The Effects of Quadrature Sampling Imbalances
on a Phase Difference Analysis Technique

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ABSTRACT

This paper studies the effects of amplitude and phase imbalances between the in-phase (I) and quadrature (Q) channels of a Digital Radio Frequency (RF) receiver on a phase analysis processing technique. Particular attention will be given to the effect that these errors have on the detection and accurate frequency resolution of simultaneous signals. Phase analysis processing of digitized time domain data to extract a radar's carrier frequency can be very reliable if the quadrature phase and 1:1 amplitude relationship can be maintained between the I and Q channels across the frequency band of interest. In practice it is very difficult to closely match the channels with respect to these parameters over wide bandwidths. The literature to present [2,3] has shown these imbalances to be the main sources of error in quadrature detectors. This paper will illustrate both through simulation and from data collected from a 250 MHz, 8bit, digital receiver, how these errors along with other anomalies affect the performance of phase analysis processing and specifically how they affect detection and determination of simultaneous signals.

1 INTRODUCTION

In the electronic warfare environment, the capability to extract a radar's carrier frequency and any intentional modulations in near real time is vitally important. Also, since this environment can have a high signal density, the capability to detect and hopefully to resolve simultaneous signals is important. Several digital processing schemes can be used to extract such signals from digitized data at the baseband level (in our case D.C. to 250 MHz). The Fast Fourier Transform (FFT) can produce a power versus spectrum plot from which one can quickly determine a radar signal's carrier frequency. However, frequency versus time information is lost in the FFT, and if a computer is used to analyze the Fourier coefficients and estimate the

modulation parameters, the processing time will be exorbitant.

Another technique (phase analysis) examines the frequency content of a sampled data sequence by calculating the change in phase between successive samples to find the instantaneous frequency. Utilizing this technique can provide frequency versus time information without most of the time consuming processing associated with the FFT. With phase processing, a signal's modulation characteristics ie. (fixed frequency, linear and nonlinear chirp, frequency stepped, phase coded) can be determined. Extraction of a signal's phase implies the use of a quadrature detection scheme which can provide the instantaneous phase. It also provides the added benefit of being able to distinguish between input frequencies above the local oscillator from those below. Therefore the 250 MHz Digital Receiver has an effective bandwidth of 500 MHz. This feature is also a benefit if the FFT is used to process one channel and the relative phase between the channels is detected (ie. +/- 90 degrees).

The quadrature detection process however, contains error sources (primarily amplitude and phase imbalances) that degrade the performance of the system. The magnitude of the effect that these errors have on phase analysis processing depends on the type of information being extracted. If for example we are only interested in intentional modulations then these errors can perhaps be tolerated. However, if the accurate determination of the carrier frequency or the detection and determination of simultaneous signals is of importance then these errors become important and ways of correcting for them must be considered.

2 QUADRATURE DETECTION AND PHASE ANALYSIS PROCESSING

The block diagram of a quadrature detector is shown in Figure 1. The input signal is divided into two paths.

In the first path the signal is mixed with the output signal from the local oscillator and is designated the I channel. The other path is mixed with a local oscillator that has been shifted by 90 degrees and is designated the Q channel. These two channels (I and Q) can be thought of as the real and imaginary components of a complex signal. To mathematically illustrate these concepts the complex notation of [1] will also be used in this paper. Therefore the input signal is represented by:

$$x(t) = \operatorname{Re}\left\{\frac{X_m(t)}{2} e^{j\omega_c t}\right\} \quad (1)$$

where $x_m(t)$ is also a complex signal which contains all of the modulation information of $x(t)$. In the ideal system, the I and Q signals would be of the form:

$$y_i(t) = \operatorname{Re}\left\{\frac{X_m(t)}{2} e^{j(\omega_c + \omega_{10})t}\right\} + \operatorname{Re}\left\{\frac{X_m(t)}{2} e^{j(\omega_c - \omega_{10})t}\right\} \quad (2)$$

and

$$y_q(t) = \operatorname{Im}\left\{\frac{X_m(t)}{2} e^{j(\omega_c + \omega_{10})t}\right\} - \operatorname{Im}\left\{\frac{X_m(t)}{2} e^{j(\omega_c - \omega_{10})t}\right\} \quad (3)$$

