Hybrid Calculation of Invariant Moments

Vicky Robinson Steven Rogers
Matthew Kabrisky James Mills

Air Force Institute of Technology

1 Abstract

A hybrid system for computing a position, scale, rotation invariant (PSRI) feature set was investigated for use in automatic pattern recognition. Specifically, the first four $(p+q)^{th}$ (zeroth, first, second, and third) ordinary moments of a set of six objects were optically calculated. The process began with the creation of computer generated detour phase holograms. The resulting holograms were then used to produce wavefronts of the form $x^p y^q$. Next, each wavefront was imaged onto the object plane. The Fourier transform (FT) of this product (object transmittance times $x^p y^q$) was optically calculated. A camera recorded the intensity pattern in the FT plane. A normalization technique was developed to account for the wide variation in beam intensities. The center value of this pattern was found using a frame grabber and an image analyzer. When scaled according to the power meter readings, the center value was the ordinary moment of the object. All of the moment values were then digitally processed to form a PSRI feature set. Finally, the Euclidean distance separation between the object's PSRI features were calculated. This indicated that the hybrid calculated invariant moments could be used for pattern recognition.

2 Introduction

A lot of research has been conducted on pattern recognition algorithms which can be used for autonomous weapons. One variable is the feature set used. The moment feature set was first investigated in 1962 by Ming-Kuei Hu (11; 8:920) and was recently shown to be suitable for tactical target recognition (6). However, it has not been extensively used because of large digital processing requirements. This constraint could be alleviated by performing part of the calculations optically. This paper details a hybrid system to calculate a position, scale, rotation invariant (PSRI) moment feature set for six objects (square, circle, triangle, tank, truck, and airplane). The ordinary moments of the objects were computed optically, then the PSRI feature set was found through digital post processing. The Euclidean distances between the moment vectors were calculated to test for pattern recognition applicability.

3 Moments

The $(p+q)^{th}$ ordinary moments of an object described by $f(x,y)$ is defined as:

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y) dx dy \quad (1)$$

The various moments contain information necessary to classify the object. For example (1:442; 6):

1. m_{00} indicates the object's scale,
2. m_{10} and m_{01} indicates the object's location, and
3. m_{20}/m_{02} indicate the object's height-to-width ratio.

Therefore, the ordinary moments are highly variable to changes in position, scale, and rotation of the object.

Mathematical manipulation of the ordinary moments can eliminate these variations. The central moment, μ_{pq} , is defined with respect to the object's centroid, resulting in shift invariance (8:925).

$$\mu_{pq} = \sum_{r=0}^p \sum_{s=0}^q \binom{p}{r} \binom{q}{s} (-\bar{x})^{p-r} (-\bar{y})^{q-s} m_{rs} \quad (2)$$

where $\bar{x} \equiv \frac{m_{10}}{m_{00}}$, $\bar{y} \equiv \frac{m_{01}}{m_{00}}$, and the combination of "a" things taken "b" at a time is defined as:

$$\binom{a}{b} \equiv \frac{a!}{b!(a-b)!} \quad (3)$$

The normalization shown below will add scale invariance (8:925).

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{(2+p+q)/2}} \quad (4)$$

Finally, rotation invariance can be produced from the algebra of invariants. The result is a PSRI feature set (2:395).

$$\phi_1 = \eta_{20} + \eta_{02} \quad (5)$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \quad (6)$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \quad (7)$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \quad (8)$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (9)$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \quad (10)$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} - \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (11)$$

4 Optical Calculations

The ordinary moments can be calculated optically with the setup shown in Figure 1 (2:395). An uniform amplitude plane wave is incident on plane P_1 . Its transmittance, $f(x,y)$, is imaged through a double Fourier transform (FT) onto plane P_2 ,

Figure 1: Optical Moment Generator

with transmittance $x^p y^q$. The FT of this product produces the following wavefront at plane P_3 .

$$U(f_x, f_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) x^p y^q e^{-j2\pi(f_x x + f_y y)} dx dy. \quad (12)$$

The on-axis value is

$$U(0, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) x^p y^q dx dy \quad (13)$$

which is, by definition, the ordinary moment of the object placed in plane P_1 (2:396).

