NUMERICAL MODELLING OF COAST GUARD BUOYS IN SHALLOW WATER

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ABSTRACT

This paper describes recent attempts by the Coast Guard R&D Center to analyze the dynamics of typical Coast Guard buoys moored with synthetic line in shallow water, using an existing numerical model. Large-scale model wavetank tests have shown that the weight and drag of the synthetic line are negligible when compared to tension forces, so that catenary effects may be ignored. This allows the use of a single linear segment in the modelling of the mooring line, thus eliminating numerical convergence problems commonly experienced in time-domain simulation of shallow water dynamics. A unique system for measuring buoy position in wavetank tests is discussed. The procedure for abstracting buoy hydrodynamic coefficients from wavetank data is outlined, along with the current capabilities of the numerical model to estimate mooring line tensions and buoy attitudes in various wave conditions.

INTRODUCTION

For several years both the Ocean Engineering Division of U.S. Coast Guard Headquarters and the U.S. Coast Guard Research and Development Center have been investigating the use of synthetic fiber lines in buoy moorings. In December of 1978 these efforts were joined with a request from the Ocean Engineering Division that the R&D Center develop a practical, analytic computer model of navigational buoys moored with a synthetic line. This development was to go hand-in-glove with practical experience that the Ocean Engineering Division would achieve by the actual use of the lines in moorings.

In particular, the Coast Guard is seeking a computer design model which will determine the vector force at the anchor for Coast Guard buoys in conditions common to their usage. These buoys range in size from 1.5 feet in diameter by 10 feet high to 9 feet in diameter by 36 feet high. The condition of usage varies from rivers to exposed coastal environments.

The model will be used in three different ways:
(1) to determine the cost (saving) of a switch

from chain moorings to synthetic line mooring, (2) to develop a field design manual for non-engineers, and (3) to assist engineers in cases where the field designs are ineffective.

WAVETANK TESTING

An essentially classical approach has been taken in developing the computer model: a literature search to identify possible existing models, preliminary data gathering to determine if existing models are adequate or if independent development is required, final model development, and validation. This paper deals primarily with the first three. The literature search identified the two general types of solutions as frequency domain and time domain and the potential difficulties with both, namely the linearizing assumptions necessary in the frequency domain and the potentially high cost per run and shallow-water resonance problems with a time domain model.

In order to provide a data base to select and evaluate computer models, a preliminary data gathering experiment was prepared. The primary goal of this data gathering was to provide experimental information for representative cases as a guide to computer model selection.

The tests were conducted at Oregon State University's Wave Research Facility. The test scheme was to tow the buoy into the waves by the mooring line thus simulating at-sea conditions as shown in figure 1. The primary measurements of interest to this project were the force measurements at the anchor. The x and z components of this force were measured using two strain gauges mounted orthogonally on a beam. In addition, a scalar force was measured with a ring gauge at the mooring attachment point. Vertical acceleration, pitch, and roll of the buoy were measured using a Humphrey gyro stabilized accelerometer and the x,y,z location of the buoy was derived from length readings of three string potentiometers located on the moving carriage. Wave profiles were taken using a sonic profiler travelling with the tow carriage in the general area of the buoy.

All ten channels of data were recorded on analog tape, simultaneously digitized at 20 samples per

second, and recorded. Thus for each test run, a time record of forces, locations, and buoy attitude was taken.

Three different buoys were tested representing the major types of Coast Guard buoys. Each buoy was evaluated under straight tow conditions, in waves with no tow, and towed into waves. Approximately 80 test runs with various wave heights, wave periods, tow speed, and mooring lengths were conducted on each buoy.

The test buoys were Froude scale models of existing buoys. Mooring line sizes were selected to nearly Froude scale the dynamic load-elongation properties of a full-scale case. This required cyclical testing of all lines on the Coast Guard R&D Center's dynamic test machine. After the model testing, the lines were subjected to cyclical loads similar to the experiment and proper load elongation relationship were developed for use in the computer model.

The data as a whole suffers from three major insufficiencies:

1. X, Y and Z position data was not always reliable due to buoy yaw. Buoy heading was initially considered as a possible measurand of the experiment but was discarded for a number of technical reasons. This selection was unfortunate because many times the buoy did not tow with the gyro roll axis pointing along the tank. The gyro thus measured some pitch as roll

and vice versa. The correct orientation of the gyro is necessary to calculate the motion of the buoy since the string potentiometers were not connected at the center of gravity or at the mooring attachment point. On each run, notes were taken on the heading of the buoy. On the majority of the tests, the data taking portion of the run was filmed.

