from deep water to the beach was one of the objectives of the Atlantic Remote Sensing Land Ocean Experiment (ARSLOE). During 23-25 October 1980 an extratropical low moved directly through the experiment area and created waves that peaked at a 5 meter significant wave height. Continuous wave records were collected at 14 sites supplemented by less frequent records at other sites. The data documents the transformation from a depth of 36 m to less than 2 m under unusually energetic conditions.

Emphasis is placed on changes in the energy spectrum at different points in time and depth. Analysis of the shape of the spectrum above the peak frequency will be made. The change in energy density in different frequency bands as the energy propagates from one depth to another will be examined.

EXPERIMENTAL SITE. The experiment was conducted offshore of the Coastal Engineering Research Center Field Research Facility at Duck, NC. (Fig. 1). The bathymetry consists of a gently sloping region from 36 to 25 meters in depth over a distance of 24 km. The bottom is nearly featureless with gentle undulations that are only a few percent of the depth. From a 25 meter to 10 meter depth (11 km), the slope is about 0.1% and there is a small oval shoal some 10 km offshore of the FRF which may be expected to produce minor refraction effects. From the 10 m depth to the shore the slope is 1% but the bathymetry in the vicinity of the FRF pier (Fig. 2) is complex due to scour. Sediment samples from the pier offshore have a median diameter of about 0.25 mm.

WAVE GAGES. The instruments deployed consisted of both in-situ and remote sensing systems. Here the in-situ devices providing surface energy spectra at various distances from shore are used (Fig. 1 & 2). The line of gages begin with the XERB buoy located 36 km from shore in about 36 m of water, with Waverider buoys deployed at decreasing depths to within 1/2 km of the pier-end. Seven Baylor resistance-type gages are situated along the pier. Extensive pre- and post-experiment calibrations of the gages

were performed.

DATA ANALYSIS. Data from the Waverider and Baylor gages were collected at the FRF by a NOVA-4 computer system. Each wave record consists of 4096 points collected at 4 Hz and is checked for bad data points, passed through a cosine-bell data window and processed through Fast Fourier Transform. Eleven spectral lines are merged into each spectral band of width 11/1024 Hz.

WAVE AND METEOROLOGICAL CONDITIONS. On October 24, a low pressure system had formed off the Georgia/Florida coast migrating slowly to the north. The following day, the system passed directly over the experiment region and by October 26 had left the area. Wind data collected hourly at the FRF pier building and at XERB is presented in Table 1 for the period of largest waves. Coincident wave data are also presented. The data show that the wind speed does not increase dramatically; however, wave height gradually increases to a peak at 1200 GMT on 25 October. Analysis of winds offshore 100 km or so indicates that the winds may have peaked near 20 m/s. Analysis of wind direction indicates that from 700 to 1000 GMT wind directions were fairly close both onshore and off. Between 1000 and 1100 GMT a front passed the FRF and between 1100 and 1200 passed the XERB buoy. After 1200 GMT the wind blew offshore.

SHALLOW WATER SPECTRA: EQUILIBRIUM RANGE. In deep water wave generation, the energy spectra at frequencies above the spectral peak attain a characteristic shape. Several investigators have proposed frequency power law forms to describe this equilibrium range (Table 2). Although the physical justifications for these power laws vary, they provide approximations to the spectrum in this range (Fig. 3) and there is evidence that there may be subranges in which they may each be applicable (Forrestal, 1981).

In shallow water Kitaigorodskii et al. (1975) proposed that the Phillip's theory must be modified to replace the deep water with the finite depth dispersion relationship. Kitaigorodskii et al. obtained:

$$E_m(f) = b g^2 (2\pi)^{-4} f^{-5} \Phi(\omega_h) \quad 1$$

as the limiting energy density where b is a constant approximately 0.0081, Φ is a dimensionless function varying monotonically between 0 and 1, and

$$\omega_h = 2\pi f (h/g) \quad 2$$

where h is the water depth. When ω_h is less than

one ϕ is approximately $\frac{1}{2} \omega_h^2$ and

$$E_m(f) = \frac{1}{2} b g h f^{-3} (2\pi)^{-2} \quad 3$$

If the Kitaigorodskii et al. form is valid, it is expected that then the shallow water equilibrium range form equivalent to JONSWAP would be

$$E_m(f) = J(f) \phi(\omega_h) \quad 4$$

where $J(f)$ is the JONSWAP form because it is proportional to Phillips form.

