The Design and Analysis of Integrated Navigation Systems Using Real INS and GPS Data

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1. ABSTRACT

An extended Kalman filter (EKF) is used to combine the information from a Global Positioning System (GPS) receiver and an Inertial Navigation System (INS) to provide a navigation solution. This research compares the results of an actual tightly-coupled GPS/INS integrated system with those results obtained in computer simulation. An analysis of various GPS models was studied to determine the best representation for a system truth model. Measurement rates and their effects upon position accuracies were studied. The results suggest that the computer simulations being used at the Air Force Institute of Technology (AFIT) produce better navigation solutions than are actually possible with real hardware.

2. INTRODUCTION

The United States Air Force (USAF) is engaged in a never-ending search for better navigation solutions. The Air Force Institute of Technology (AFIT) supports this quest by addressing some of the issues which impede progress, require technical explanation, or necessitate further research. Several issues are associated with the integration of a Global Positioning System (GPS) and an Inertial Navigation System (INS) in an extended Kalman filter (EKF) for both computer simulation analysis and actual hardware integration. This paper focuses on GPS error models in the context of a tightly-coupled integration of an Inertial Navigation System (INS) and a Global Positioning System (GPS). In simulation, GPS error models are used in both the system truth and the filter models. Normally, but not always, the greater the number of states the better the estimation of the errors by the model. Several error models were used in the EKF to compare performance.

Due to a lack of hardware, previous AFIT research was

restricted to computer simulations [10]. Unlike past AFIT research, this effort was conducted with actual hardware, in addition to the simulation. The objective of the research was to lay the groundwork for AFIT's real-time, tightly-coupled GPS/INS integrated navigation system and to compare the performance of various GPS models for more accurate simulation results [3].

3. BACKGROUND

The GPS/INS integration is an advantageous combination, especially for military applications. The GPS/INS integration exploits the individual strengths and offsets the weaknesses of each stand-alone system. For instance, the GPS maintains accuracy over a long period of time, while the INS continually drifts from its initial alignment. The GPS is susceptible to jamming, yet the INS is immune to externally radiated interference. Finally, the GPS performs poorly in highly dynamic environments due to the shadowing of satellites and high rates of change in doppler; however, the INS has excellent performance in high dynamic flight scenarios. These complementary characteristics have been well documented throughout the technical literature [5,7,11].

The complementary characteristics of each system can be exploited by combining the systems in an extended Kalman filter (EKF). A brief summary of Kalman filtering theory will be presented, but for more detail the volumes written by Maybeck are excellent references [8]. A Kalman filter is an optimal recursive data processing algorithm which is based upon linear models. The GPS receiver and INS error state models consist of a set of linear state-space differential equations and nonlinear measurement equations. These nonlinearities prevent the use of a linear Kalman filter. Because of this linearity constraint, an EKF is used. The basic idea of the EKF is to relinearize about each state estimate once it has been computed. This relinearizing about newly

declared nominals at each sample time enhances the adequacy of the linearization process.

In an EKF, the state models are a set of nonlinear continuous-time differential equations of the form:

$$\dot{x}(t) = f[x(t), t] + G(t) w(t) \quad (1)$$

where $f[x(t), t]$ is the state dynamics vector which in general is a nonlinear function of the state vector $x(t)$ and of time t . In the GPS/INS integration the state dynamics vector is a linearized model in both the system truth model and the filter models. $G(t)$ is a noise distribution matrix which for this research is an identity matrix without loss of generality. The vector represented by $w(t)$ is a zero-mean white Gaussian noise vector process with a noise strength given by:

$$E\{w(t)w^T(t+\tau)\} = Q(t)\delta(\tau) \quad (2)$$

The measurements of the system are assumed to be discrete-time measurement updates of the form:

$$z(t_i) = h[x(t_i), t_i] + v(t_i) \quad (3)$$

where $z(t_i)$ is the measurement available at time t_i , h is a known vector which is a function of the states and time. The vector h can be either linear or nonlinear. For the GPS/INS integration, h is a nonlinear function of the state vector and time due to the nonlinear nature of the GPS measurements. The vector $v(t_i)$ represents a zero-mean, white Gaussian noise process with noise covariance:

$$E\{v(t_i)v^T(t_j)\} = \begin{cases} R(t_i) & t_i = t_j \\ 0 & t_i \neq t_j \end{cases} \quad (4)$$