2. Real-world effects (difference in waves, wave reflection, difference in buoy position, etc.) caused a large deviation in the force measurements between waves. Typically, the standard deviation of the peak tension measurements during any test was 15 percent of the average peak tension. Deviations were unusually large during tests with low currents and large waves.
3. Dropouts in wave data. Interference from the carriage drive motor caused difficulties with the digital record of the sonic wave profile.

Despite these problems, the data were more than adequate to select between the program types. The frequency domain approach was considered first by virtue of its potential economy.

FREQUENCY AND TIME DOMAIN MODELS

A frequency domain program generally calculates a digital approximation of the exact solution to a

FIGURE 1. Test Schematic

set of linearized differential equations describing the buoy mooring. Various schemes are available to avoid the restriction to purely linear damping forces, and frequency domain solutions allow the use of frequency-dependent hydrodynamic coefficients and random sea inputs. To achieve this, however, it is almost universally assumed that sinusoidal buoy and mooring line outputs result from sinusoidal wave inputs, and furthermore that the total dynamic response to waves and current can be viewed as a small perturbation about the static (current only) configuration.

Quantitative testing of such a small perturbation assumption is difficult; however, a qualitative evaluation of sinusoidal response superimposed on the static solution was undertaken.

This evaluation showed:

1. The tension responses in the test moorings were not sinusoidal.
2. The dynamic component was not a perturbation about the static load since in 75 percent of the cases, the mean load (average of maximum and minimum) was more than twice the tow only (static) condition.
3. The amplitude of the tension is very large when compared to the mean. In 75 percent of the cases, the amplitude tension (one-half dynamic range) was greater than 90 percent of the average load.
4. Minimum tensions of zero are common. Twenty-nine percent of the cases encountered tensions of zero during the wave trough.

Thus the area of interest is not well described by solutions assumed by a frequency domain model. Further consideration of such models was therefore discontinued and attention was concentrated toward existing time domain solutions.

It was immediately recognized that evaluation of the time domain approach would require full operation of a model. The efficiency and accuracy of the model chosen would govern future work. The model selected was one developed by Dr. Nath (1) to analyze drifting buoys.

The Nath program is a two-dimensional lumped mass model which features empirical damping input, choice of linear or stream function wave theory and power function model of line elasticity. The program is buoy shape specific and requires multiple input coefficients in its calculations. These are (1) laminar and turbulent hydrodynamic drag coefficient for each longitudinal buoy section, (2) added mass coefficient for each longitudinal section, (3) Froude drag coefficient for each longitudinal section that may pierce the water, (4) one hydrodynamic and one Froude drag coefficient for axial motion, and (5) an axial added mass coefficient.

COEFFICIENT DEVELOPMENT

Textbook values for drag and added mass could have been used as input if only the economy of operation were to be tested. However, previous experience (1) indicated that accurate results would require empirical development of these coefficients. This need had been anticipated and considerable effort was made during testing to develop the required data. Three types of tests were available in selecting coefficients (1) heave impulse response tests, (2) pitch impulse response tests, and (3) tow-only tests at several speeds.

To select these coefficients, the program was first set up to model the two types of impulse response tests. With approximate values for drag and added mass coefficients, the simulated pitch and heave frequencies were within about 20% of actual but the damping prediction was unsatisfactory. The heave response, with the buoy moving purely axially, is independent of all coefficients except axial hydrodynamic and Froude drag and axial added mass. As expected, varying the latter coefficient provided a continuous adjustment of heave frequency so only a few runs were required to determine its proper value.

The heave impulse response test alone was not sufficient to clearly distinguish between the vertical Froude and hydrodynamic drag, so no determination of an empirical formula for the Froude force could be made. Since a linear heave Froude drag versus velocity relationship had been used successfully by Nath (1), this was retained, and the hydrodynamic and Froude coefficients manipulated until the program accurately portrayed the actual heave damping.

Since so many of the lateral direction coefficients have an influence on the pitch response, it was felt that the tow test data should be considered next. This would provide a better idea of the drag coefficients. The lateral added mass coefficients theoretically have no influence on steady-state tow response, so these could be tuned later.