This system requires a different mask for each moment. The method used to produce the masks for this research was the detour phase computer generated hologram (5). With this method, the paper on which the hologram was printed was divided into equally spaced cells. The desired wavefront, $A(x, y)e^{j\phi(x, y)}$, was sampled at the center of each cell. A rectangle was then printed inside each cell, having the dimensions and position calculated as follows (5:127, 133).

$$h_{nm} = A_{nm} d_y \quad (14)$$

$$w_{nm} = w \quad (15)$$

$$c_{nm} = d_x(2\pi n + \phi_{nm})/2\pi \quad (16)$$

where nm represents the cell located in column n and row m of the hologram, d_x and d_y are the sampled spacing in the x and y directions, respectively, h_{nm} is the height of the rectangle, w is the width of the rectangle, c_{nm} is the locations of the center of the rectangle, ϕ_{nm} is the sampled phase, A_{nm} is the sampled amplitude, and n is an integer. Since the wavefront needed for moment generation had no phase ($x^p y^q$), ϕ was set to zero in equation 16. The result is $c_{nm} = nd_x$ which means that each rectangle was centered in the appropriate cell. The amplitude was normalized so that the maximum value across the entire wavefront was one. Thus, the maximum rectangle height was the height of the cell. To obtain the best tradeoff between intensity and wavefront quality, the width of every

rectangle was one half the cell width (7:498-499). The CGH was printed onto a transparency and photoreduced onto a high resolution glass slide. The final product transmitted light through the rectangles.

The hologram was placed in plane P_1 of the optical system shown in Figure 2. An uniform amplitude plane wave was incident on the hologram. The FT of the hologram transmittance was located at plane P_2 . Since the hologram was a sampled wavefront, the FT was periodic. A spatial filter, which passed only one order of the transform, was placed in plane P_2 and a second FT taken, resulting in the desired wavefront at P_3 .

Figure 2: Optical Wavefront Generator

The photoreduced CGH used in the research had 100×100 cells and was approximately 1×1 cm. The resultant wavefront was approximately $.6 \times .6$ cm. A CCD camera was placed at plane P_3 and the wavefront recorded with a VCR. The tape was then taken to a frame grabber system. Image-Pro software by Data Translation Inc., was used to obtain intensity line profiles at various positions across the wavefronts. The CGH for producing the wavefront "x" is shown in Figure 3. The printer threshold resulted in rectangles of

Figure 3: x CGH: Linear Ramp in Intensity

uniform height along the CGH edges. Thus, the wavefront had uniform intensity edges. However, the center was a good ramp (Figure 4). The calculation of the seven PSRI fea-

*Note: Dashed line represents the desired profile.

Figure 4: Resultant wavefront From "x" CGH - x Direction

tures required the production of the first four orders of the ordinary moments (zeroth, first, second, and third). Since the wavefront intensities varied with position, the same hologram could be used to produce a variation in both the x and y direction by rotating the mask. Therefore, five CGHs were created (x , x^2 , xy , x^2y , and x^3). The zeroth order moment was calculated using a 1×1 cm transmitting square.

5 Experiment

The ordinary moments were calculated with the setup shown in Figure 5. The largest object dimension was 1.6 cm. This necessitated the use of lens L_3 to magnify the $x^p y^q$ wavefront. The wavefront at plane P_4 was 1.8×1.8 cm. For a given moment, the appropriate hologram was centered in plane P_1 , and the wavefront recorded at plane P_3 . The camera was then moved to plane P_6 , the objects sequentially centered in plane P_4 , and the resultant intensity patterns (plane P_6) recorded onto the VCR. The frame grabber and Image-Pro software were again used to analyze the data. The threshold utility was used to remove the background energy, leaving only the center portion of the intensity pattern, which was subsequently enlarged. The intensity value of the center pixel was used to determine the ordinary moment of the object.

The intensities of the moments, both for a given object and for the different objects, varied greatly. To record all of the values with one camera and avoid saturating the line profile, a beam-splitter and neutral density filters were placed in

the system. The portion of the beam deflected by the beam-splitter was directed into a power meter. The meter reading was recorded for the beam without filters (P_b). The filters were then placed in the system and the meter reading (P_a) noted as the intensity pattern was recorded onto the VCR. Using these data, the actual value of the ordinary moment was:

$$M = \frac{P_b(m + P_a)}{P_a} \quad (17)$$

where m is the center pixel intensity value found with the Image-Pro software.