The state estimate and covariance are propagated from time t_i to the next sample time t_{i+1} through the integration of the following equations:

$$\hat{x}(t/t_i) = f[\hat{x}(t/t_i), t] \quad (5)$$

$$\begin{aligned} \dot{P}(t/t_i) &= F[t, \hat{x}(t/t_i)]P(t/t_i) \\ &+ P(t/t_i)F^T[t, \hat{x}(t/t_i)] + G(t)Q(t)G^T(t) \end{aligned} \quad (6)$$

With the incorporation of discrete-time measurements, $z(t_i)$, the EKF is accomplished through the following

equation:

$$\hat{x}(t_i^+) = \hat{x}(t_i^-) + K(t_i) \{z_i - h[\hat{x}(t_i^-), t_i]\} \quad (7)$$

$$P(t_i^+) = P(t_i^-) - K(t_i)H^T[t_i, \hat{x}(t_i^-)]P(t_i^-) \quad (8)$$

$$\begin{aligned} K(t_i) &= P(t_i^-)H^T[t_i, \hat{x}(t_i^-)] \{H[t_i, \hat{x}(t_i^-)] \\ &P(t_i^-)H^T[t_i, \hat{x}(t_i^-)] + R(t_i)\}^{-1} \end{aligned} \quad (9)$$

where,

$$F[t; x_n(t)] = \left. \frac{\partial f[x(t), t]}{\partial x} \right|_{x=x_n(t)} \quad (10)$$

and

$$H[t; x_n(t_i)] = \left. \frac{\partial h[x, t_i]}{\partial x} \right|_{x=x_n(t_i)} \quad (11)$$

The superscript, $-$, indicates the value at a time before the incorporation of a measurement, and the superscript, $+$, is used to signify a value after a measurement has been taken.

4. TIGHTLY-COUPLED INTEGRATION

Although there are numerous integration methods with the Kalman filter, this research uses a tightly-coupled feedforward approach. The tightly-coupled implementation uses the most basic of information from each sensor (sometimes referred to as raw or unprocessed information). The raw information, measured pseudoranges and satellite ephemeris data, is used directly in one EKF. Loosely-coupled GPS/INS integration schemes normally have two Kalman filters. The loosely-coupled integration uses the position outputs of a Kalman filter solely dedicated to a particular sensor as input to a master EKF. The literature has pointed out potential stability problems caused by filters driving filters. This instability problem arises because the cross-correlation and temporal correlation of data are not adequately modeled or compensated for in the GPS/INS integrated filter's design. Most loosely-coupled designs reduce the measurement updates to around one every ten seconds. This technique stretches out the time between measurements and assists in decorrelating the incoming data. Thus, a tightly-coupled approach was chosen for this research because it provides a more optimal solution than the loosely-coupled.

Since this effort had never been undertaken at AFIT, a

feedforward scheme was used. The term *feedforward* in this context refers to the fact that the GPS/INS solution was not fed back to reset the INS or aid the GPS receiver. By implementing a feedforward type of approach, the GPS and INS outputs could be collected, stored and post-processed at a later date. The ability to do post-processing made the integration proceed more smoothly. In the feedforward configuration, the INS operates independently. In other words, the INS errors will continue to grow larger with time. Kalman filtering theory is based upon linear models; the accuracy of the integrated navigation solution is directly related to the adequacy of the linearized error models. In simulation, the feedforward implementation works well because the system truth models are linearized as well as the filter model. Since both the system truth and filter models are linear, large INS errors can be accurately estimated by the filter. However in the real world, the actual system error dynamics are nonlinear and linear models are unable to estimate INS errors properly. The extended Kalman filter solution is only as good as the models being employed. By conducting both a simulation and hardware integration, the differences caused by nonlinearities should become more apparent.