To determine the Froude drag relation, all the velocity versus tension data was gathered together and polynomials of different degrees were regressed onto it. The best fit was given by a fourth degree polynomial with very small odd order contributions. This was slightly simplified to

$$F_D = 1.41 V^2 + 0.13 V^4$$

where F_D = total drag force and V = tow velocity.

Since one well-known source (2) indicated that Froude drag might be proportional to V^4 , the regression results seemed encouraging. Next, a check was made to see if the V^2 coefficient (1.41) resulted in a reasonable hydrodynamic drag coefficient. The underwater projected area of the

buoy was calculated for each tow tank run (using the draft data collected), and averaged. The result was a coefficient C_D of 0.58. As an overall figure, this compared quite favorably with drag data collected previously on a full scale 8x26 buoy (3).

Several program runs were then made using a V^4 -dependent Froude drag formula and the coefficients obtained above. Very good tension results were obtained. It was also found that by varying the contributions of individual sectional coefficients to the total hydrodynamic drag coefficient of 0.58 the pitch attitude of the buoy could be properly simulated as well.

Attention was then returned to the pitch step response. When the lateral added mass coefficients were adjusted to obtain the proper frequency of oscillation, the damping was surprisingly close - within 10 percent.

In an attempt to further refine coefficient values, it was found that adjustments were in many instances interactive. This made it very difficult to choose a logical sequence of changes which would optimize the program's simulation of all the tests. Furthermore, the finite resolution of the data acquisition system and the unavoidable experimental scatter impose a noise floor, making adjustment beyond a certain point a questionable undertaking.

The hydrodynamic and viscoelastic properties of the mooring line as summarized in drag and added mass coefficients and a stress-strain relation are also necessary program inputs. Sensitivity tests showed that with lines of small diameter and short length, as used in the scale model tests, even wide variations in drag and added mass coefficients had little effect on program predictions. Therefore, an arbitrary value of 1.0 was used for both in simulations of the wavetank experiments. Some full size buoy-mooring configurations of interest will undoubtedly require more precise data.

The program's sensitivity to the elasticity of the line, on the other hand, is substantial. This makes it especially important to have accurate stress-strain relations in any validation attempt. As mentioned previously, the lines used in the wavetank tests were tested under loadings similar to those actually experienced. These took the form of rather narrow width triangular tension pulses separated by longer periods of low tension. When compared to results obtained while using pure sinusoidal loading, stiffness increases of as much as 20 percent were occasionally recorded. Further work in this area with a variety of line types would be very useful.

It should be pointed out that characterizing the line with a simplified stress-strain relation, such as the presently used formula $T = a\epsilon^b$, completely overlooks the hysteresis effect prevalent in most synthetic lines. The three-parameter

Maxwell model, which does account for energy storage, would be a better choice, but is difficult to implement efficiently in a time domain analysis. The tension history of the line must be known to use such a scheme, requiring a fair amount of additional computer memory.

OTHER PROGRAM INPUTS

While the buoy coefficients were being developed, insight was gained in two other areas where program options existed. First, because the program follows the usual procedure of modelling the mooring line as a collection of short straight line segments joined end-to-end there was the problem of deciding how many segments to use. Second, one of two wave theories could be used to determine water particle velocities and accelerations, as well as the actual waveform. Both areas were potentially important in determining the overall efficiency and accuracy of the model.

It is generally assumed that greater accuracy can be obtained when using a large number of mooring line segments since a larger number of line configurations can be approximated. This is especially important when higher order vibrational modes are of interest. On the other hand, computer costs increase very quickly with more segments, since in addition to the increase in calculations per time step, the time step must generally be made smaller to avoid instabilities. (The time step should be less than the transit time of a compression wave along the shortest segment.)

The solution to this dilemma, at least for the purposes of this project, sprang from close observation of the mooring line during the wavetank tests. There it was noticed that whenever the test lines were even slightly loaded the familiar gravity and drag induced catenary shape was quite flat. Within the limits of tension measurement accuracies, it also appeared that the tension forces at the buoy were close to those at the bottom of the mooring line. Subsequent analysis of the force beam data using Pode's tables verified that the difference in line angles along the length of the line was generally less than a few degrees when the tension was above 10-25 pounds (depending on the line size). Since the submerged weight of the line and its total drag force contribution were at least one, and usually several orders of magnitude smaller than the hydrodynamic forces on the buoy, it seems reasonable that the line would have very little effect on buoy motion except during those periods when the curvature was drawn out of it. As a result, it appeared that it would be possible to simulate the critical portions of the line's dynamic response with a single linear element.

