EFFECTS OF SURFACE REFLECTION ON BEARING TRACKER PERFORMANCE

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ABSTRACT

Recent at-sea testing of a passive sonar bearing tracker indicated that received signal propagation paths which include a surface reflection can result in a substantial increase in bearing error variance. Measurements at high signal-to-noise ratio showed excessive bearing error variance compared to both theory and other measurements when measured Depression/Elevation angle indicated a surface reflected path. It is hypothesized that wavefront scattering due to surface waves is the mechanism for this increase.

Based on a Pierson-Moskowitz surface wave spectrum, a simplified acoustic propagation model, the actual test geometry and the parameters of the broadband tracker, effects of surface reflection on tracker performance were analyzed. Resulting work yielded a simple equation which relates random error to surface reflection dependent on (1) test geometry, (2) wind velocity and direction, (3) spatial correlation coefficients between specular points, and (4) the phase center separation between half-beams. Results of calculations show good agreement with the experimental data.

1.0 BACKGROUND

A strong correlation between unusually high bearing error scatter (random error) and broadband target signals reflecting off the ocean surface was observed during a recent at-sea sonar tracking experiment. Figure 1 shows the bearing error as measured by the sonar bearing tracker, while Figure 2 shows the measured received Depression/Elevation (D/E) angle as measured by the D/E tracker. In Figure 2, the D/E angles above the centerline indicate a surface-reflected path, while D/E angles below the centerline indicate a direct path. A comparison of Figures 1 and 2 reveals that the unusually high bearing error scatter correlates with the surface-reflected path received signal. The sound velocity profile and the test geometry were consistent with both direct and surface-reflected paths. In addition, the environmental factors of a sea state 3 and 12-knot winds resulted in a rough ocean surface.

In light of the above facts, it is hypothesized that the large bearing error scatter at high re-

ceived signal strength is the result of surface-wave reflections. The remainder of this article consists of a simplified analysis of the surface-wave scattering problem and its effect on a hypothetical, broadband bearing tracker.

2.0 THEORETICAL ANALYSIS

Based on a Pierson-Moskowitz surface spectrum, a simplified acoustic propagation model, the actual test geometry and the consideration of the broadband tracking loop, effects of surface reflection on tracker performance were analyzed. Resulting work yielded a simple equation which relates random error to surface reflection as functions of (1) test geometry, (2) wind velocity and direction, (3) spatial correlation coefficients between specular points, and (4) the phase center separation between tracker half-beams.

Surface Reflection Propagation Model

Figure 3 represents the signal propagation model used for the analyses. The X, Y, Z coordinates were chosen such that the X Y plane at the axes' origin is parallel to the local horizon plane and coincided with the mean free surface (a surface which contains the mean wave height). The Z axis was chosen positive downward. A source at depth Z_g emitting an acoustic broadband signal, hit the surface at two points separated by a distance, Y, simultaneously; the reflected signal then reached receivers R_1 and R_2 representing the two tracker half-apertures. Because of the wave motion, however, the actual points of reflection were at $Z_1 = h(X_1, Y_1, t)$ and $Z_2 = h(X_2, Y_2, t)$ where $h(X, Y, t)$ represented the instantaneous random surface wave height. A fixed point on the surface, $h(X, Y, t)$, is defined by the Pierson-Moskowitz amplitude wave spectrum.

$$h^2(W) = .0081 g^2 W^{-5} e^{-\frac{.74g^4}{v^4 W^4}} \quad (1)$$

where v = wind speed in ft/sec
 W = spectral frequency in rad/sec
 g = acceleration constant

