Modeling and Control of an Electro-Hydrostatic Actuator*

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Abstract

Advanced Electro-Hydrostatic Actuators offer a high degree of maintainability and combat survivability of aircraft flight control systems, because all the components necessary to operate the actuator are collocated with the actuator. The Quantitative Feedback Theory robust control method is used to design a controller for the actuator. The impact of parameter variations, sensor noise, and flight condition variability are explicitly considered in the design process. The controller utilizes a two loop feedback controls structure. The resulting linear design is not only robust with respect to plant parameter variations, but is also insensitive to the effects of sensor noise. Furthermore, the actuator's phase lag is reduced by incorporating phase constraints in the QFT design paradigm; thus, improving the performance of the overall flight control system.

1 Introduction

Actuators are a critical subsystem in any flight control systems; however, robust automatic con-

trol of actuators has not fully been explored in the past. This research focuses on the modeling and control of an advanced Electro-Hydrostatic Actuator (EHA). Electric motors have a limited torque-to-mass ratio, due to the finite and limited magnetic flux density that can be generated [2]. High pressure hydraulic systems, with the system pressure of 2000 to 5000 psi, can generate high forces resulting in higher torque-to-mass ratios than in electric motors. Generally, high pressure hydraulic systems are stiffer against the load than electric motors. Hence, the EHA employs a hybrid approach, where a brushless DC motor drives a hydraulic pump to circulate high pressure fluid into the piston chamber. Thus, the DC motor internal to the EHA converts electrical power into mechanical power. It is the pump that converts this mechanical power into hydraulic power. The hydraulic power, acting against the piston, is converted to mechanical power capable of moving large flight control surfaces.

A linear mathematical model of the EHA is derived in Section 2. The models for the individual EHA components: motor, pump, piston, flight control surface (inertia), and hinge moment are explained. The EHA model is rearranged to form a two loop feedback controls structure. The controller design process is described in Section 3. The inner loop controller is designed to increase the outer loop robustness. The outer loop controller and prefilter are designed to enforce the ro-

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Figure 1: Motor Model

bust tracking of commands. The controller design is validated by simulations. In Section 4, the sensitivities of the EHA controller and of an alternative high gain control system to sensor noise, are analyzed. The QFT design is compared against the high gain design for sensor noise and bias handling characteristics. In Section 5, the EHA is incorporated in a flight control system and its performance is compared against the performances of a flight control system with first-order and fourth-order conventional hydraulic actuator models.

2 Actuator Modeling

2.1 Motor

Motors with the rotor moment of inertia J_m [$in \cdot lb \cdot sec^2$] and electro-mechanical damping B_m [$in \cdot lb \cdot sec$] (see Fig. 1), are subject to variations in output torque and subsequent fluctuations in rotor speed ω_m . Since the torque due to the load counteracts the torque generated by the motor, perturbation may also be caused by variations of the load torque. This relationship is expressed as,

$$\tau_e(s) = \tau_{cmd}(s) - \tau_{load}(s) \quad (1)$$

where τ_{load} is the load torque due to the differential pressure of the fluids. This results in a first order transfer function

$$\frac{\omega_m(s)}{\tau_e(s)} = \frac{1}{J_ms + B_m} \quad (2)$$

2.2 Pump and Fluid

The flow rate Q_m generated by the pump is proportional to the motor speed

$$Q_m = \frac{D_m}{2\pi} \omega_m, \quad (3)$$

where D_m [in^3/rev] represents the pump displacement constant.

The flow rate of the hydraulic fluid is primarily dependent on two factors: change in chamber volume and change in pressure due to the compressibility effect of the fluid [2]. The chamber volume changes as the piston moves through the chamber at speed $\dot{X}_p$. The flow rate due to the changes in chamber volume is then expressed as $\pm A \dot{X}_p$. Secondary fluid effects include fluid compressibility effects, internal leakage flow, and external leakage flow. The EHA designers elected to model these secondary effects with a first-order differential equation

$$\delta Q(s) = Q_m(s) - A s X_p = K_s s P(s) + C_t P(s) \quad (4)$$

This results in a transfer function

$$\frac{P(s)}{\delta Q(s)} = \frac{1}{K_s s + C_t} \quad (5)$$

2.3 Piston and Flight Control Surface

The pressure developed by the pump and fluid acts on the piston surface (see Fig. 2), causing the RAM to extend or retract. This force then generates a torque through a hinge to deflect the control surface. This torque has to overcome two load components: control surface inertia and aerodynamic loads. The aerodynamic load only occurs in flight, when the air pressure over the control surface applies aerodynamic reaction forces to it. The aerodynamic load is determined by three factors: the surface area of the flight control surface, dynamic pressure which varies with altitude and airspeed, and the surface's relative angle to the wind. The surface's angle to the wind depends on the surface deflection angle and on the aircraft's angle of attack.