An integrated system with a feedback implementation estimates the errors in the inertial system, but then feeds these errors back to the INS in the form of corrections. This method does not allow the inertial errors to grow unchecked, and the adequacy of the linearized model is enhanced [8]. Over the long term, the feedback implementation is more accurate than the feedforward approach for actual hardware implementations. A disadvantage of the feedback method is that the INS is dependent on the EKF estimates. Without proper integrity monitoring, this reliance on the EKF could allow the INS to be reset to erroneous positions and possibly become unstable.

5. PROCEDURE

The equipment used in this research was a Litton LN-94 ring laser strapdown system and NAVSTAR's XR-4PC and XR-5PC GPS receivers. The GPS receivers are capable of receiving the unclassified GPS code only; therefore, selective availability was a factor in determining navigation solutions. These receivers are PC-based and readily make the pseudoranges and satellite ephemeris data available to the user. The XR-4PC is a two-channel receiver [12]. The XR-5PC is a six-channel receiver also providing information on up to eight satellites in view [13].

The first step in combining the GPS and INS was to con-

duct the integration in simulation. The simulation was performed in MATLAB code. This code was named MATSOFE [4], and directly patterned after the USAF simulation software, Multimode Simulation for Optimal Filter Evaluation (MSOFE) [1]. A static scenario was selected as a starting point to avoid timing difficulties. The duration of each run was 17 minutes. After completing the simulation, the MATLAB code was enhanced to accept the LN-94 and GPS receivers actual data and accomplish the post-processing. The simulations consisted of a ten run Monte Carlo analysis, while the hardware integration was composed of ten sets of data taken on different days of the week at various times.

6. OBJECTIVE

The objective of this research was to determine the accuracy of the various GPS error models in estimating the errors. The error models' performances with real data versus computer simulation results were of additional interest. As stated earlier, the test was done under static conditions. This allowed the same INS data file to be used with each GPS error model. Also, the GPS antenna location and INS were collocated at a surveyed position, so that the errors could be determined.

7. GPS ERROR MODELS

Two LN-94 and four GPS error models were studied in this research. Recent AFIT research has employed a 39-state INS error model in conjunction with a 30-state GPS error model as the system truth model for computer simulations [10]. A portion of this research was directed at a comparison of the 30-state model with three other models to see how closely each filter estimated the errors. The 2-state model containing receiver clock bias and drift error states is used in the filter model, since these states are the dominant sources of error.

Four GPS error models were used in this research: the 30-state system truth, the 22-state, the 10-state, and the 2-state filter models. The state sizes are given assuming four satellites are being used. In reality, some of the actual data taken from the hardware had eight satellites available. All available data was used in the tightly-coupled integration; therefore, in the case of eight satellites, the 30-state model is actually a 58-state model. The following model descriptions will clarify this point further. If further description is necessary please refer to the thesis document [3].

7.1 30-State GPS Model

The 30-state GPS system truth model is comprised of five types of error sources. By far the most significant of these errors are the user clock bias and the user clock drift. The remaining 28 states is composed of seven error states for each of the four satellites. These seven errors are composed of the code loop, tropospheric, ionospheric, satellite clock, and three position errors in an orthogonal frame. The code loop and atmospheric errors are modeled as first-order Markov processes. The remaining errors are modeled as random constants. (See Table 2.)

7.2 22-State GPS Model

The 22-state model is very similar to the 30-state model. The difference between the two models is that the three directional error states in the 30-state model are combined into one line-of-sight error. The 22-state model is an experiment by this researcher. The model is a melding of the 10-state and 30-state models. Since the pseudorange comprises only one line-of-sight measurement, it seems reasonable that all of the satellite line-of-sight error can be represented in one state. Of course, the dynamics driving noise will be somewhat different, and the error variance used is just the sum of the three directional variances given in the 30-state model. (See Table 2.)

7.3 10-State GPS Model

The basis for this model was taken from the MSOFE GPS/INS integration sample problem. The model has not been used in past AFIT research, but was used in the development of the MATLAB m-files used in writing MATSOFE. The model contains the receiver related errors, such as user clock drift and receiver noises in the first two states. The remaining errors are the satellite-related (satellite clock and orbital ephemeris) and path-related (tropospheric and ionospheric delays) errors, which are lumped into the last eight states. The last eight states (two per each of four satellites) are simple pseudorange and pseudorange-rate error. (See Table 3.)