Low pass filters then remove the $\omega_c + \omega_{10}$ components. These signals are then sampled by and ADC (analog/digital converter) at a rate greater than or equal to four times the highest input signal frequency. Given that the quadrature and in-phase samples are denoted by Q_m and I_m ($m = 1, 2, \dots$) respectively, the instantaneous phase is:

$$\theta_m = \tan^{-1}\left(\frac{Q_m}{I_m}\right) \quad (4)$$

$$\theta_{m-1} = \tan^{-1}\left(\frac{Q_{m-1}}{I_{m-1}}\right) \quad (5)$$

and the instantaneous frequency is:

$$f_m = \frac{\theta_m - \theta_{m-1}}{2\pi(t_m - t_{m-1})} \quad (6)$$

where t_m is the sample time.

Thus the analysis of phase difference versus time illustrates how the frequency content of a signal varies and theoretically we should be able to derive the carrier frequency on a sample by sample basis.

3 SYSTEM SIMULATION

This section concentrates on phase and gain errors caused by the RF signal paths, inequalities in the mixers and the effects of the A/D conversion process. The derivation of the error signal is identical to that given in [1] with the exception that the phase and gain errors are frequency dependent.

In order to gain some insight into the effect of these errors a computer simulation of the quadrature detector and an ideal 8 bit A/D converter was generated in which both the amplitude and phase errors were lumped into the quadrature channel such that the Q channel had the form:

$$y_q(t) = \operatorname{Im}\left\{(1 + \beta(\omega_c)) \frac{X_m(t)}{2} e^{j[(\omega_c - \omega_{10})t + \sigma(\omega_c)]}\right\} \quad (7)$$

where β is the amplitude mismatch and σ is the phase mismatch.

A convenient way of illustrating the effect of amplitude and phase errors on the complex signal made up of $y_i(t)$ and $y_q(t)$ is to represent it as a phasor rotating in the complex plane. Figure 2a. shows the phasor diagram of a 50 MHz sinusoidal input signal without amplitude and phase errors. As can be seen, the vector traces out a circular path. Figure 2b. shows the effect of an 8 degree phase error on the phasor diagram. The error distorts the coordinate system and gives the appearance that the vector traces out an ellipse at 45 degrees with respect to the I and Q axis. An amplitude ratio (I/Q) of 0.9 is shown in Figure 2c. In this case the phasor appears to trace an ellipse whose major axis is coincident with the I axis.

In the ideal case, the phase difference of an unmodulated, single, sinusoidal input signal would be constant with time. When amplitude and phase errors are introduced, they create a time varying phase difference which is shown in Figures 3a. and 3b. Thus the phase difference and consequently the instantaneous frequency for a given point in time can now take on a range of values. Some representative numbers appear in the table below.

Table 1.

Pk/Pk Deviation	Figure 3a.	Figure 3b.
Phase Difference	0.0616 radians	0.1074 radians
Frequency	9.7980 MHz	17.1003 MHz

As shown in Table 1. small mismatches in phase and amplitude and create large deviations in the calculated frequency value. During simulation, two methods were utilized to compensate for the mismatches. In the first method an average of the phase difference was used to calculate a frequency estimate. This method is computationally very simple but the mismatches are not corrected. A discussion of this method is given in the experimental section. The second utilized an orthogonalization procedure [2] in which the I and Q channels are multiplied by correction coefficients. The relationship used to derive the corrected signals is:

$$\begin{bmatrix} I_c \\ Q_c \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix} \quad (8)$$

where:

$$a = \frac{\cos(\sigma)}{1+\beta} \quad (9)$$

and

$$b = \frac{-\sin(\sigma)}{1+\beta} \quad (10)$$

Application of this procedure to the 50 Mhz signal (phase and amplitude mismatch; 8 degrees and 0.1 respectively) produced the following results:

Table 2.

