USING STOCHASTIC MODELS TO EFFECTIVELY SCHEDULE HARDWARE TEST EFFORTS

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ABSTRACT - *This paper identifies a stochastic method via which to plan a schedule of qualification or reliability testing of electronic hardware. The technique utilizes a Markov model to anticipate scheduling delays, exhibiting as its inherent advantages the following:*

- (1) mean time between failures (MTBF) need not be known to generate the corresponding Markov statistics;*
- (2) model anticipates random bottlenecks in scheduling that a deterministic model cannot;*
- (3) model readily avails itself to computation of Markov statistics that estimate the time to completion of all t tests (and its variance), in addition to the time to completion of each qualification or reliability test (and the respective variances of each test); and*
- (4) a Markov model used in (3) has been translated into a synopsis of a proposed software configuration, to function as guideline via which to generate software that would execute the desired Markov matrix computations, in the event the time to test for a large group of DUT's is to be determined.*

In the event that qualification or reliability testing results in failure of hardware devices under test (DUT's), late arrival of DUT's, or other tardy completion of essential testing procedures, a deterministic math model cannot account for these random events to any appreciable extent. A more effective alternative for estimating the random occurrence of scheduling bottlenecks makes use of a stochastic approach that employs Markov statistics to predict the incidence of scheduling delays.

This stochastic method utilizes a Markov model in much the same way as a tracking algorithm would use it to model its scan, acquisition, and tracking states. For example, the tracking algorithm of Reference [7] uses Markov statistics to estimate the time a noncoherent optical receiver resides in each of its scan, acquisition, and tracking states. The Markov model that seemed to most closely depict the tracking aspects of this laser intersatellite communication link was a finite absorbing Markov chain. In the case of this optical tracking model, the 0-th state corresponded to the scan mode, the intermediate states to the acquisition mode,

and the final, or N-th state, to the tracking mode. The N-th state is also an "absorbing" Markov state because, once entered, it is nearly inescapable.

Whereas the N+1 Markov states in the referenced Markov tracking model represent progressive stages of the scan, acquisition, and track modes for the subject optical receiver, the N+1 Markov states in a qualification test model will analogously represent the various environmental and electromagnetic interference (EMI) tests to which a device under test is submitted (Figure 1). Qualification would in this application be symbolized by the N-th, or "absorbing", Markov state (because once qualified, a DUT would not revert to an "unqualified" state. Similarly, in the reliability testing application of the finite absorbing Markov chain model, the N+1 states correspond to the various reliability tests to which the DUT is submitted. The final, or "absorbing" state, is then the state of "reliability".

As there was a probabilistic criterion to justify the transition from one state to the next in the aforementioned laser communication tracking model, so there must also be for a Markov model of hardware qualification or reliability. This criterion for each Markov state transition will be a "failure threshold" that, if not exceeded, allows the DUT to progress to the next Markov state, i.e., the next qualification or reliability test. In those cases where the "failure threshold" is exceeded, a DUT exhibits critical failure, and is submitted for repair and/or maintenance. Critical failures are depicted graphically on the Markov state diagram of Figure 1 by arrows that circle back into the same state from which they previously came. The probability of failure is denoted by a "P" with a subscript "f". Similarly, passing a test is indicated by a "P" with a subscript "s" (for survival).

This probabilistic criterion for transition to the next testing state is the most crucial aspect of the Markov model. Once the suitable criterion is established, the Markov matrix definitions will be used to compute the mean and variance of test cycles in each test, and the mean and variance of the number of total test cycles for each testing process. Since two electronic modules will generally exhibit similarly valued statistics when manufactured by the same vendor to perform the same function, approximately equivalent means

and variances of test cycles will likely occur in these two matching electronic modules. The “failure threshold” here is equated to mean time between failure (MTBF) for each respective test. If the MTBF is not exceeded, then the DUT progresses to the next Markov state of testing. If, on the other hand, the MTBF is exceeded, the DUT fails that test, and the test must be repeated. Please see Figure 1 for a graphical description of the Markov model for testing processes. Additionally, Figure 2 will depict in the Q submatrix of the canonical P-matrix (“P” for probability matrix) the probability of failure on the main diagonal elements, and the probability of survival on the diagonal immediately below the main diagonal of the same Q submatrix. The MTBF for qualification or reliability testing can be computed relatively conveniently if the probability density function for test failures is assumed to be exponential and of the form:

$$f(t) = k(\exp(-kt)) \quad (1)$$

, where k is the failure rate in units of failures per hour and t is the time (Reference [8], page 78). Unstressed failure rates can be found for components of most systems, and have been estimated or tabulated, or fit to regression curve. When the failure rates indigenous to reliability testing have been established, the failure rates associated with more stressful qualification testing may then be estimated via stress derating factor equations (Ref. [9]), or by referring to stress derating curves. If either of these methods for estimating failure rates under stressful conditions is not available, stressed failure rates have been noted by Bazovsky (Ref. [8]) to be at least ten times greater than the unstressed failure rates. As a result of Bazovsky’s observation, a failure rate of ten times the failure rate for reliability testing provides a conservative estimate of the failure rate associated with qualification testing of any DUT.