The f^{-3} dependence in shallow water is seen in field data (Kitaigorodskii et al. 1975, Thornton, 1977) and laboratory data (Ou, 1980). However, there is other evidence that it may not hold universally (Black, 1978, Sawaragi and Iwata, 1980). The ARSLOE storm data set provides an opportunity to examine the equilibrium range under high energy conditions at a range of progressively shallow depths. Here, all the spectra are single peaked.

The analysis of spectral shape was made by assuming that the spectrum from the peak, f_p , to twice that frequency is described by eq. 1. The hypothesis is tested through a regression analysis. If the observed spectrum is denoted by $E(f)$, the hypothesis requires that

$$E_o(f) = b g^2 f^{-5} \phi(\omega_h) (2\pi)^{-4} \quad 5$$

in the range f_p to $2f_p$ or that

$$g^{-2} (2\pi)^4 f^5 E_o(f) = b = \text{constant}. \quad 6$$

Since $E_o(f)$ is a statistical estimate, estimation of b at each frequency provides slightly different values; therefore, b will be presumed to be a function of f . If a relationship other than eq. 1 holds and has a variation with frequency, $b(f)$ will not be a constant but will have a correlation with f .

For each spectrum eq. 6 is formed and $b(f)$ computed. A linear regression is then performed of $b(f)$ against f and the slope and correlation calculated. For hypothesis to be valid, the correlation (and thereby explained variance) should be zero. The hypothesis on correlation can be tested with a t-test.

Restraint of the range to $(f_p, 2f_p)$ is justified on a number of grounds. Kitaigorodskii et al.'s equation is valid only for a linear, finite depth dispersion relation; above $2f_p$, harmonics of the spectral peak are expected to be important. The effect of the dimensionless function ϕ is to act more strongly for low frequencies than for high frequencies. Testing at too high frequencies weights the analysis to a deep water condition.

The results of the correlation analysis are summarized by gage in Table 3. Also shown are results for an analysis equivalent to eq. 6 based on the Phillips form and on equation eq. 3. The percent variance explained (correlation squared times 100 percent) was calculated for each spectrum and then averaged by gage and proposed equilibrium range form. In most cases 10 spectral frequency bands were in the range f_p and $2f_p$. For 10 bands the correlation coefficient must be less than 0.63 (or explained variance less than 40%) to accept

the hypothesis that b is a constant based on a 5 percent level of significance.

For the gages 710, 630, 620, and 625 the average value of the explained variance is less than 40 percent for the Kitaigorodskii et al. form or its f^{-3} asymptote. When each spectrum analysis for these gages is examined individually the hypothesis for the Phillips form is rarely accepted. Because the wind speed does not vary much the Toba form was not expected to vary much from that shown in Fig. 3. It appears an inadequate description of the shallow water spectra. The Kitaigorodskii et al. equation describes the shallow water spectra for both active sea and decaying sea during the storm better than any other form.

For gages in less than a 9 meter depth, results in Table 1 indicate that none of the proposed forms consistently fit the data. Examination of the bathymetry around the FRF pier (Fig. 2) suggested an explanation. The bathymetry is dominated by a large scour hole. Considering the response of a wave field propagating into the hole suggests that the lower frequency region of the spectrum will see a reduction in energy level due to refraction while the higher frequency components are initially unaffected. Nearshore where the bottom slope becomes steep, the lower frequency waves shoal up reaching saturation again. The intermediate frequency waves will still see refractive effects and there can still be higher frequency waves unaffected by the hole. Such a f^{-3} , non- f^{-3} , followed by a f^{-3} region is indeed observed (Fig. 4). The results seem highly influenced by refraction but still retain vestiges of a Kitaigorodskii et al. form.

VARIATION OF THE EQUILIBRIUM RANGE CONSTANT. One output of the analysis is the mean value of b . According to Kitaigorodskii et al. b should have a value near 0.0081. ARSLOE results suggest that b is different from 0.0081 and increases with decreasing depth (Fig. 5) for both the active and decaying wind sea cases.

