Co-Designed Radar-Communication Using Linear Frequency Modulation Waveform

Michael J. Nowak[1], Zhiping Zhang[2], Yang Qu[2], Dimitri A. Dessources[2], Michael Wicks[3], and Zhiqiang Wu[2]
Air Force Research Laboratory[1], Wright State University[2], University of Dayton Research Institute[3]

***Abstract*—As electromagnetic spectrum availability shrinks, there is growing interest in combining multiple functions, such as radar and communications signals, into a single multipurpose waveform. Historically mixed-modulation has used orthogonal separation of different message signals in different dimensions such as time or frequency. This research explores an alternative approach of implementing an in-band, mixed-modulated waveform that combines surveillance radar and communication functions into a single signal. The contribution of this research is the use of reduced phase-angle binary phase shift keying (BPSK) along with overlapped (channelized) spread-spectrum phase discretes based on pseudorandom noise sequences to encode multiple messages in a single pulse. The resulting mixed-modualted signal provides a low data rate communications message while minimizing the effect on radar performance. For the purpose of this research, radar performance will be evaluated in terms of power spectral density, matched filter auto-correlation for target detection, and the ambiguity function.**

***Index Terms*—Radar Communication, Linear Frequency Modulation, M-sequence, Reduced Phase Magnitude**

I. Introduction

The expansion of consumer and wireless devices has placed increasing demand on signal bandwidth. This increased demand on a finite resource is in turn driving interest in more efficient ways to use available bandwidth. One approach is through mixed modulation of existing waveforms to achieve complementary objectives.

Historically mixed modulated signals have been designed to include some degree of orthogonality between the different waveforms to preclude interference. Such approaches usually involve use of time division multiple access (TDMA) or code division multiple access (CDMA), or the use of orthogonal frequency division multiplexing (OFDM). With the evolution of digital signal processing another approach is mixed modulation through intended modulation on pulse (IMOP). In the past IMOP was challenging due to both the obvious cross interference concerns and the limits of signal processing technology [1]. However, advances in both digital electronics and signal processing have opened the door to re-exploring IMOP approaches.

One option in implementing an IMOP scheme is the use of binary M-sequences with small magnitude phase discrete changes. Previous work in this area has been reported by Kowatsch [2] [3] and Song [4]. However, the focus of their work was in the use of spread spectrum pseudonoise and linear frequency modulated (LFM) waveforms primarily as a communications means and there were no limitations placed on the magnitude of the associated phase change. As a direct result, the deleterious effect on the ambiguity function of the LFM pulse was not considered.

In order to reduce the effects of the phase discretes and improve message recovery, this paper will explore the use of binary M-sequences and relaxed phase discrete magnitudes (e.g. $\phi_\delta \ll \frac{\pi}{2}$). M-sequences exhibit high in-phase and low out-of-phase auto-correlation properties and have lengths that can satisfy the required duration to endure adequate bit energy at the receiver. In addition, the length of longer, higher order M-sequences will allow relaxed phase discrete magnitudes through integration of the message at the receiver.

The additional benefit of the M-sequences is the possibility of channelized bit streams by selecting preferred pair M-sequences such that two message channels can be encoded into a single pulse. Through judicious selection of primitive polynomials and seeds, preferred pairs of M-sequences can be generated that have high auto-correlation properties but low cross-correlation.

Specifically, the contribution of this paper is to explore hybrid signals that transmit both radar and communications information in a single radar pulse width through a series of short duration phase changes to the LFM radar pulse (phase discretes) that encode the information in-band with the radar signal. The challenge is to find a modulation scheme that provides adequate signal efficiency and a satisfactory bit error rate while minimizing radar degradation. As will be seen, the combination of spread spectrum encoding combined with smaller phase changes provides a balance between communications signal efficacy and satisfactory radar performance.

The paper is organized as follows: Section II introduce the digitally sampled LFM waveform. Section III explains intentional modulation on pulse (IMOP) to embed communication message in the LFM waveform via M-sequences and channelization. Section IV and section V discuss the performance of reduced phase embedded communication message and LFM radar waveform respectively. In section VII, the results of numerical simulation using representative radar performance parameters are provided. Finally, section VIII provides a summary of the findings.