The wave action causes each ray to travel a distance which differs from the nominal path as shown in Figure 4 and is given by

$$d_1 = \sqrt{R^2 + (z_S + z_R + 2z_1)^2} - \sqrt{R^2 + (z_S + z_R)^2}$$

$$\approx 2z_1(z_R + z_S)/R$$

$$d_2 = \sqrt{R^2 + (z_S + z_R + 2z_2)^2} - \sqrt{R^2 + (z_S + z_R)^2}$$

$$\approx 2z_2(z_R + z_S)/R \quad (2)$$

where z_1, z_2 = instantaneous random surface wave heights

z_S = source depth

z_R = receiver depth

R = horizontal range between the source and receiver

In turn, different distances travelled produced a net time shift in the time of arrival (TOA) of the signal of the two half-beams. The time of arrival differences between the two half-beams are converted by the tracker into bearings. Thus, surface-wave induced time shifts will result in bearing fluctuations.

To carry the analysis further, the nominal reflection point, X_N , can be calculated from Figure 4 and is given below.

$$X_N = \frac{z_S}{z_R + z_S} \cdot R \quad (3)$$

Also, from Figure 5, the specular separation (Y) between nominal reflection points is approximated by

$$Y \approx (d/R) X_N$$

or, using Equation (3)

$$Y \approx \frac{z_S}{z_R + z_S} \cdot d \quad (4)$$

For a hypothetical test geometry and tracking array the following parameters can be assumed: Range $R = 6000$ ft, Source Depth $z_S = 400$ ft, Receiver Depth $z_R = 100$ ft, and Dipole Spacing $d = 5$ ft

Using Equation (2) to (4) one obtains the parameters

$$X_N = 4800 \text{ ft and } Y \approx 4 \text{ ft,}$$

as a basis for the subsequent numerical calculation.

The quantity h_{RMS} , the Root Mean Square value of the wave height, can be calculated by integrating the squared wave amplitude spectrum as given in Equation (1). Performing the integration and substituting the value for the acceleration constant ($g=32.17 \text{ ft/sec}^2$) yields the desired expression

$$h_{RMS} = 1.62 \times 10^{-3} V^2 \text{ (ft)} \quad (5)$$

Note: Surface spectrum has an effective downwind and crosswind direction with correlation lengths given respectively by reference (1) as

$$l_{DW} = .0385V^2 \text{ and } l_{CW} = .0769V^2$$

For example, at a sea state 3 with wind speed of 12 knots (20 ft/sec), which approximates the conditions during the experimental sea test, one obtains the numerical results $h_{RMS} = .65$ ft, $l_{DW} \approx 15.4$ ft, and $l_{CW} \approx 30.76$ ft. If the broadband acoustic noise sources were radiating into the wind, the specular reflection points for both receivers are highly correlated since

$$Y = 4 \text{ ft} \ll l_{CW} = 30.76 \text{ ft} \quad (6)$$

From Equation (2), the surface-induced time shift at a particular specular point is given by

$$\tau = 2 \cdot \frac{z_R + z_S}{R \cdot c} \cdot h(X, Y, t) \quad (7)$$

and the RMS value of τ (σ_τ) is given by

$$\sigma_\tau = 2 \cdot \frac{z_R + z_S}{R \cdot c} \cdot h_{RMS} \quad (8)$$

where c is the speed of sound in water

3.0 TRACKING LOOP ANALYSIS

Let the received signals at the two tracker half-beams be written as

$$R_1(t) = S(t - t_1 - \tau_1) + n_1$$

$$R_2(t) = S(t - t_2 - \tau_2 + \tau_D) + n_2 \quad (9)$$

where

S = target signal

t_1 = TOA at R_1

t_2 = TOA at R_2

τ_1 = Surface-induced time shift for receiver 1

τ_2 = Surface induced time shift for receiver 2

τ_D = half-beam feedback time delay from tracking loop

n_1, n_2 = noise received at each receiver.