7.4 2-State GPS Model

The 2-state filter is rather small when compared to the 30-state system truth model. A model reduction of this order is possible because of the size of the user clock bias and drift errors with respect to the remaining states. Due to the filter order reduction, the dynamics driving noise, Q , must be increased to compensate for added uncertainty in the model. These are normally the only

two GPS error states used in integrated filters.

8. INS ERROR MODELS

The system truth and filter models discussed below were derived from the 93-state Litton model [6]. The first 13 states consist of position, velocity, attitude, and vertical channel errors. These states are classified as *general* errors corresponding to standard Pinson error model states and states associated with barometric altimeter aiding of the vertical channel. The *trend* states are modeled as first-order Markov processes and represent gyro, accelerometer, and baroaltimeter errors. The gyro and accelerometer bias errors are modeled as random constants. Six accelerometer and gyro initial thermal transients are set up as first-order Markov processes, and the gyro compliance error states are seen as biases in the system truth model.

8.1 39-State INS Model

AFIT often uses the 39-state model as the system truth in simulation. Although the 93-state model is the most accurate characterization available, the high degree of dimensionality makes it computationally burdensome. Previous AFIT research conducted by Lewantowicz and Negast have demonstrated that the 39-state is a sufficient substitute for the 93-state model without a significant loss of accuracy [7].

8.2 11-State INS Model

The 11-state filter model is used in both the hardware integration and computer simulation extended Kalman filters. The 11-state INS error model consists of the X,Y, and Z components of vector angle from the true to computer frame; the X,Y, and Z components of vector angle from the true to platform frame; X,Y, and Z components of error in computer velocity; the error in vehicle altitude above the reference ellipsoid; and the error in lagged inertial altitude.

9. RESULTS

The results are divided into two parts: (1) the performance of the GPS error models and (2) the effect of an increased measurement update rate. The research studied other aspects of GPS/INS integration, but due to space constraints, they are not presented here.

9.1 Filter Performances

The navigation solution was determined with the EKF using five different filters. The first four filters

employed the 39-state INS model in conjunction with each of the previously described GPS models (30-state, 22-state, and 10-state.) The fifth filter consisted of the 11-state INS and 2-state GPS filter. Table 1 presents the results of each GPS model used in the EKF with real data.

Of course, the EKF outperformed the individual INS and GPS over the 17 minute run. At the completion of the run, the INS had drifted nearly 1000 ft in both latitude and longitude while the results from Table 1 show that the best GPS-only navigation solution had nearly twice the errors of the lowest order filter (13 states). Thus as expected, the EKF provides a better position solution than either system alone.

Table 1: Filter Comparison

Filter	Temporally Averaged Mean Error (ft) ^a	Temporally Averaged Standard Deviation
69-state (39-INS + 30-GPS)	118.17	69.93
61-state (39-INS + 22-GPS)	112.94	52.99
49-state (39-INS + 10-GPS)	117.27	57.95
41-state (39-INS + 2-GPS)	137.54	61.25
13-state (11-INS + 2-GPS)	140.37	75.29
XR-4PC(only)	252.42	24.25
XR-5PC(only)	318.34	62.34

- a. In order to compare the performances of each filter, a temporally averaged mean error was calculated. The errors from each of the ten data sets associated with each measurement time were summed together and then the mean error over the complete run interval was calculated. The temporally averaged mean error for latitude, longitude, and altitude was squared, summed together, and the square root taken. This left a scalar term which was used to compare performance.

There were some unexpected results from this research. The largest state filter, the 69-state, which in

past AFIT research has been used as the system truth model in simulation, did not follow the errors of the GPS receiver as well as the 61-state and 49-state models [10]. This is an example of the fact that the sheer number of error states does not necessarily make a better filter model. For instance, the INS models were also compared in the 41-state and 13-state filters and a reduction of 28 INS states represented only a 2.83 ft increase in temporally averaged mean error. Of course, there are several variables which come into play. For instance, this research used a static scenario. A dynamic flight profile would most certainly increase the observability of some states in the INS error model and increase the error margins between the 11- and 41-state INS model. Overmodeling seems to have occurred in the case of the 69-state GPS error representation. When too many inaccurately modeled states are used, the filter can be overly constrained by trying to fit noise to the specified parameters.