Figure 3: Bare EHA Block Diagram

of air above and below the flight control surface applies a torque on the hinge assembly, which in turn adds back-pressure to the piston. This hinge moment is modeled as follows:

$$\tau_L = \bar{q} S_t R_h (Ch_\alpha \frac{\alpha(s)}{\Theta_L(s)} + Ch_\delta) \Theta_L \quad (17)$$

where S_t denotes the surface area of the control surface, $\bar{q}$ denotes the aerodynamic pressure, and Ch_α and Ch_δ denote the hinge moment coefficients of the control surface with respect to angle of attack and surface deflection.

The aerodynamic load's dynamic pressure $\bar{q}$ is a function of airspeed U_0 and air density ρ , as shown in Eq. (18) [7]. The air density in a standard atmosphere drops exponentially with increasing altitude. The dynamic pressure $\bar{q}$ increases as the Mach number gets higher and altitude gets lower.

$$\bar{q} = \frac{1}{2} \rho U_0^2 \quad (18)$$

The hinge moment for modern fighters with stabilator, where $Ch_\alpha \approx Ch_\delta$, is modeled by

$$\tau_L = \bar{q} S_t R_h Ch_\delta (\frac{\alpha(s)}{\Theta_L(s)} + 1) \Theta_L \quad (19)$$

2.5 Short Period Approximation

If the aircraft's forward speed is assumed constant (i.e., $u \approx 0$), the X force equation can be neglected since it does not significantly contribute to the short period oscillation [1]. Thereby, the short

period approximation of the aircraft's longitudinal channel is extracted and is written as,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} z\alpha & zq \\ m\alpha & mq \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} z\delta_e \\ m\delta_e \end{bmatrix} \delta_e \quad (20)$$

This yields a second-order minimum-phase transfer function of the form:

$$\frac{\alpha(s)}{\delta_e(s)} = \frac{-K(s+a)}{(s+b)(s+c)} \quad (21)$$

For the frequency of interest, the short period approximation closely resembles the full state model. As seen in Fig. 4, the phase and attenuation characteristics are closely matched at high frequency. The solid lines represent the full state model frequency response and '+' lines represent the short period model frequency response. The approximation, as seen in the figure, is not valid for frequencies below 0.5 rad/sec due to the effects from the slow phugoid mode.

3 Compensator Design

In order to proceed with the QFT design, the block model as shown in Fig. 3 must first be transformed to a state space model. Using block manipulation rules, Fig. 3 is manipulated to form the standard inner and outer QFT loop structures shown in Fig. 5. The inner loop controls the angular rate of the motor while the outer loop controls

Figure 4: Comparison of Full and Short Period Approximation for the Longitudinal Equations of Motion

the surface deflection. Since tracking in the inner loop is not important, for its function is to reduce the uncertainty level of the outer loop, the inner loop prefilter can be set to unity. The tracking performance enforcement is relegated to the outer loop's prefilter F_Θ .

3.1 Inner Loop Control

The purpose of the inner loop design is to facilitate the design of a robust outer loop, without the problems associated with excessive gains. The inner loop robustness effectively shrinks the region of uncertainty (i.e., the template size) of the outer loop, while also increasing the outer loop gain margin. The inner loop controls the angular rate of the electric motor. The direction of the motor rotation determines the direction of flow of the hydraulic fluid, and ultimately the piston direction. The rise time is the dominant design constraint in the inner loop.

Figure 6 shows the flight conditions considered. Both subsonic and supersonic points are tested to determine the maximum template width. While the dynamic pressure parameter $\bar{q}$ plays the most significant role in enlarging the template size, the aircraft damping and natural frequencies for given altitude and airspeed also play a significant

Figure 5: Two Feedback Loops in QFT Two Degrees of Freedom Structure

Figure 6: A typical modern fighter flight envelope and the flight conditions considered for the design

role. Two flight conditions ac41, (10,000 ft altitude and Mach 1.1), and ac28, (50,000 ft altitude and Mach 0.78), determine the perimeter of the plant templates. Since the flight conditions ac28 and ac41 result in the largest template size, the EHA parameters are varied to enlarge the templates described by these flight conditions. The largest template is at the frequency of 5 rad/sec, with about 22 degrees in width and 6 dB in height.

