New Ranging Algorithm for FM/CW Radars

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Abstract—A new ranging algorithm for frequency modulated/continuous wave (FM/CW) radars is given. This algorithm is near optimal, very robust, does not depend on the type of modulation, and is fairly easy to compute. An example of an older algorithm based on sinusoidal frequency modulation (FM) used on the MK-95 CW radar is given to illustrate shortcomings in typical older algorithms and for comparison purposes. The characteristics of the new ranging algorithm and test results from the MK-95 radar are provided. The actual target range and the one calculated using the new ranging algorithm differed by less than 1%.

I. INTRODUCTION

Continuous wave (CW) radars have been used in many applications throughout the history of radar. In many applications a periodic modulation on the transmitted signal is used for the purposes of extracting range. Fundamental descriptions of CW radar exist in the literature [1]. In the past, simple algorithms were used on the target echoes obtained from frequency modulated/continuous wave (FM/CW) radars to extract range. These were not necessarily optimal and suffered from failure conditions. They were used because the hardware and/or digital processing was very limited at the time. In this paper we review a typical older ranging method which was used on the MK-95 radar and discuss its characteristics and shortcomings. A new ranging algorithm is discussed which is near optimum, very robust and does not depend on the type of modulation used. Data was collected with the new algorithm implemented on the MK-95 radar.

II. DESCRIPTION OF FM/CW RADAR TARGET ECHO SIGNAL

A mathematical description of an FM/CW radar target echo signal using sinusoidal modulation is given. The transmitted signal μ_t for sinusoid modulation is

$$\mu_t = U_t \sin(\omega_c t + \frac{\Delta f}{f_m} \sin\omega_m t) \quad (1)$$

where

U_t = magnitude
ω_c = radian carrier frequency
ω_m = radian modulation frequency
Δf = frequency deviation
t = time
f_m and f_c are frequencies related to ω_m, ω_c, by 2π.

The received signal is time delayed by

$$T = T_0 + (2v/c)t \quad (2)$$

where

T_0 = time delay at t=0,
v = target velocity, and
c = speed of light.

The received signal is then

$$\mu_r = U_r \sin[\omega_c(t-T) + \frac{\Delta f}{f_m} \sin(\omega_m(t-T)) + \varphi] \quad (3)$$

where

U_r = magnitude, and
φ = arbitrary phase.

Mixing both the transmitted and received signal to baseband the signals are

$$\hat{\mu}_t = U_t \exp\{j \frac{\Delta f}{f_m} \sin\omega_m t\} \quad (4)$$

$$\hat{\mu}_r = U_r \exp\{j[-\omega_c T + \frac{\Delta f}{f_m} \sin(\omega_m(t-T)) + \varphi]\} \quad (5)$$

Forming the product and applying a trigonometric identity, results in

$$\mu_d = \hat{\mu}_r^* \hat{\mu}_t = U_d \exp\{j[\omega_c T + (2\frac{\Delta f}{f_m} \sin(\omega_m T/2) \cdot \cos(\omega_m(t-T/2))) + \varphi]\} \quad (6)$$

where

the asterisk denotes the complex conjugate and
U_d is the magnitude of the baseband echo.

This expression is simplified to

$$\mu_d = U_d \exp\{j[\omega_d t + D\cos(\omega_m(t-T_0/2)) + \alpha]\} \quad (7)$$

where

ω_d = radian Doppler
α = arbitrary phase

$$\frac{2v}{c}\omega_m \approx 0, \text{ and}$$

$$D = \frac{2\Delta f}{f_m}\sin(\omega_m T_0 / 2).$$

D is the amplitude of the sinusoidal FM phase component which is a function of range. This expression can be expanded into a Fourier series, which results in

$$\begin{aligned}\mu_d = U_d\{&J_0(D)\exp[j(\omega_d + \beta)]\\ &+J_1(D)\exp[j(\omega_d t + \omega_m(t - T_0/2) + \beta)]\\ &+J_1(D)\exp[j(\omega_d t - \omega_m(t - T_0/2) + \beta)]\\ &+\textit{higher order terms}\}\end{aligned} \quad (8)$$

The expression for μ_d contains spectral energy from the original Doppler line in the first term followed by the the upper and lower sidebands. The sidebands, produced by the sinusoidal FM, have amplitudes related to a series of Bessel functions, $J_0(D)$, $J_1(D)$, ... The higher order terms contain Bessel functions of higher order. The constant, β, is an arbitrary phase term. To extract range, which is proportional to T_0, the phase between the Doppler line and a sideband is measured. Define

$$X = (\text{upper sideband})\cdot(\text{Doppler line})^* = \exp(j\omega_m T_0 / 2)$$

and solve for T_0

$$T_0 = \frac{2}{\omega_m}\tan^{-1}\frac{X_{imag}}{X_{real}} \quad . \quad (9)$$

The range is then

$$R = \frac{c}{2\pi f_m}\tan^{-1}\frac{X_{imag}}{X_{real}}. \quad (10)$$

The target Doppler is the frequency of the center of the Doppler line.