SHALLOW WATER SPECTRUM: FORWARD FACE. Studies of deepwater wave growth have shown that most of the net growth is at frequencies near and lower than the peak frequency - the forward face of the spectrum. Hasselmann (1962) provided an explanation in terms of the transfer of energy by resonant nonlinear wave-wave interactions. In shallow water refraction, shoaling, bottom friction, percolation and direct atmospheric processes may be important. An analysis attempting to balance all these mechanisms is most difficult. With the ARSLOE storm data, the gross changes in the forward face and at the peak frequency are analyzed to see if any patterns emerge that can guide more detailed analyses.

The data are summarized in the following way. At 710, 630, 620, and 625 wave spectra were calculated from the first 1024 seconds of a 20 minute wave record every 20 minutes from 700 to 1530 GMT. To increase statistical stability, a sequence of wave spectra was then constructed by a running average of 3 consecutive spectra. Consequently every 11/1024 hz frequency band has approximately

65 degrees of freedom. For XERB data obtained once per hour the number of degrees of freedom are 20.

The assumption is made that the XERB record typifies the spectrum of the wave field propagating toward the coast. This seems justified by size of the storm, general flatness of the bathymetry, and the analyzed wind characteristics. It takes approximately 1 hour for the waves to propagate from XERB to gage 710 and another 30 minutes so to reach the shore assuming a direct path of propagation. Given a value for $E(f)$ at XERB it is possible to estimate the corresponding times at which the energy should reach the other gages. For a frequency it is possible to plot the ratio of energy at a gage site to that at XERB. This was done for frequencies of .08, .09, and .10 hertz. Also plotted is an estimate of refracted and shoaled component based on a directional spectral refraction model with monotonically decreasing depth, thereby ignoring the small shoal between gage 710 and 630.

From 720 to 1020 GMT on October 25, the frequency f_m was 0.10 hz. At all times during active wind growth, frontal passage, and offshore wind the pattern of change in this band was for a decrease with depth. (Fig. 6-A). At the 10 meter depth, the values were normally 40% or less of the initial value and considerably less than the refracted and shoaled value. For this gage the 90% confidence bands are about $\pm 20\%$. The probability that the initial spectral estimate at XERB is this low is less than 5% given a chi-square distribution. The pattern of change is consistent however with this spectral band reaching the Kitaigorodskii et al. form with decreasing depth. It is also possible to postulate that the loss may be due to bottom friction and percolation, although such a loss would be somewhat dramatic.

Figure 6-B provides the results for the .09 hertz band. During the early periods of the active wind growth (720 and 920 GMT), the energy at the intermediate depths (15 to 25 m) are 200% to 300% of the initial values at XERB. In the 1020 GMT case the maximum increase is of the order of 170%. It appears that the spectral value in these cases increases as depths initially decrease, then eventually is clipped. In the 1220 and 1320 cases the wind is turning to an offshore direction and becomes uncoupled to most of the energetic portions of the spectrum. The spectral densities decrease in these cases in a pattern not too different from the 0.10 hz case.

Analyses of confidence bands for the spectra suggest that the increases observed are not likely due to statistical instability. In the 720 GMT case, if the XERB value is assumed accurate, the probability that the gage 710 ratio is about 2 is less than 1%. If the gage 710 value is correct, then the probability that the XERB values are mis-estimated too low is less than 5%. Likewise a combination of both values are off to give the observed value in general leads to probability of less than 5%. The only other possibility is for the XERB gage to be biased too low with the observed value an accidentally high value. This seems contradicted by the data because it would imply that for sea cases the observed value is always far too low but the swell case it is far too high. It would also suggest that the

frequency response of the buoy tended to be opposite for .09 hz and a .10 hz band. Fig. 6-C indicates that the same trends shown for the .09 case are evident for .08 hz. Thus, it appears that the growth of the forward face of the spectrum during active wind growth is real and statistically significant. In all cases, the pattern of spectrum evolution appears to significantly deviate from the patterns suggested by refraction and shoaling.

