A/I SUPERVISOR FOR F-4 WEAPON COEFFICIENT OPTIMIZATION

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ABSTRACT

Weapon coefficient optimization is essential for accurate bomb delivery. The existing technique is unreliable and requires overnight computations. A hybrid technique of non-linear regression analysis coupled with gradient descent was developed, reducing computation time to 2 or 3 minutes. An additional Artificially Intelligent supervisor was employed to detect algorithm instabilities, make corrections, and re-start the process. The supervisor employs the first law of good Electrical Engineering (when the thing goes unstable, turn down the gain). The new algorithm not only produces better solutions, but operates without operator assistance. The result will be more accurate weapon delivery.

HISTORICAL PERSPECTIVE

Ballistics coefficients for the F-4 program were traditionally derived from an Air Force program which takes a long time to run, requires repeated submissions, and sometimes requires up to two weeks before reasonable answers are obtained. In this application, intelligence is required to insure that the calculations are meaningful, and guide the algorithm through the choice of initial values and control parameters. In the past this process was performed by "expert" engineers. This project, along with 20 others were targeted as problems for application of Artificial Intelligence (A/I).

The author is a member of the Neural Networks group for F-16 software development. Neural Networks, a subset of A/I, deal primarily with paradigms which determine their own rules as opposed to the rule-driven applications of more traditional A/I. Although this application requires the more traditional rule-driven approach, the author felt that a solution was obtainable, and a software specification was issued in September.

BACKGROUND

This problem is a special case of a class of problems which deal with the minimization of an energy function, defined as a summed square error between a set of data and a model.

$$J = \sum (\text{data} - \text{model})^2 / 2 \quad (1)$$

In the F-4 application, the data consists of a set of bombing tables, and the model consists of a ballistic model containing release conditions, drag parameters and separation effects.

In the traditional approach, one begins with an initial "guess" for the set of ballistics parameters. The ballistic parameters are varied one at a time. A simulation is performed for each entry in the bombing table, and the results are compared to the entries in the bombing table. An improved value for each parameter is obtained which reduces the energy function in Equation (1). This lengthy process can extend to overnight, and convergence to the appropriate values is not always achieved, requiring repeated submissions.

THE NON-LINEAR REGRESSION APPROACH

An alternative approach to this problem is a non-linear regression analysis program. The data is related to the model by an Equation,

$$y_i = f(\{x_{ij}\}, \{b_j\}) + e_i \quad (2)$$

where

y_i is the i th sample from the bombing table,
 $\{x_{ik}\}$ represents the set of release conditions,
 $\{b_j\}$ represents the set of ballistic parameters which need to be determined,
 f is the prediction made by the model corresponding to the set release conditions and ballistic parameters,
 e_i is an error term. Error is usually present because of modeling errors, noise in the data, and possible errors in the data.

The simulation program provides a prediction corresponding to the data in Equation (2),

$$\hat{y}_i = f(x_{ij}, \hat{b}_j) \quad (3)$$

Subtracting Equation (3) from Equation (2),

$$y_i - \hat{y}_i = f(\{x_{ij}\}, \{b_j\}) - f(\{x_{ij}\}, \{\hat{b}_j\}) + e_i \quad (4)$$

Taking a first order Taylor Series approximation, Equation (4) can be approximated by

$$y_i - \hat{y}_i \approx \sum_{j=1}^k \frac{\partial f}{\partial b_j} (b_j - \hat{b}_j) + e_i + e_i' \quad (5)$$

The second error term appears because of possible non-linearities in the model.

Equation (5) can be expressed in matrix form,

$$\Delta Y = H (B - \hat{B}) + E \quad (6)$$

The minimum energy solution for Equation (6) is provided by the regression analysis solution,

$$B - \hat{B} = (H^T H)^{-1} H^T \Delta Y \quad (7)$$

The process is repeated to reduce the effects of non-linearities in the model. As the estimate approaches the correct solution, the non-linear effects go to zero.