Since the expected value of the exponential failure density , otherwise known as the mean time between failure (MTBF), is 1/k, the MTBF for stressed conditions (such as qualification testing) would be (again, according to Bazovsky) no more than one tenth of the MTBF at unstressed conditions. Alternatively, the MTBF’s for qualification tests can be determined by relating them to MTBF’s for reliability testing via either the failure derating curves or the aforementioned stress derating factor equations, since the failure rate is the reciprocal of the MTBF. Also worthy of note is that the MTBF of exponential failure density is 1/k, its standard deviation is 1/k, and the corresponding variance is the square of 1/k.

A chief convenience of using a Markov model to simulate reliability or qualification testing is that the system failure rate need not be known in order to compute the mean and variance of times in each of the test states. This is so because the integral of the failure density, with upper and

lower bounds of zero and MTBF, is $1 - \exp(-k(\text{MTBF}))$. Essentially when the failure rate, k, and the MTBF cancel, yielding the result that the probability of survival, Ps, during testing is $1 - 1/e$, and the probability of failure, Pf, is $1/e$. By virtue of this cancellation, an initial estimate of total time to test may be computed before more precise MTBF’s or failure rates are available.

For purposes of simplification, the author will assume that stochastic independence of qualification tests, reliability tests, and DUT’s is, for the most part, realistic. Certainly DUT types that are subjected to testing are independent of one another. However, for a given DUT, the damage impact of some tests on the DUT may demonstrate some degree of interaction, i.e., damage exacted in one test may accelerate damage impact of a forthcoming test. In the interest of computational simplicity, it will be presumed that any such interactions between qualification tests or reliability tests are negligible. With this simplifying assumption in hand, the mean and variance of time for each test of a particular type of DUT are additive, so that the total estimate of time to complete testing is additive.

An Example of a Markov Model Computation for Reliability or Qualification Tests

Before this computation can be discussed to any degree of detail, the ground rules that define the basis of the Markov model should be stated. As mentioned earlier, the model used here is a finite absorbing Markov chain. The Markov matrix definitions that are fundamental to the matrix computations are provided in Reference [11]. Reference [11] explains how the canonical P-matrix is utilized to derive the four matrices that will define in statistical terms how a specific reliability or qualification test can be characterized. These four matrices are the N, N2, T, and T2 matrices, and they provide the following information with respect to reliability test or qualification test processes:

- (1) N is the mean number of test cycles that a particular DUT resides within a particular Markov test state,
- (2) N2 indicates the variance of residence time in which a DUT resides in a particular test state,
- (3) T provides the mean total number of test cycles to which a DUT is submitted during the overall testing process,
- (4) T2 offers the variance of total residence time for the overall test process to which a DUT is submitted.

As indicated earlier on this page the integral of equation (1), when integrated from zero to the MTBF (where $\text{MTBF} = 1/k$) yields the probability of test survival, Ps, or $1 - 1/e$. Since the probability of failure, Pf, during a test cycle is achieved when the MTBF is exceeded, the integral of equation (1) for that case is $1/e$. Both the Pf and Ps results will appear in the Q-submatrix of this Markov matrix computation. The P-

matrix is then manipulated via the row matrix operations to yield $N = [I - Q]^{**}(-1)$, an inverse matrix. Results are shown in Figure 2 for a testing process with four total tests. N_2 , T , and T_2 are similarly derived via row operations, as indicated in Figure 2.

Another attractive aspect of Markov computations is their amenability to induction for N Markov test states. For example, in Figure 2 four test states were available, where $N=4$. However, if $N=10$ for the Markov model of Figure 1, the Markov matrices could be induced from the calculation for the $N=4$ case. This advantage greatly streamlines the determination of Markov model results.

A Proposed Software Configuration to Do Markov Modeling of Testing Processes

Using the basics of Markov statistics, the concept of failure threshold, P_f , and P_s , software requirements for various software modules that perform the calculations heretofore identified are now proposed. These proposed modules, and their specific functions to model and/or schedule reliability or qualification testing, are as follows: (1) the DUT Arrival Module, (2) Exponentially Distributed Random Variable Generator Module, (3) Test Hardware Allocation Module, and (4) the Finite Absorbing Markov Chain Module. The first module's primary functions would be to statistically simulate Poisson arrivals of DUT's (where interarrival times of DUT's of the same type are exponentially distributed), prioritize allocation of hardware test resources, and affix time tags to the arriving DUT's via a universal time clock.