The inner loop compensator should be of low order. A unity forward gain is sufficient to satisfy the inner loop optimal bounds, but it also requires the outer loop gain to be high. Through several design iteration, a gain of 10 is used in the inner

Figure 7: Inner Loop Compensator Design - The loop transmission is steered through the trough, as required.

loop which reduces the gain required in the outer loop. A pole at the origin is introduced to create a type 1 system, to ensure tracking of a step input. This pole also allows the nominal loop to fall between the crevices of the optimal bounds, as can be seen in Fig. 7. The nominal plant transmission at 1 rad/sec can be situated as low as 48 dB by using the phase and magnitude information, instead of 74 dB by using the magnitude information only. This equates to a gain reduction of about 20. The inner loop compensator transfer function is

$$G_m(s) = \frac{1000(s + 15)}{s(s + 1500)} \quad (22)$$

3.2 Outer Loop Control

A common model of conventional aircraft actuators is of fourth-order, with the dominant pole located at about -20 [6]. This point of reference is used to build the upper and lower tracking bounds. Thus, tracking bounds that satisfy both the time and frequency specifications are defined as

$$T_{ru}(s) = \frac{270(s + 50)(s + 400)}{(s + 45)(s + 60)(s + 2000)} \quad (23)$$

and

$$T_{rl}(s) = \frac{200}{(s + 10)(s + 20)} \quad (24)$$

Figure 8: Outer Loop Compensator Design

It is reasonable to conclude that the actuator is quite stiff from the load end; hence, the external disturbance can be ignored (i.e., $d_2(t) = 0$). The EHA should provide adequate internal disturbance (i.e., $d_1(t) = u_{-1}(t)$) rejection by attenuating the disturbance input by -20 dB or more. Hence, the disturbance bound specification is defined as

$$T_D(s) = \frac{5}{s + 50} \quad (25)$$

The flight conditions that define the perimeter of the plant templates are ac28 and ac41, at the opposite ends of the envelope. A DC gain of 60 is required to bring the nominal open loop transmission above the respective optimal bounds shown in Fig. 8. A pole at 8.5 rad/sec and a zero at 14.5 rad/sec are added to meet the optimal and stability bounds. A pair of complex poles are added at $-200 \pm j200$ to swing the transmission line across the -180 degree phase line. Figure 8 shows that the nominal loop transmission satisfies all the required bounds. The resulting compensator's transfer function is shown in Eq. (26).

$$G_\Theta(s) = \frac{2.813793 * 10^6(s + 14.5)}{(s + 8.5)(s^2 + 400s + 80000)} \quad (26)$$

A prefilter with third order zeros over third order poles is required to satisfy the bounds. A zero and two poles at high frequency are required to satisfy the frequency specifications, magnitude as well as phase. The resulting prefilter transfer function is shown in Eq. (27).

Figure 9: Outer Loop Prefilter Design

$$F_{\phi}(s) = \frac{(s + 3.6)(s + 8)(s + 4444.4)}{(s + 3.2)(s + 200)(s + 200)} \quad (27)$$

The time and frequency responses are simulated using the Matlab Simulink block models, not the reduced order transfer functions used in the controller design process. Figures 10 to 12 show the system response in both the time and frequency domains. As expected, Fig. 10 shows that the disturbance input is rejected at levels well below the specification. Figure 11 shows the time domain response of the system to a unit step input. The system shows a slight oscillation, primarily due to the pair of complex poles near the origin from the aircraft short period module. However, the rise and settling times are quite fast. All plants settle to 98% of the final value within 0.3 seconds. The oscillations and violation of the bounds are deemed acceptable, since the magnitude of the violations are relatively small.

The small-signal frequency responses are obtained by commanding the individual plants with a sinusoid input having a magnitude of 0.5% of the maximum piston travel and frequencies varying from 0.5 rad/sec to 200 rad/sec. The peaks of both the reference and the output signals are first detected, then the attenuation and phase between the peaks are computed. By repeating the process over a predefined set of frequencies, a set of 'Bode plot' data is derived. The dashed lines in both the magnitude and phase plots are the minimum values allowed in the EHA specifications. As can be seen in Fig. 12, all plants stay well above the specification boundaries.