Figure 6 provides a simplified block diagram of a broadband tracking loop. The function of the Time Delay detector is to measure the time delay difference between the two half-beam signals. This time difference is filtered by a low-pass filter and fed back to one of the half-beams such that the Time Delay detector output will remain close to zero. The surface-induced time shift has a frequency spectrum directly related to the wave spectrum (Figure 7) and lies almost entirely within the pass band of the tracking loop at high signal-to-noise ratios. Thus, for a good degree of approximation one can assume the tracker will track the target as well as the surface induced bearing fluctuation.

Therefore, one can write

$$\tau_D = (\tau_1 - \tau_2) + (\tau_1 - \tau_2) \quad (10)$$

and the output bearing is given by

$$\theta = (180/\pi) (c/d) \tau_D \quad (\text{in degrees}) \quad (11)$$

where c is the speed of sound, and d is the dipole spacing.

Writing the true bearing and bearing fluctuation as

$$\text{True Bearing} = \theta_T = \frac{180}{\pi} \cdot \frac{c}{d} \cdot (\tau_1 - \tau_2)$$

$$\text{Bearing fluctuation} = \delta = \frac{180}{\pi} \cdot \frac{c}{d} \cdot (\tau_1 - \tau_2) \quad (12)$$

One obtains

$$\theta = \theta_T + \delta \quad (13)$$

Since τ_1 and τ_2 are zero mean processes, one further obtains

$$\bar{\delta} = 0 \quad (14)$$

and

$$\sigma_\delta = (180/\pi) (c/d) \sigma_T \sqrt{2(1 - \rho(Y, \vec{V}))} \quad (15)$$

Using Equations (5) and (8), one can express Equation (15) as

$$\sigma_\delta = .186V^2 \cdot \left(\frac{Z_R + Z_S}{R \cdot d} \right) \sqrt{2(1 - \rho(Y, \vec{V}))} \quad (\text{deg}) \quad (16)$$

where $\rho(Y) = \tau_1 \tau_2 / \sigma_T^2$ is the spatial correlation coefficient at two specular points separated by a distance of Y feet.

Equation (16) expresses the surface-induced random error as functions of wind speed (V), dipole spacing (d), source depth (Z_S), receiver depth (Z_R), horizontal range (R) and the spatial correlation coefficient (ρ). Equation (16) is plotted in Figure 8 with wind speed = 10, 15, and 20 knots for our hypothetical geometry and array. It is seen that spatial correlation could cause a great degradation on tracker performance at high signal-to-noise ratios. Fortunately, the specular separation distance, Y , is small so that the correlation coefficient remains close to 1.

For a wind speed of 12 knots and a specular separation of 4 feet, it was estimated that the crosswind and downwind correlations are given respectively by $\rho_{CW} = .973$ and $\rho_{DW} = .833$ (reference 1). Using these values, one can estimate surface-induced random errors between $\sigma_\delta = .3$ and $\sigma_\delta = .7$ degrees for our hypothetical geometry and tracker. In a relative sense, the above data agrees well with the actual at-sea experiment.

4.0 SUMMARY AND CONCLUSIONS

The results reported herein represent a brief study of the surface wave reflection problem. It was found that spatial correlation between reflection points on the ocean surface could dramatically degrade the tracker performance. However, extensive literature research in this area yielded very lit-

tle information on surface correlation. It is recommended that further investigations be made in this area to better understand the underlying relation and its effect on the tracker and its related system performance.

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REFERENCE

1. Roderick, William I., Doppler Spectra of Bistatic Reverberation From the Sea Surface, Naval Underwater Systems Center Technical Report 6031, 8 May 1979.

Figure 1. Bearing Error Versus Relative Bearing for a Spherical Array

Figure 2. D/E Angle Versus Relative Bearing for a Spherical Array

Figure 3. Surface Reflections

Figure 4. Surface-Induced Time Delay Model

Figure 5. Top View

Figure 6. Block Diagram of a Simplified Broadband Tracker

Figure 7. Pierson-Moskowitz Wave Spectrum (Sea State 3; Wind Speed=15 knots)

Figure 8. Surface-Induced Random Error Versus Correlation Coefficient