The second module's primary responsibility is one of generating all exponentially distributed random variables.

The third software module is the Test Hardware Allocation Module, whose main functions are as follows: (1) coordinate and control all other software modules as the module's executive function dictates, (2) maintain a universal time clock for the simulation, whose time is available to the other modules as needed, (3) maintain current status and/or availability of all resources in the simulation, referenced to the universal time clock, (4) invoke Module (2) to either estimate the time to repair/maintain a critically failed DUT or the time to a test failure, and (5) time reference repairs, tests, and other events that are important to a discrete event simulation such as this.

The fourth and final software module is termed the Finite Absorbing Markov Chain Module. Its responsibilities to the simulation are: (1) to compute Markov statistics for one or several "failure thresholds", (2) to pass the results of Markov computations to the Test Hardware Allocation Module, i.e., Module (3).

A diagram of a proposed software configuration is depicted in Figure 3, as also are some of the proposed configuration's software variables.

Summary of Findings

Use of such a stochastic model to schedule hardware testing, while blessed with many computational conveniences, is also inclined to yield a more accurate scheduling model than would a deterministic, discrete event simulation because it is better at anticipating random delays -- and is well-suited to software application.

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FIGURE 1: (N+1)-STATE FINITE ABSORBING MARKOV MODEL FOR A QUALIFICATION OR RELIABILITY TESTING PROCESS

FIGURE 2: COMPUTING MARKOV STATISTICS FOR FOUR QUALIFICATION OR RELIABILITY TESTS VIA MATRIX OPERATIONS ON THE Q-SUBMATRIX OF THE OF THE MARKOV CANONICAL P-MATRIX

state =	4	3	2	1	0
4	1	0	0	0	0
3	$P_{S,34}(t < \text{MTBF})$	$P_{F,33}(t > \text{MTBF})$	0	0	0
2	0	$P_{S,23}(t < \text{MTBF})$	$P_{F,22}(t > \text{MTBF})$	0	0
1	0	0	$P_{S,12}(t < \text{MTBF})$	$P_{F,11}(t > \text{MTBF})$	0
0	0	0	0	$P_{S,01}(t < \text{MTBF})$	$P_{F,00}(t > \text{MTBF})$

R-submatrix
Q-submatrix

where i is the source row and j is the destination column of the canonical P-matrix

$$Q = \begin{bmatrix} .368 & 0 & 0 & 0 \\ .632 & .368 & 0 & 0 \\ 0 & .632 & .368 & 0 \\ 0 & 0 & .632 & .368 \end{bmatrix}$$

$$N = [I - Q]^{-1} = \begin{bmatrix} 1/.632 & 0 & 0 & 0 \\ 1/.632 & 1/.632 & 0 & 0 \\ 1/.632 & 1/.632 & 1/.632 & 0 \\ 1/.632 & 1/.632 & 1/.632 & 1/.632 \end{bmatrix}$$

$$N2 = N[Ndg - I] \cdot Nsq = \begin{bmatrix} .922 & 0 & 0 & 0 \\ .922 & .922 & 0 & 0 \\ .922 & .922 & .922 & 0 \\ .922 & .922 & .922 & .922 \end{bmatrix}$$

$$T = NE = \begin{bmatrix} 1/.632 & 0 & 0 & 0 \\ 1/.632 & 1/.632 & 0 & 0 \\ 1/.632 & 1/.632 & 1/.632 & 0 \\ 1/.632 & 1/.632 & 1/.632 & 1/.632 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.632 \\ 2.632 \\ 3.632 \\ 4.632 \end{bmatrix} \begin{matrix} \text{State 3} \\ \text{State 2} \\ \text{State 1} \\ \text{State 0} \end{matrix}$$

$$T2 = [2N - I] \cdot T - Tsq = \begin{bmatrix} 2.165 & 0 & 0 & 0 \\ 2.165 & 2.165 & 0 & 0 \\ 2.165 & 2.165 & 2.165 & 0 \\ 2.165 & 2.165 & 2.165 & 2.165 \end{bmatrix} \begin{bmatrix} 1/.632 \\ 2/.632 \\ 3/.632 \\ 4/.632 \end{bmatrix} = \begin{bmatrix} 2.504 \\ 10.014 \\ 22.532 \\ 40.058 \end{bmatrix} \begin{bmatrix} .922 \\ .263 \\ 1.978 \\ 5.802 \end{bmatrix} \begin{matrix} \text{State 3} \\ \text{State 2} \\ \text{State 1} \\ \text{State 0} \end{matrix}$$

FIGURE 3: PROSPECTIVE CONFIGURATION OF SOFTWARE TO STOCHASTICALLY
MODEL A QUALIFICATION OR RELIABILITY TEST PROGRAM