It is beyond the scope of this paper to try and balance all the diverse source terms. However, a few observations are warranted. If the loss of energy in the .10 hz band is dominated by bottom friction or percolation, it is difficult to understand the very large growth rates seen at .08 hz and .09 hz bands where bottom friction is expected to be greater. Two mechanisms have been proposed to add energy - nonlinear wave-wave interactions and direct atmospheric input to the component. This portion of the frequency spectrum if it were deep water would not be expected to receive direct input. In shallow water however the phase speed of the waves approaches $\sqrt{gh}$ which for 25 meter depth is 16 meters/second and for 17 meters is 13 meters/second. Thus for winds of about 15 m/sec and water depths of 17 meters or less all of the spectrum may receive direct atmospheric input.

SUMMARY AND CONCLUSIONS. The ARSLOE storm provided an extensive data set by which to examine several aspects of wave transformation across decreasing water depths. The major conclusions that can be drawn from the data set are (1) that the equilibrium range appears described by the form suggested by Kitaigorodskii et al. if the b-coefficient is allowed to vary, (2) during the period of active wind growth examined, energy levels markedly increased on the forward face of the spectrum, and (3) during the period of decaying wind sea the energy levels appear to decrease on the forward face. The applicability of the Kitaigorodskii et al. form is of great usefulness in wave modeling and allows development of simple methods for estimating depth limited wave energy (Vincent, 1981; Grosskopf and Vincent, 1982).

More fundamental, the patterns of change in the spectrum do not appear to follow the classic patterns of refraction, shoaling and losses to bottom friction and percolation. Consequently, engineering techniques that are based on such an approach may be in substantial error particularly if there is active wind growth. Considerable effort is still required to understand the physics that control spectral evolution in shallow water. Additional data sets need to be examined or collected to obtain variations in bottom conditions and slopes to understand the processes under a wider range of geophysical conditions.

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TABLE 1
Winds and Waves
October 25, 1980

TIME (GMT)	NEARSHORE WIND		OFFSHORE WIND		WAVE HEIGHT ³	
	SPEED ¹	DIRECTION ²	SPEED ¹	DIRECTION ²	XERB	620
0300	16.1	238.5	13.0	258.1	3.13	
0400	16.1	238.5	13.3	257.0	3.26	
0500	13.5	238.5	13.7	253.1	3.15	2.77
0600	11.9	238.5	10.7	261.6	3.27	3.10
0700	11.4	261.0	10.4	266.5	3.42	3.20
0800	13.5	283.5	11.6	287.1	3.96	3.10
0900	14.6	283.5	12.4	288.6	3.72	3.70
1000	14.0	306.0	14.1	308.2	4.53	3.70
1100	8.8	13.5	16.2	311.8	4.98	4.01
1200	9.9	13.5	12.7	348.6	5.06	3.80
1300	10.4	36.0	12.8	14.8	4.36	3.30
1400	8.8	13.5	14.4	42.0	4.35	3.10
1500	9.9	13.5	14.1	39.4	4.21	2.90
1600	10.4	13.5	13.9	31.1	3.29	--
1m/s ² 0 _{TN} 3 _m						

TABLE 2
DEEP WATER EQUILIBRIUM RANGE FORMS

Phillips (1958) $E_m(f) = bg^2 f^{-5} (2\pi)^{-4}$

Hasselmann et al. (1973) $E_m(f) = bg^2 f^{-5} (2\pi)^{-4} \times (-1.25 (f/f_m)^{-4} \times \gamma \exp(-\frac{1}{2}(f - f_m)^2 \sigma^{-2} f_m^{-2}))$

Toba (1973) $E_m(f) = bguf^{-4}$

TABLE 3
REGRESSION ANALYSIS RESULTS FOR PERCENT VARIANCE EXPLAINED

GAGE	MEAN			f ⁻³
	DEPTH	PHILLIPS	KITAIGORODSKII, et al.	
XERB	36	43	17	48
716	25	53	28	8
630	18	46	13	10
620	15	55	24	18
625	9	68	35	32
655	5	70	60	59
615	2	72	60	59

A value greater than 40% is required to reject hypothesis that there is no correlation with frequency.

Figure 1. Location of Offshore Buoys. Bathymetry is shown in units of feet.

Figure 2. Gages and Pier Bathymetry. Depths are given in units of meters.

Figure 3. Equilibrium Range Forms. Coefficients have been taken to maximize fit.

Figure 4. Plots of Normalized Spectra

Figure 6. Changes in Energy Density in Selected Spectral Bands with Depth. Refracted and shoaled value given by dashed line.

Figure 5. Variation of b - coefficient.