II. Linear Frequency Modulated (LFM) Waveform

Since it is one of the more common radar waveforms, this paper considers a LFM signal as the baseline radar waveform. The basic LFM pulse linearly sweeps a frequency band of width B over a pulse width of t_p. As a result the instantaneous frequency of the pulse changes linearly from $0 \leq t \leq t_p$. Since frequency is the time derivative of the phase angle, the instantaneous phase angle of the LFM signal, $\phi(t)$, can be expressed as

$$\phi(t) = \int \left(\dot{B}_w t + f_0\right) dt = \frac{\dot{B}_w}{2} t^2 + f_0 t + \phi_I \qquad (1)$$

where $\dot{B}_w$ is the bandwidth rate of change, f_0 is the initial frequency, and the constant of integration, ϕ_I, is the initial phase angle which can be assumed to be zero without loss of generality.

Let's consider a discretely sampled LFM radar pulse. In next section, we show how to implement the proposed embedded modulation through the sampled LFM pulse. For a pulse length of time t_p, the sampling frequency, f_s, determines the number of samples, n_p, in a transmitted pulse:

$$n_p = f_s t_p \qquad (2)$$

or, in terms of time

$$t_i = \frac{n_i}{f_s} \quad \text{where} \quad 0 \leq t_i \leq t_p, \quad i = 1, 2, 3, \ldots, n_p \qquad (3)$$

From this, the LFM instantaneous phase angle can be recast in terms of the sample number, n, as opposed to time.

$$\phi_L(n) = \frac{\dot{B}_w}{2}\left(\frac{n}{f_s}\right)^2 + f_0 \frac{n}{f_s} \qquad (4)$$

The benefit of LFM signals for radar detection lies in its excellent performance on autocorrelation and Doppler tolerance. The detial will be illustrated in section V.

III. Intentional Modulation on Pulse (IMOP)

As opposed to other 'orthogonal' mixed modulation approaches such as code or time division multiple access (CDMA or TDMA), this paper will address intentional modulation of the radar signal by the communications signal, or in-band modulation. This can be viewed as using the radar signal, comprised of a radar pulse, as the carrier and the communications message as the modulating signal. Further, the use of traditional BPSK encoding that relied on large phase magnitude changes created cross interference problems with the primary (radar) signal that argued against using in-band modulation.

The difference in the approach taken in this paper is to use small angle binary phase shift keying (BPSK) to minimize cross-interference. Further, the use of spread spectrum encoding is considered both to improve bit signal to noise ratio (SNR) through integration and to explore the possibility of low data rate, channelized communications.

Implementation of the mixed modulation is done by first digitizing the LFM signal into a vector of instantaneous phase angles, $X_L(n)$. Separately, the desired communications message is encoded into a vector of phase changes, $X_M(n)$. The digitized LFM signal is phase shifted by the communications vector using element-wise multiplication. The resulting mixed-modulated, LFM-communications signal is then converted back to an analog signal and unconverted to a higher power carrier wave for transmission.

The means of encoding the communications message is by using binary M-sequences to encode each bit. As pseudorandom binary sequences, M-sequences of length $2^m - 1$ are produced using primitive polynomial functions of degree m with a binary-valued, $(0, 1)$, seed of length m [5][6]. Encoding of an individual bit is done by multiplying the M-sequence by the appropriate phase change, ϕ_δ, corresponding to a '1' or '0'. The resulting message vector, $\mathbf{M_1(n)}$ or $\mathbf{M_2(n)}$, is composed of sequentially adjoining a series of the phase modified M-sequences.

The primary benefit of M-sequences for the mixed modulated waveform lies in their length and correlation properties. Individual M-sequences exhibit high auto-correlation values while minimizing out-of-phase (e.g. $\tau \neq 0$) auto-correlation values. Further, "preferred pairs" of M-sequences generated by different starting seeds also exhibit excellent cross-correlation between the paired bit sequences.

Since the "preferred pairs" of M-sequences are selected so as to minimize cross-correlation between the overlapping bit sequence, a channelized approach can be used to simultaneously transmit two bit sequences. In a channelized system two distinct digital message vectors are created that represent the instantaneous phase angle at each sample instant. The communication message vector, $\phi_M(n)$, that is modulated on the LFM carrier consists of the sum of the two individual message vectors. Since each individual message vector consists of a series of phase changes of $\pm\phi_\delta$, the sum of each vector element will be one of three values: $-2\phi_\delta$, 0, or $2\phi_\delta$. By keeping ϕ_δ small, it should be possible to keep the overall phase discrete modulation change at a single sample within a tolerable phase shift.