	Pk/Pk Deviation
Phase Difference	0.0116 radians
Frequency	1.8393 MHz

The problem with the orthogonalization procedure is that the correction coefficients must be derived. Since the input signal frequency is not known a priori and the imbalances are frequency dependent, this becomes a complex problem.

Random noise on the digitized I and Q signals also has an effect on the processing and can be divided into two types; analog and digital. Analog noise arises from signal conditioning, sample and hold circuitry and of course the input signal. Digital noise is due to the inherent quantization noise of the A/D converter. If the quantization noise is considered to be white and uncorrelated with the input, then we can lump the analog and digital components together into an effective noise. This noise can also be represented as quadrature components. This noise effectively spreads out the positions of the vectors in the phasor diagram over loci surrounding the noiseless points [4]. More will be said about noise factors in the final section.

4 SIMULTANEOUS SIGNALS

In the perfectly balanced quadrature detector the two signal condition can be detected from the phase difference data by detecting the phase spiking that occurs when multiple signals are present. The introduction of amplitude and phase errors into the system creates ambiguities in this detection process. Although many different cases (ie. different combinations of amplitude and frequency) can be studied, we have chosen three cases to illustrate the effects of amplitude and phase errors. In case one, the amplitudes of the two signals ($f_1 = 50$ Mhz and $f_2 = 150$ Mhz) are very close (amplitude ratio f_2 to f_1 of 0.98). Figure 4a. shows the phase difference in the balanced case. Introduction of an 8 degree phase error is shown in 4b. As can be seen, the effect on the detection process is minimal. In case two, the signal

frequencies are the same as above, however, f_2 is now 20 dB down from f_1 . Figure 5. Shows the phase difference of the balanced case. Note that this is very similar to Figure 3b. which illustrated an 8 degree phase error. In the phasor domain (Figure 6.) we see what appears to be a large amplitude imbalance. Thus as the signals' amplitude separation increases, the simultaneous signal case becomes indistinguishable from the single frequency phase error situation. In the final case, the amplitudes of the two signals are equal and the phase difference is a constant, with an amplitude equal to the average of the two signals' phase differences. Amplitude and phase errors in the system would give the appearance of the simultaneous signal condition but the frequency derived would be the average of the two inputs or 100 Mhz.

In cases 2 and 3, ambiguities develop when amplitude and phase errors are present. For proper detection of the simultaneous signal case these errors must be eliminated or reduced significantly. The accurate resolution of two or more simultaneous signals utilizing the phase spiking information is an area that requires further research.

5 EXPERIMENTAL SETUP AND RESULTS

Figure 7. shows the experimental setup. The input was a CW signal set at 3000 MHz and the tuner frequency was varied from 2750 to 3250 MHz by P.C. control through an IEEE-488 interface. The output of the tuner into the quadrature detector was down-converted to 750 to 1250 Mhz. Inside the quadrature detector the signal was split and mixed with a 1000 MHz local oscillator signal. The I and Q channels were then filtered (300 Mhz cutoff) and sampled at a one giga-sample per second rate by an HP54111D Digital Oscilloscope. The digital oscilloscope has an 8K high speed memory which was down-loaded to a Microvax for further processing.