Figure 10: Outer Loop Time Domain Unit Step Disturbance Response

Figure 11: Outer Loop Time Domain Unit Step Input Response

Figure 12: Outer Loop Small-Signal Frequency Domain Sinusoidal Input Response

Figure 13: Addition of LVDT Sensor Noise in QFT Feedback Structure

4 Sensor Noise Effects

In this design, two observable states, the pump motor's angular rate and the piston position, are measured and used for the synthesis of feedback controls. Tachometers are used to sense the rotational velocities, while Linear Variable Differential Transformers (LVDT), embedded inside the actuator piston chamber, are used to sense the piston RAM movement [2]. All sensors introduce some degree of inaccuracy. There are two basic types of measurement errors that are dealt with in this paper: random and bias. With random errors, the signals can take on any values within a band of finite size from the nominal values. Sensor bias errors entail offsets from the nominal value by a constant amount. The sensor errors can either be introduced gradually over the lifetime of the sensor (i.e., wear and tear) or introduced abruptly due to component failures. The degree of bias errors or random noise determines the width of the error band.

It is a well known fact that a high system gain tends to amplify the effects of sensor noise; hence, the most effective countermeasure to sensor noise is to decrease the control system's loop gain as much as possible, while still meeting specifications, e.g., tracking and disturbance rejection [7]. QFT is an ideal design method to economically satisfy this balance. The troughs in the composite bounds on the Nichols chart can be exploited during loop shaping, and they allow for substantial gain reductions.

4.1 Random Sensor Noise Error

The advantages of the QFT design can easily be seen in Figs. 14 through 16, which simulate the effects of LVDT sensor noise (random error) in the QFT structure as shown in Fig. 13. The maximum error toleration level of typical aircraft quality actuator RAM LVDTs is about one percent [8]. This uncertain feedback quantity can then cause chattering, or noise, on the output as the feedback control system tries to zero out the error. Unit step responses with 1% sensor uncertainties of both the high forward gain and QFT designs are shown in Fig. 14. In Figs. 14 and 15, the figures (a) are the high forward gain design responses, while the figures (b) are the QFT design responses. The zero-noise responses for the respective designs are shown as dashed lines in all the figures. As can be seen from the figures, the QFT design substantially outperforms the high gain design. The effect of noise is almost negligible in the QFT design, while noise effects are quite visible in the high gain design.

The differences are even more startling as the signal to noise ratio is decreased from 100 to 1. Even with the sensor noise level equal to that of the commanded input, the QFT design nominally tracks the unit step command. But with the high gain design, the output is fairly chaotic, with the output amplified to about six times the commanded signal.

A noise error function shown of IAE type [3] in Eq. (28) is used to quantitatively compare the controller performances.

$$E = \int_0^1 \text{abs}(y_{\text{nominal}}(t) - y_{\text{noise}}(t)) dt \quad (28)$$

The noise error function measures the deviation of the non-zero noise response from the nominal (i.e., zero noise) response.

4.2 Sensor Bias Error

The QFT design handles sensor bias errors better than the high gain design. If the feedback signal is offset from the actual output level due to a sensor bias, control systems will chase after an offset steady state value rather than the commanded

Figure 14: Unit Step Response with LVDT Signal to Noise Ratio of 100.

Figure 17: Unit Step Response with Sensor Bias Error of $\pm 1\%$

Figure 15: Unit Step Response with LVDT Signal to Noise Ratio of 1.

Figure 18: Unit Step Response with Sensor Bias Error of $\pm 10\%$

Figure 16: Noise Error Functions due to the Changes in Signal to Noise Ratio

value. Even with just 1% sensor bias level, the output steady state error of the high gain design, shown in Fig. 17(a), is about $\pm 15\%$ while the error for the QFT design, shown in Fig. 17(b), is about $\pm 1\%$.

The problem worsens in Fig. 18, where the sensor bias error of about $\pm 10\%$ is applied. As seen in Fig. 18(b), the QFT design limits the error to about $\pm 10\%$, while the high gain design, as seen in Fig. 18(a), amplifies the error to about $\pm 100\%$. While the QFT design limits the output error to the level of the bias, the high gain design amplifies the effect of the bias by 10 to 15 times. Again, the

QFT design is better at handling sensor bias than the high gain design.