IV. Performance of Reduced Phase Embedded Communication Message

The hybrid combination of a radar and communications signal is beneficial in ensuring adequate bit energy at the receiver. In a monostatic radar, the transmitted pulse must have adequate signal energy to complete a round-trip path to and from the radar. This creates a reduction in signal energy by $\frac{1}{r^4}$ at the radar receiver. The communications signal on the other hand, is processed at the target receiver so it undergoes a one-way, $\frac{1}{r^2}$ reduction. Coupled with the normal high carrier power associated with most radars, this works to an advantage for the communications signal.

Still, each series of phase adjusted basis vectors corresponding to a single encoded bit must be long enough to have an adequate bit signal to noise ratio (SNR) at the receiver. In turn, the length of the M-sequence basis vector, n_λ, required to transmit each bit directly affects the number of message bits in a pulse (e.g. pulse bit density).

At the same time, the objective is to minimize degradation to the radar pulse performance while maximizing pulse bit density. Traditional communications methods, such as BPSK, use large phase changes ($\pm$ 90 degrees) in order to encode this information. However, large abrupt phase changes increases signal bandwidth, introduces undesirable sidelobes and adversely affects the performance of the radar. Therefore, smaller magnitude phase discretes combined with the longer M-sequence encoding will be considered. The following subsections provide more details on a closed form equation to compute required bit-sample length, n_λ.

A. Reduced Phase Margin Bit Error Rate

BPSK implementations typically maximize the distance between the '1' and '0' bits to improve detection and reduce bit error rates during message recovery. Bit error rate (BER) can be expressed as the distance between the different bit values measured in the I-Q plane. The general expression for the BER of a BPSK signal with bits separated by d_{min} is the following

$$[BER] = \mathbf{Q}\left(\frac{d_{min}/2}{\sigma}\right) \tag{5}$$

where the $\mathbf{Q}$-function is defined as

$$\mathbf{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-z^2}{2}} dz \tag{6}$$

Figure 1 depicts a situation where the separation between bits varies as a function of phase angle. The bit separation is a function of the magnitude of the phase difference, ϕ_δ and the signal bit energy, E_b: $d_{min} = 2\sqrt{E_b}\sin\phi_\delta$. In this case the signal energy is a function of the M-sequence bit duration (e.g. bit-sample length, n_λ). For 180 degree phase difference BPSK encoding $\phi_\delta = 90$ and $d_{min} = 2\sqrt{E_b}$.

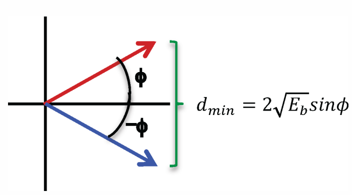

Fig. 1. Reduced Phase Angle Bit Distance

In addition, the signal is assumed to be affected by additive white Gaussian noise (AWGN) with $\sigma^2 = \frac{N_0}{2}$. Substituting into the BER equation

$$[BER] = \mathbf{Q}\left(\frac{2\sqrt{E_b}\sin(\phi_\delta)/2}{\sqrt{\frac{N_0}{2}}}\right) = \mathbf{Q}\left(\sqrt{\frac{2E_b}{N_0}}\sin(\phi_\delta)\right) \tag{7}$$

The equation can be expressed in terms of the required bit energy to signal ratio as a function of desired BER and phase magnitude change:

$$\frac{E_b}{N_0} = \frac{1}{2}\left[\frac{\mathbf{Q^{-1}[BER]}}{\sin\phi_\delta}\right]^2 \tag{8}$$