The magnitude of the Bessel functions depends D which is a function of the modulation index $\Delta f / f_m$ and the range time delay T_0. To illustrate the variability in the Bessel functions, consider a target flying toward the radar. The values of $J_0(D)$ and $J_1(D)$ are plotted in Fig. 1 as a function of D from 0 to 5. This corresponds to ranges of 0 to 59.8 km (32.3 nmi) and a fixed frequency deviation Δf of 2 kHz. We see that there are nulls in the Doppler line at D=2.405 which corresponds to a range of 28.7 km (15.5 nmi). The sideband null occurs for D=3.832 which is a range of 45.7 km (24.7 nmi). This points out the need to change the modulation index for some ranges. In fact, the optimal points for D are between 0.5 to 1.5.

An additional problem occurs if the target is under acceleration which changes the character of the signals. The compensation for this effect is beyond the scope of this paper and is not relevant to the new algorithm.

III. NEW RANGING ALGORITHM

The new ranging algorithm is based on a large bank of matched filters. A large set of cross-correlations is performed where each stored signal to be cross-correlated with the received signal μ_d represents a different target velocity, range, and acceleration. The maximum output of all the cross-correlators represents the replica that best fits the received echo data. Therefore, the associated range, velocity, and acceleration of the target are those of the replica with the largest cross correlation value.

If the cross-correlation algorithm is applied directly to the time domain echo signal, the computing required is extremely large and probably prohibitive as well as it is very sensitive to phase errors in the stored replica relative to the transmitted signal. The first step in deriving the new algorithm is to take the Fourier transform of the echo data. The magnitude and phase of the Fourier transform of a target echo created by a repeater using the MK 95 radar is shown in Fig. 2.

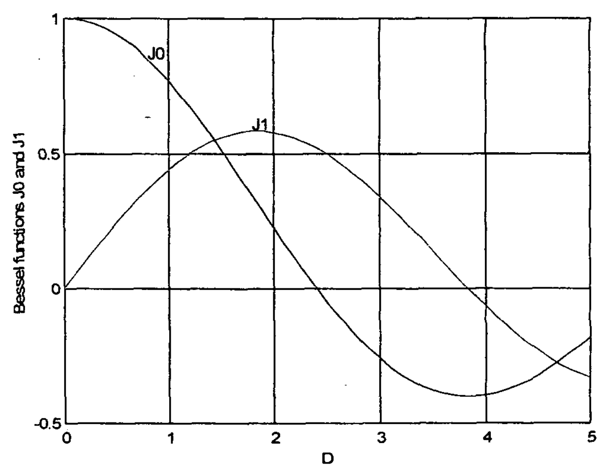

Fig. 1 Bessel functions $J_0(D)$ and $J_1(D)$

The parameters are
f_m = 86.2 Hz,
Δf = 2 kHz,
range = 15.0 km,
velocity = 370.8 m/s,
acceleration = 0 m/s^2,
time duration of processing = 23.2 ms, and
sampling pri = 5.659 μs.

For this case the velocity resolution is approximately 0.64 m/s. Observing Fig. 2, the target echo spectrum does not span many points and consequently the cross correlation function is much more easily computed. It is necessary to search over range and velocity to achieve a correct result in range. However, the target Doppler can be centered at a predetermined filter location, eliminating the need to search in velocity.

It is important to center the target accurately to reduce range errors and to eliminate any correlations performed in velocity. By examining the phase of the target in the time domain, the target's center frequency can be determined. Once the target's center frequency is known, it can be placed in a known position in a data vector for correlating in range with the library of reference vectors. The reference vectors are calculated and stored for the ranges of interest. The stored reference vectors are spectral signatures of the target echo calculated and conjugated for ranges from 0 to 91.5 km (50 nmi).

To determine the target's center frequency, 64 points are extracted from the spectrum. Since most of the energy of the target echo is contained within a few points around the target Doppler, no information was lost and reduces the amount of computation needed. The inverse Fourier transform is then calculated and the time domain phase term extracted. Shown in (11) is the expression for the phase of the target echo.