The use of actual data revealed some characteristics that were not established in the original GPS error models. The GPS receivers had noticeable jumps in the user clock. These jumps were rather commonplace in occurrence, but their magnitudes varied considerably. These sudden receiver clock bias shifts were more noticeable in the XR-4PC than the XR-5PC. The presence of these clock bias jumps was only detectable on the residuals; the EKF reacted quickly enough that the position estimates were relatively unaffected.

9.2 Measurement Update Rate

Because all of the GPS ephemeris and pseudorange data was collected for post-processing, a beginning measurement update rate was selected at 0.05Hz. This kept the data files memory requirements to a manageable size. Since a Sun SPARC20 workstation was being used, the measurement update rate was not a factor from a computational loading standpoint; however, with AFIT's upcoming mobile, real-time EKF, the computation power must be considered. The research considered the trade-offs between computational loading and navigational accuracy.

By taking measurements at a higher frequency, the Kalman filter has less time to propagate before the next update. While a greater frequency of updates should improve accuracy, it most certainly will increase computation time. A test case was run to determine how much accuracy could be gained, by reducing the measurement update period from 20 seconds to 10 seconds. The 13-state filter was used for this study and data was collected from the XR-4PC and XR-5PC receivers. The

results showed that by increasing the frequency of measurement updates from 0.05Hz to 0.1Hz resulted in a decrease in temporally averaged mean error. The 0.05Hz or 20 second period measurement had a temporally averaged mean error of 108.89 ft, while the 0.1Hz or 10 second period measurement updates was at 98.58 ft. The GPS-only solution had a temporally averaged mean error of 262.90ft.

10. CONCLUSIONS

Using real data, the 22-state and 10-state GPS models outperformed the 30-state model. The 30-state GPS model may be overmodeled. The 22-state model's use of line-of-sight errors are more representative of pseudorange errors than the larger 30-state's X,Y, and Z components of error. The number of states in an error model does not necessarily dictate that it more accurately depicts the real world and may actually increase estimation error.

Measurement update frequency is directly proportional to computational burden and inversely related to estimation error. The quicker the updates, the less error propagation time, and thus, lower errors. By doubling the measurement update frequency in this research, a gain of approximately 10% in temporally averaged mean error was gained at a considerable increase in computation time. Therefore the measurement update frequency is dependent upon the system application's accuracy requirements and computing power available. Since this is a tightly-coupled system, the previous statements may not directly apply to a loosely-coupled integration.

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12. APPENDIX

Table 2: 30- and 22-State GPS Error Models

State Number	Definition
1	User Clock Bias
2	User Clock Drift
3	SV 1 code loop error
4	SV 1 tropospheric error
5	SV 1 ionospheric error
6	SV 1 clock error
7	SV 1 X component of position error*
8	SV 1 Y component of position error*
9	SV 1 Z component of position error*
10	SV 2 code loop error
11	SV 2 tropospheric error
12	SV 2 ionospheric error
13	SV 2 clock error
14	SV 2 X component of position error*
15	SV 2 Y component of position error*
16	SV 2 Z component of position error*
17	SV 3 code loop error
18	SV 3 tropospheric error
19	SV 3 ionospheric error
20	SV 3 clock error
21	SV 3 X component of position error*
22	SV 3 Y component of position error*
23	SV 3 Z component of position error*
24	SV 4 code loop error
25	SV 4 tropospheric error
26	SV 4 ionospheric error

Table 2: 30- and 22-State GPS Error Models

State Number	Definition
27	SV 4 clock error
28	SV 4 X component of position error*
29	SV 4 Y component of position error*
30	SV 4 Z component of position error*

* The 22-state model combines the X, Y, and Z components of position error into one line-of-sight error for each satellite

Table 3: 10-State GPS Error Model

State Number	Definition
1	User Clock Bias
2	User clock drift
3	SV 1 range error
4	SV 1 range-rate error
5	SV 2 range error
6	SV 2 range-rate error
7	SV 3 range error
8	SV 3 range-rate error
9	SV 4 range error
10	SV 4 range-rate error