Program runs were made as the buoy coefficients were being developed (and with their final values) using from 1 to 4 segments to model the line. Aside from a staggering increase in run time when using multiple segments, no substantial difference

in tension or buoy motion results were seen. Therefore, the use of a single linear element in the line modelling was adopted in all further work with the program and all the results presented later were obtained using this simplification.

The choice of wave theory presented a different sort of problem. Experimentation with the stream function formulation was restricted since it appears that coefficients in the determining expressions are tabulated only for a few "representative" waves. Since the major basis for a choice between theories was through comparison between computer and wavetank results and since none of the tabulated waves matched those used at the wavetank, no real conclusions have yet been reached. Although the overall tension results (presented below) using the linear theory are respectable, tests using stream function waves with parameters similar to those of waves seen at OSU often show more realistic buoy trajectories. Since the trajectory extremes are directly related to tension, it appears that for some waves the nonlinear stream function expressions might provide more accurate tension predictions. However, program runtimes are considerably longer using this theory so any future work will still have to address the cost/accuracy tradeoff.

COMPARISON OF COMPUTER AND EXPERIMENTAL RESULTS

With the buoy and line coefficients obtained as previously described, the program was run to simulate a wide variety of full dynamic (waves and current) situations. For purposes of preliminary comparison, a group of nine experimental tests involving the model 8x26-foot (a 1/4-scale model) buoy and the medium length mooring were chosen as representative.

Figure 2 shows the range of environmental conditions considered and the actual tension comparisons. (Both the experimental and simulation results use the average of the peak tensions seen during each wave cycle, hence the absolute maximum peak for any run is larger than the value shown.) In judging the degree of correspondence between experimental and simulated tensions portrayed in figure 2, it must be kept in mind that shortcomings in the experimental setup and normal experimental scatter prevent the assignment of a single tension number to a particular set of wave and tow conditions. Standard deviations about mean peak tension values were on the order of 15% of the mean for a typical run. In fact, two runs with virtually identical wave and tow conditions showed a difference of more than 10% in average peak tension. This sort of variability obviously makes it difficult to claim any level of accuracy for the computer program.

In general, the results shown in figure 2 are encouraging. There is a fairly broad range of environmental conditions represented by the middle portion of the figure over which good agreement between the computer model and the OSU test results has been obtained. There is, however, a

EXPERIMENTAL VS. COMPUTER MODEL AVERAGE PEAK LINE TENSION 8 X 26 FT. BUOY - MEDIUM MOORING

Figure 2. Experimental Versus Simulated Tensions

general tendency for the program to overpredict at the low tensions and underpredict at high tensions.

MODELLING PROBLEMS

The most serious problem encountered with the computer program is directly related to its discrete-time structure. While the approximations inherent in assuming a particular wave theory or line model can be smoothed out somewhat by suitable buoy or line coefficients, the only obvious way of diminishing the effect of time-stepping is to reduce the time step, thus increasing cost.

The program now uses a combination of predictor-corrector and Runge-Kutta integration algorithms which allow a dynamic adjustment of time step dependent on predictor-corrector differences. This procedure yields a satisfactory degree of accuracy and efficiency when the process changes slowly compared to the time step. However, when rapid movement of the buoy (as was seen in the model tests) is experienced, the program's assumption that this movement is constant over the time step can cause serious problems. While in real life the mooring line restraint on the buoy would increase with time as the distance between the buoy and sinker increased, the program simply calculates a new buoy position according to the mooring

line and water forces extant at the beginning of any time interval. This often leads to exaggerated buoy motion and, as the line tension is calculated directly from buoy position, exaggerated tension peaks as well.

The second problem inherent in a computer program operating in the time domain is the difficulty in validating or applying the program in random seas. As was pointed out earlier, system nonlinearities preclude the superposition of response to sinusoids. Since the program requires water particle velocities and accelerations of every space-time point that the buoy and mooring line will occupy, and these points are not known a priori, a sizeable quantity of information is required. This makes it difficult to use a preliminary analysis and tabulation of water motion within a measured or assumed complex waveform.

A finite approximation of the spectrum of a particular sea profile might be used to specify a reasonably sized collection of sine wave generators whose vector-added outputs would constitute the water particle velocities and accelerations. Such a scheme would be satisfactory with regard to computing and storage demands, but its efficacy from the modelling point of view is uncertain.

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