LIMITATION OF NON-LINEAR REGRESSION

The non-linear approach sometimes becomes unstable when deriving ballistic coefficients with highly correlated effects. Equation (7) requires the inversion of a large matrix. This inversion process can become ill-conditioned, producing erroneous answers, and causing the software to fail.

GRADIENT SEARCH TECHNIQUES

Equation (7) is actually evaluated using intermediate matrices,

$$\Delta B = C^{-1} \nabla F \quad (8)$$

where the sample covariance matrix, C , and the summed gradient vector, ∇F , are calculated on-the-fly as one performs simulations for the entries in the bomb table.

Gradient search techniques provide an update of the form

$$\Delta B = L \nabla F \quad (9)$$

where L is a diagonal matrix. In classical filtering techniques, L is referred to as the gain factor. Gradient search techniques do not have the fatal flaw of instability exhibited by the non-linear approach. Unfortunately, they tend to be extremely slow.

Let D denote the diagonal matrix

$$D_{jk} = C_{jj} \quad \text{for } k = j \quad (10)$$

$$= 0 \quad \text{for } k \neq j \quad (11)$$

Consider the estimate, analogous to Equation (8),

$$\Delta B = D^{-1} \nabla F \quad (12)$$

The solution in Equation (12) is similar to the technique employed in the traditional F-4 software. It regresses on each ballistic coefficient separately. Equation (12) is also a special case of gradient search. Gradient searches have the drawback of being extremely slow, but do not become ill-conditioned.

HYBRID TECHNIQUE

Equation (10) provides an obvious selection of the coefficients to be employed for gradient search. Furthermore, the similarity between Equation (8) and Equation (12) suggests a hybrid technique,

$$\Delta B = m [(1-l)C + lD]^{-1} \nabla F \quad (13)$$

where l, m are numbers between zero and one. A supervisor is required to select values of l, m which will insure convergence of the algorithm, and yet do so in a timely manner.

SUPERVISED LEARNING

The supervisor keeps track of a history of the energy function for each pass through the bombing table. In addition, the software prescribes an allowable range for the ballistic coefficients (this feature already exists in the traditional software). The value of l will begin at a nominal value of between 0 and 5%, and a relaxation factor, m between 80% and 100%. One initially attempts a solution close to the non-linear regression analysis solution.

At the end of each pass, the update provided by Equation (13) is attempted. Ill-conditioning of the inversion process can be eliminated by increasing the factor l . This will slow convergence, avoids the alternative of failure and re-submission.

FIRST LAW OF GOOD ELECTRICAL ENGINEERING

The first law of good electrical engineering can be stated as follows: "When the thing goes unstable, turn down the gain". Instability can be recognized when the energy function increases from pass to pass. When this occurs, one reduces the gain factor (m) by a factor of 10% to 20%, and re-initiates the process until stability is achieved. Additionally, the change suggested by Equation (13) may produce answers outside the physically realizable values of the parameters. This also calls for a reduction in the the gain factor.

IMPLEMENTATION

The implementation required one follow-up visit with the programmer. Several program "bugs" were detected and removed.

PERFORMANCE

As anticipated, the run time was reduced from over-night to a few minutes. What was not anticipated was the improvement in the final solutions! The resulting error for weapon GBU-12 was reduced by 50%. Theoretically, the traditional software should be capable of obtaining the optimal solution; practically, the computational resource requirements may preclude this. An attempt was made to improve or verify the new GBU-12 solution with the original software. Unfortunately, the traditional software failed to converge in the time allotted.

CONCLUSION

A complete verification of the hybrid algorithm would require evaluation against a variety of weapons. Unfortunately, this is beyond the scope and funding of the current F-4 effort. The new software does, however, provide the analysis with a new tool for deriving ballistic parameters as required by the F-4 Users Group. This was the goal of the original task which must therefore be considered as a success.

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