The meter readings were not stable, so $\pm$ variations were recorded. With this information, three ordinary moment values were determined. The minimum value was calculated with the minimum P_b and the maximum P_a , the nominal value with the nominal P_a and P_b , and the maximum value with the maximum P_b and the minimum P_a . Using equations 2-11, the PSRI feature set was then calculated (Table 1). The min, nom, and max labels refer to the ordinary moments used for these calculations.

6 Data Analysis

After being calculated, the PSRI feature set was tested for pattern recognition potential. The testing consisted of determining whether the PSRI vectors were separable using the Euclidean distance.

The Minkowski distance between two vectors is defined as (4):

$$M_N = \sqrt[N]{\sum_{i=1}^M |x_i - y_i|^N} \quad (18)$$

where N is an integer. When $N = 2$, the Minkowski distance is termed the Euclidean distance.

The Euclidean distance between each object's nominal vector and all other vectors was calculated (Table 2). An error occurred if the distance to any other object vector was less than the distance to the maximum and minimum vectors of the given object. (The errors are indicated by an asterisk

in Table 2). There were four errors, resulting in a 95.5% separation. These results indicated that the hybrid method implemented in this research produced a feature set usable for pattern recognition.

Figure 5: Experimental Setup

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7
Square							
min	7.11	30.25	0.00	0.02	0.00	0.11	0.00
nom	6.51	23.03	0.00	0.01	0.00	0.07	0.00
max	5.90	16.90	0.00	0.01	0.00	0.04	0.00
Circle							
min	11.02	24.21	0.00	0.05	0.00	0.26	0.00
nom	9.91	21.77	0.00	0.04	0.00	0.17	0.00
max	8.77	18.81	0.00	0.02	0.00	0.10	0.00
Triangle							
min	15.50	104.65	0.97	1.87	2.24	19.07	1.14
nom	14.63	90.44	0.70	1.34	1.17	12.75	0.56
max	13.41	72.12	0.47	0.89	0.53	7.56	0.24
Truck							
min	46.98	1903.58	31.56	50.36	1653.46	2091.11	1138.60
nom	43.52	1429.48	22.96	33.60	801.91	1282.75	626.62
max	38.80	1123.05	16.41	24.23	397.20	785.14	274.81
Tank							
min	59.42	3357.86	110.02	83.15	7916.16	4818.06	760.73
nom	56.77	2814.79	80.61	62.66	4437.30	3320.50	378.26
max	52.89	2217.30	52.31	41.34	1922.12	1921.41	-33.90
Plane							
min	31.77	680.05	0.38	5.06	6.83	131.64	1.50
nom	28.39	515.99	0.25	3.60	3.27	81.50	0.85
max	24.82	377.10	0.15	2.38	1.37	46.10	0.41

Table 1: PSRI Moments

	Square nom	Circle nom	Triangle nom	Truck nom	Tank nom	Plane nom
Square min	7.25	8.93	61.99	2154.49	6214.84	493.00
nom	0.00	3.63	69.10	2159.22	6218.10	500.15
max	6.16	6.32	75.17	2163.25	6220.88	506.23
Circle min	*4.67	2.68	67.52	2158.27	6217.44	498.77
nom	*3.63	0.00	70.00	2159.92	6218.59	501.23
max	*4.79	3.18	73.00	2161.91	6219.97	504.21
Triangle min	84.34	85.25	15.63	2094.08	6169.92	416.26
nom	69.10	70.00	0.00	2107.50	6180.38	431.30
max	50.15	51.02	19.10	2122.66	6191.75	450.26
Truck min	3456.07	3456.66	3411.25	1365.94	*3266.84	3159.91
nom	2159.22	2159.92	2107.50	0.00	4399.26	1818.96
max	1435.82	1436.65	1377.05	793.26	5062.51	1046.25
Tank min	9879.29	9879.63	9849.49	8176.66	3845.47	9681.00
nom	6218.10	6218.59	6180.38	4399.26	0.00	5965.68
max	3494.09	3494.78	3444.34	1649.46	2968.45	3157.16
Plane min	670.60	671.69	601.76	1706.23	5874.29	171.62
nom	500.15	501.23	431.30	1818.96	5965.68	0.00
max	357.54	358.61	288.78	1916.18	6041.14	143.39

Table 2: PSRI Moment Distances

* ERROR

7 Bibliography

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