The FM modulation term and an arbitrary offset of the target's center from zero are included. Note that this is similar to the expression for the phase term in (7), the only difference is that ω_d is now an arbitrary value, depending on where the target was placed in the 64 point spectrum.

$$\Phi(t) = \omega_d t + D\cos\omega_m(t - T_0/2) + \alpha \tag{11}$$

where

ω_d = Target center frequency in 64 point spectrum
t = time
ω_m = modulation frequency
α= arbitrary phase

Taking advantage that $\Phi(t)$ has a periodic component t_p such that t_p=$2\pi/\omega_m$ and some averaging is necessary, the slope, ω_d, can be calculated:

$$\omega_d = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \{[\Phi(t + t_p) - \Phi(t)] / t_p\} dt \tag{12}$$

Shown in Fig. 3 is the unwrapped phase term $\Phi(t)$ of (11) for the target shown in Fig.2. The phase term $\Phi(t)$ can be corrected to place the target in a known position for use by the cross correlator. The cross correlator no longer needs to search in velocity, only in range [2].

The next step after correcting the phase $\Phi(t)$ is to recreate the time series and produce the spectrum data vector for use with the cross-correlator. Shown below is the expression used to calculate the correlation values

$$C(r) = \left| \hat{R}(r) \bullet \hat{T} \right| \tag{13}$$

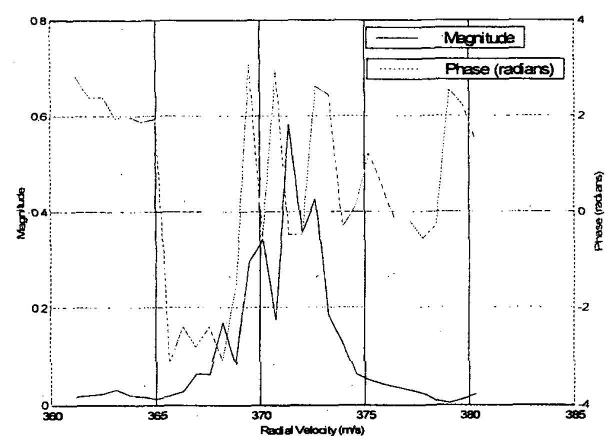

Fig 2. Spectrum of example target echo

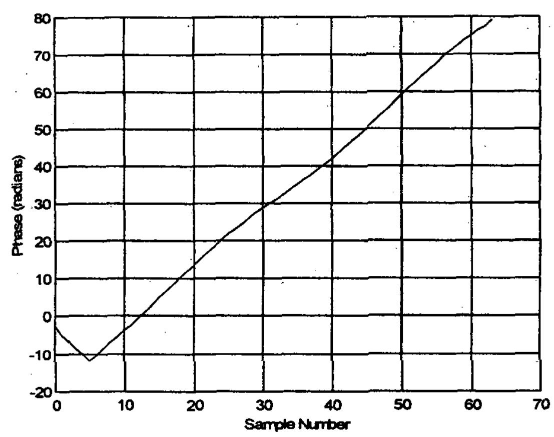

Fig. 3 Unwrapped phase of target $\Phi(t)$

where

$\hat{R}(r)$ = unit reference vector,

$\hat{T}$ = unit target data vector,

C(r) = correlation values , and

r = range.

The correlation values are calculated by finding the magnitude of the dot product of the reference vectors with the data vector. C(r) will peak when the best match is made and should be near the range of the target. However, the lower the SNR of the data vector, the poorer the match with the reference vectors. This causes the range values to spread out around the expected value of the range. Thus it is necessary to provide some averaging of the range.

Shown in Fig. 4 is a comparison of the reference vector for a range of 16.1 km and a target at a range of 15.0 km. In magnitude alone the two signatures agree very well with each other. The reference vector and the data vector consisting of the target from Fig. 1 are both normalized to 1 for ease o comparison. The target centers are placed at filter zero an‹ the rest of the spectral components only being displayed +/ 20 filters around the target center. The target data consiste‹ of a repeater 152 meters from the MK 95 radar with a dela; line equivalent to 15.0 km and SNR of 30 dB.

