ELECTROMAGNETIC STIMULATION OF A MYELINATED AXON

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Abstract

We present a mathematical model of electromagnetic stimulation of a peripheral nerve axon in a limb. The model predicts the distribution of transmembrane potential along the axon, and the shape of the tissue volume that is stimulated. We show that threshold stimulus strength is inversely proportional to the square of the axon diameter.

Introduction

Electromagnetic stimulation offers a non-invasive and relatively painless method of diagnosing some neurological disorders [1]. Below, we present a model of electromagnetic stimulation of a peripheral nerve within a limb. An RLC circuit models the current source and stimulating coil. Maxwell's equations predict the electric field distribution within the limb. A cable equation describes the response of the mammalian peripheral axon to this electric field. The shape of the stimulating "cathode" within the limb and relationships among stimulus strength, axon diameter and its depth within the tissue are predicted.

Model

The internodal region of the axon is modelled as a passive cable whose transmembrane potential, V , is governed by:

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} - \tau \frac{\partial V}{\partial t} - (V - V_r) = \lambda^2 \frac{\partial \epsilon_x(x,t)}{\partial x} \quad (1)$$

The electric field in the axial direction of the axon, ϵ_x , arises from electromagnetic induction [3] and induced charge at the air/tissue interface [4]. Above, V_r is the rest potential (-80 mV), λ is the space constant of the fiber,

$$\lambda = d_i \sqrt{\frac{\rho_m}{8\rho_a} \ln\left(\frac{d_o}{d_i}\right)} \quad (2)$$

[5] and τ is the membrane time constant,

$$\tau = \epsilon_0 \kappa \rho_m \quad (3)$$

In Eqs. (2) and (3) ρ_m and ρ_a are the resistivities of myelin and axoplasm, respectively, κ is the dielectric constant of myelin, ϵ_0 is the permittivity of free space, d_o is the outer diameter of the myelin sheath, and d_i is the diameter of the axon membrane [5].

Each node of Ranvier is modelled as a current source that contributes I_n to the membrane current,

$$I_n = \pi d_i \delta \left(g_{Na} m^2 h (V - E_{Na}) + g_L (V - E_L) + c_n \frac{\partial V}{\partial t} \right) \quad (4)$$

Nodal membrane properties are listed in Table 1. We used the gating kinetics of Sweeney et al. [6] after reconciling them

with Chiu and Ritchie [7]. Relationships between node separation, Λ , and axon outer diameter, d_o , ($\Lambda = 100 d_o$), and inner (d_i) and outer diameter ($d_i/d_o = 0.6$) are given in [6,8].

The axon is oriented axially within a cylindrical, homogeneous volume conductor whose radius is 3 cm. The stimulating coil has 14 turns, a radius of 4.5 cm, and carries a current that rises to 4000 Amps in 150 μ s. It lies 0.5 cm above the cylinder, as shown in Figure 1.

Figure 1. A schematic diagram of the cylindrical arm, the nerve axon and the stimulating coil.

The response of an axon ($d_o = 10 \mu$ m) to a supra-threshold electromagnetic stimulus is shown in Figure 2. Two action potentials propagate in opposite directions at 45 m/sec. These waves do not originate below the center of the coil ($x=0$), where the electric field, $\epsilon_x(x,t)$, is a maximum, but at a point where the negative electric field gradient, $-\partial \epsilon_x(x,t)/\partial x$, is a maximum [2].

Figure 2. A contour plot of transmembrane potential (mV) during a supra-threshold electromagnetic stimulation event.

The predicted relationship between threshold stimulus strength and axon diameter is shown in Figure 3. Threshold stimulus strength is measured by the initial voltage across the capacitor plates, V_0 , just sufficient to produce an action potential. It is inversely proportional to the square of the axon diameter.

Figure 3. Threshold capacitor voltage vs. axon diameter.

Finally, we locate the boundary that separates regions of sub- and supra-threshold response of nerve axons within a limb. For the circular stimulating coil and cylindrical arm shown in Fig. 1, we calculate the effective "volume of stimulation" [9], whose boundary we call the "virtual cathode". Figure 4 shows the approximate shape of the volume of stimulation, which is represented by a volume in which $-\partial\epsilon_x/\partial x$ at $t=0$ is greater than 600 mV/cm^2 . Nerve axons whose diameters are $20 \mu\text{m}$ or larger and pass through this volume are expected to be stimulated.

Figure 4. An approximation of the volume of stimulation in a cylindrical arm produced by a circular stimulating coil.

TABLE 1

R	Stimulating circuit resistance	0.47 Ohms
L	Stimulating coil inductance	20 μH
C	Stimulating circuit capacitance	3100 μF
V_0	Voltage across capacitor plates	2000 Volts
E_{Na}	Sodium Nernst Potential @ 37C	35.35 mV
E_{L}	Leak Nernst Potential @ 37C	-80.01 mV
g_{Na}	Sodium Conductance	1,445 mS/cm^2
g_{L}	Leakage Conductance	128 mS/cm^2
c_n	Nodal Capacitance per sq. cm	2.5 $\mu\text{F/cm}^2$
ρ_a	Resistivity of Axoplasm	$5.47 \times 10^{-2} \text{ kOhm-cm}$
ρ_m	Resistivity of Myelin	$7.4 \times 10^5 \text{ kOhm-cm}$
κ	Dielectric Constant of Myelin	7
ϵ_0	Permittivity of a vacuum	$8.85 \times 10^{-8} \text{ F/m}$
δ	Width of node of Ranvier	$1.5 \times 10^{-4} \text{ cm}$

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