The first experiment was to determine what the amplitude and phase errors of the system were and how they varied with frequency. A software approach to this problem was chosen rather than using hardware [5]. A 1024 point FFT was taken of each channel in 1 Mhz steps from 2999 MHz to 2750 MHz. By using the real and imaginary components of the FFT for each channel, relative phase and amplitude information was derived. The relative phase and amplitude ratio is defined to be:

$$\theta_r(\omega) - \theta_q(\omega) - \theta_i(\omega) \quad (11)$$

and

$$A_r = 20\log_{10}\left(\frac{Y_q(\omega)}{Y_r(\omega)}\right) \quad (12)$$

Figure 8a. and 8b. show the results of this procedure. It should be noted that this quadrature detector was made from off the shelf components. Much better amplitude and phase matching can be achieved with custom designs [6].

After the amplitude and phase errors were determined, the sinusoidal input data was processed to extract the carrier frequency. As in the simulation, 50 Mhz is used as a representative example. The phase difference plot is shown in Figure 9. and some numerical values are given below.

Table 3.

	Pk/Pk Deviation
Phase Difference	0.1890 radians
Frequency	30.0834 MHz

The phase difference contains the same type of time varying behavior along with an added high frequency noise component. Figure 10. is the phasor plot which shows the amplitude and phase errors and also the noise perturbation that was described in a previous section. The source of this noise component could be a combination of phase noise on the input signal, noise introduced by the signal conditioning circuitry in each channel of the ADC, and limitations of the dynamic accuracy of the ADC.

The dynamic accuracy of an ADC can be illustrated by the use of effective bits which is defined to be:

$$\text{effective bits} = N - \log_2 \left(\frac{\text{actual rms error}}{\text{ideal rms error}} \right) \quad (13)$$

where N is the number of bits, ideal rms error is assumed to be the quantization noise generated by an ideal ADC and the actual error is the difference between an actual data record of a digitized sinewave and a software generated best fit sinewave. Nonlinear effects such as harmonic distortion, aperture uncertainty, and noise cause a reduction in the number of effective bits.

The number of effective bits for the ADC used in the experimental setup falls off from a high of about 7 bits (below 1 Mhz) to a low of 5.2 bits when sampling at 1 GHz [8].

The orthogonalization procedure that was described in a previous section can be implemented practically in a narrowband receiver. In a wideband EW receiver however, this implementation is not practical in its present form because the input signal frequency is not known a priori. For the experimental setup, a much less complex method of estimating the carrier frequency was implemented which averaged the phase difference data. For the test case shown in Figure 9. the estimated carrier frequency was 50.2067 Mhz which was calculated by averaging 40 ns of data. If an estimate of the primary carrier frequency can be derived then a set of precalculated correction

coefficients can be applied to the input data in an effort to orthogonalize the I and Q channels and subsequently to detect the ambiguous simultaneous signal condition.

6 CONCLUSIONS

This paper has illustrated through simulation and experiment the effects of amplitude and phase errors on a phase analysis processing technique. It was shown that these error effects can appear to be identical to a simultaneous signal condition. Thus using this technique in a practical EW receiver could cause incorrect conclusions unless the amplitude separation of the signals' is small. Much more research needs to be done in this area and in the area of resolving two signals with the phase difference information. In a narrowband receiver, the orthogonalization procedure can be utilized to correct for amplitude and phase errors. In a wideband EW receiver however, the implementation of such a technique is very complex because the imbalances are frequency dependent and the input signal is not known a priori. A much less complex procedure of estimating the carrier frequency was used in this paper. By deriving a frequency estimate for the primary carrier by averaging, previously calculated correction coefficients can be applied to the I and Q channels in an effort to detect the ambiguous simultaneous signal condition. Other factors influencing phase processing were also discussed, including random noise on the signal and ADC effects.

References

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Fig 1. Quadrature Detector

Fig 2c. Amplitude ratio 0.9 (Q/I)

Fig 2a. Without amplitude and phase errors

Fig 3a. Amplitude ratio 0.9 (Q/I)

Fig 2b. 8 degree phase imbalance

Fig 3b. 8 degree phase imbalance

Fig 8b. Phase accuracy response

Fig 9. Experimental: 50 MHz

Fig 10. Experimental: 50 MHz