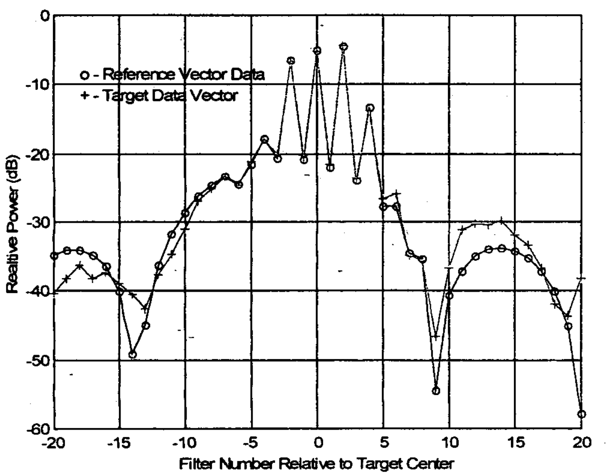

Fig 4. Measured target signature and calculated library reference signature from the MK-95 radar for the cases of range = 15.0 km, acceleration = 0 G.

Shown in Fig. 5 were the correlation values as a function of range resulting from correlating the data vector in Fig. 2 with the complete set of library reference vectors. The maximum correlation occurring at 16.1 km. The peak correlation value is also a figure of merit as to the quality of the match. The closer the peak correlation value is to one, the better it matches with the reference vector in magnitude and phase.

This new algorithm is easy to compute. For the example given, only 64 spectral points need to be stored. A search for the maximum cross correlation can be divided into two parts: a coarse and a fine. The coarse could search in 9.26 km (5 nmi) increments. A fine search about the best coarse point can be performed over 1.8 km (1 nmi). Searches of this type in modern processors are simple and not very time intensive.

The new ranging algorithm was tested using a fly-by aircraft. The target was a Falcon 10 which is a twin engine turbofan business jet. The target was flying at an altitude of 4.6 km (15000 ft). Shown in Fig. 6 is the actual range data from the identification friend or foe (IFF) system and the smoothed range data from the radar.

In the figure, the smoothed ranging data was in error by a scale factor of 3.8%. The ranging algorithm had not been calibrated against the internal calibration signal. The corrected range error, after calibration was less than 1%.

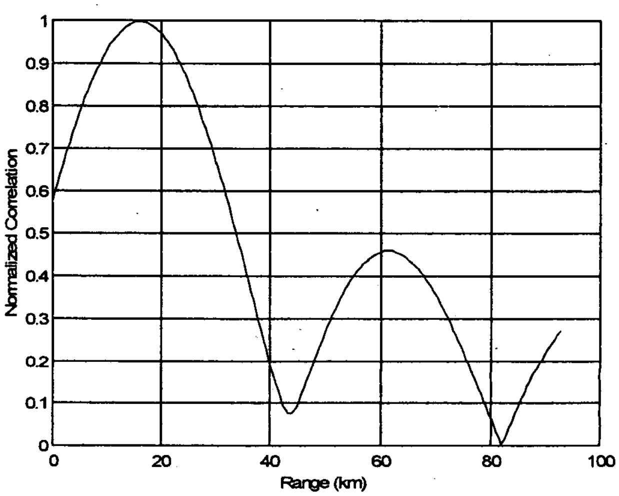

Fig 5. Correlation as a function of range for 15.0 km repeater target with the reference vector set.

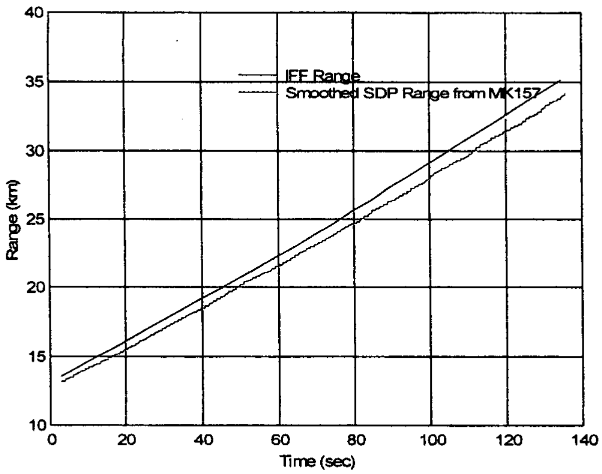

Fig. 6 Flight test results using the Falcon 10 flying at 4.6 km (15000 ft) altitude

IV. SUMMARY

A new ranging algorithm for FM/CW radars was described. This algorithm is nearly optimal, very robust, does not depend on the type of modulation used, and is relatively easy to compute. As background material, a conventional ranging algorithm for sinusoidal modulation and its shortcomings were discussed. The old algorithm is sensitive to phase errors, acceleration, velocity, requires a changing modulation index, and is modulation dependent. The new algorithm avoids these problems and works very effectively on the MK95 radar.

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