5 Flight Control System

5.1 Pitch Control System

The true test of the actuator design is to evaluate the actuator performance as an integral component of a flight control system. The flight control system is designed initially with the commonly used first-order actuator model $\frac{20}{s+20}$ [1]. The results are then compared against the fourth-order F-16 stabilator actuator and EHA models. The fourth-order conventional actuator model is

$$\frac{\delta_e}{\delta_{cmd}} = \frac{(20.2)(71.4^2)(144.8)}{(s+20.2)(s^2+105.1s+71.4^2)(s+144.8)} = \frac{N_{act}}{D_{act}} \quad (29)$$

The first-order actuator model is frequently used in text book settings to simplify the model. However, the first-order model does not correctly represent the high frequency phase lag of the actual physical system [6]. A fourth-order actuator model, such as Eq. (29) which accurately describes the phase characteristics of the actual physical system, should be used to design flight control systems. However, the conventional actuator control systems have undesirable amount of phase lag at the upper end of the bandwidth. Thus, the EHA control system is designed to reduce the high bandwidth frequency phase lag. The reduced phase lag can minimize the chances of actuator rate saturation. The EHA model accurately predicts the performance of the physical system and behaves much like a first-order model.

5.2 Simulations

The time domain simulations of the flight control system using different actuator models are shown in Fig. 20. The dotted, dashed, and solid lines correspond to the results for the first-order conventional, the fourth-order conventional, and the EHA models, respectively. Time domain responses among three actuator models are similar and the responses are slightly underdamped. The

Figure 19: A Pitch Control Flight Control System

Actuator Type	t_r (sec)	M_p	t_p (sec)	t_s (sec)
1st Conv.	0.38	1.032	0.85	1.10
EHA	0.36	1.044	0.75	0.95
4th Conv.	0.31	1.030	0.68	0.78

Table 1: The Pitch Flight Control System FOM

FOM are shown in Table 1.

The conventional fourth-order actuator model displays more realistic phase information than the first-order model. The phase lag difference between the first-order and fourth-order models at the high end of the aircraft bandwidth can be as large as 50 degrees [6].

Frequency simulations of the pitch control systems are shown in Fig. 21. The dash-dotted, dashed, and solid lines correspond to the results from the first-order conventional, the fourth-order conventional, and the EHA models, respectively. The first-order and EHA models behave in a similar manner, but the fourth-order model exhibits much more lag at the high bandwidth frequency.

Some lead is added to meet the given frequency domain specifications; specifications which the conventional fourth-order actuator (model) could not meet. Indeed, the conventional fourth-order actuator model suffers from excessive phase lag at high frequency. The phase lead is achieved by low frequency zeros, reducing the phase lag at high fre-

Figure 20: Time Domain Simulations of the Pitch Control System at Mach 0.7 and 30,000 ft Altitude

Figure 21: Frequency Domain Simulations of the Pitch Control System at Mach 0.7 and 30,000 ft Altitude

quencies to within specifications. The main short-fall of using the first-order model is that it does not accurately describe the frequency domain behavior of the actuator. The EHA model, composed of the actuator and compensator components, behaves like the first-order actuator model, yet fully describes its behavior.

6 Conclusions

The Electro-Hydrostatic Actuator offers easier maintainability and higher degree of combat survivability over conventional actuators.

The sub-component dynamic models are used to construct the required EHA model, resulting in a complex feedback structure shown in Fig. 3. A

mathematical model of an advanced EHA is derived from basic device principles. The mathematical model that completely describes the EHA dynamics is required to design a control system. The use of QFT for the control of the actuator establishes a rigorous design procedure and facilitates other EHA control system designs in the future.

The QFT controller design is compared against the high gain design for sensor noise handling characteristics. The effects of sensor noise are almost negligible in the QFT control system response while the effects of sensor noise dominate the high gain control system response. The QFT controller nominally tracks the unit step command even with the signal to noise ratio of 1, while the high gain design fails to track the command.

The EHA model used in a simple flight control system is compared against the same flight control system with the first-order and fourth-order conventional hydraulic actuator models. Some lead is added to the EHA control system to meet the given flight control system frequency domain specifications; specifications which conventional fourth-order actuator (model) can not meet. Thus, the flight control system with the EHA model is more stable with respect to gain variations than the flight control system with a fourth-order conventional actuator model. Furthermore, the amount of lag at the upper bandwidth frequency is reduced with the EHA model, which minimizes the chances for actuator rate saturation. Hence, the controlled EHA behaves like a first-order actuator model, and yet the fidelity of the actuator model has not sacrificed.

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