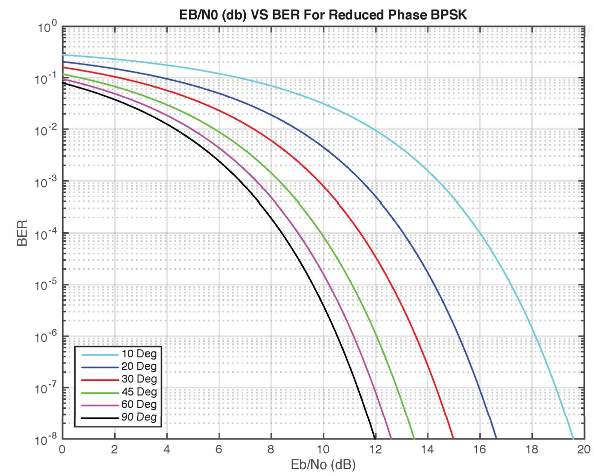

Fig. 2. Reduced Phase Angle Bit Error Rate

Figure 2 depicts the relationship between reducing the phase magnitude and the increasing change in $\frac{E_b}{N_0}$. In the figure, the 90 degree line is identical to the traditional BPSK method where the bits are separated by 180 degrees (e.g. $\pm$ 90 degree phase change) and the $\mathbf{Q}$-function reduces to the standard value of $\mathbf{Q}\left(\sqrt{\frac{2E_b}{N_0}}\right)$. As angle decreases the required bit signal-to-noise value increases for a comparable BER. This in turn requires a longer bit duration in the pulse and a lower bit throughput per pulse. Still, for low signal rate communications such as administrative communications, the use of reduced phase magnitude IMOP of a radar pulse might provide a satisfactory solution.

Using either equation (8) or the plot in Figure 2, the required SNR for a specific BER and phase angle can be determined. This, in turn, drives the next question of bit-sample length in order to achieve the desired SNR value.

B. Radar Range Equation - Bit Energy

With the required E_b/N_0 value established, the required sample length can be determined. The LFM signal is a constant envelope waveform so the energy in one bit can be expressed as

$$E_b = \frac{V^2}{2}T_b = P_R T_b \tag{9}$$

where T_b is the time duration of the bit phase discrete, V is the magnitude of the received LFM waveform, and P_R is the bit power at the receiver relative to a 1Ω resistance. Since the communication message is digitized, the pulse consists of a series of discrete time values, or samples, whose time duration depends on the sampling rate, f_s, of the analog-digital converter.

To achieve enough bit energy at the receiver, the duration of the phase change for a single bit must extend over a set period of time (e.g. set number of samples). The time duration, T_b, of a bit is merely the product of the number of samples and the sample duration, $\frac{1}{f_s}$, of each sample. Therefore, the energy expression for a single *transmitted* bit can be rewritten as

$$E_T(\lambda) = P_T \frac{n_\lambda}{f_s} \tag{10}$$

where n_λ is the number of samples that make up the bit, f_s is the sampling rate, and G_T is the transmitter gain.

The effect of distance on the received energy must also be considered since radar energy falls off as a square of the distance. Unlike the radar pulse, the communication message is received and processed at the receiver so the one-way radar range equation, or Friis equation, can be used:

$$E_R = \frac{E_T G_T A_e}{4\pi r^2 L} \tag{11}$$

In this equation E_R is the received energy at the aircraft, G_T is the transmitter aperture gain, r is the radar to aircraft distance, and L are energy losses. Further, the effective antenna aperture can be expressed as:

$$A_e = \frac{G_R \lambda^2}{4\pi} \tag{12}$$

where G_R is the receiver aperture gain and λ is the signal wavelength. Incorporating both A_e and equation (10) for the bit energy, E_T, into the one way range equation allows rewriting of the bit energy, E_b, at the receiver as

$$E_b(n_\lambda, r) = \frac{P_T n_\lambda G_T G_R \lambda^2}{(4\pi r)^2 L f_s} = \left[\frac{P_T G A_e}{(4\pi)^2 L f_s}\right]\left(\frac{n_\lambda}{r^2}\right) \tag{13}$$

The first bracketed term in the equation is determined by the performance characteristics of the radar transmitter while the second parenthetical term determines the energy of the communication message pulse at the receiver as a function of sample length and range.

Message bit detection and probability of misidentifying a message bit depends on signal-to-noise ratio. For message recovery the bit energy to noise power spectral density is crucial in ensuring an acceptable BER. For thermal noise, the noise power spectral density, N_0, can be expressed as

$$N_0 = \kappa T_0 F \tag{14}$$

where F is the receiver noise factor, κ is Boltzmann's constant ($1.3806488 \times 10^{-23} m^2 kg s^{-2} K^{-1}$) and T_0 is the antenna aperture noise temperature in degrees Kelvin (normally assumed to be 290 degrees).

Rewriting the energy equation in terms of required bit SNR and sample length:

$$\frac{E_b}{N_0} = \frac{P_T n_\lambda G_T G_R \lambda^2}{(4\pi r)^2 L f_s \kappa T_0 F} \geq [SNR]_{req} \tag{15}$$

$$n_\lambda \geq \frac{[SNR]_{req}(4\pi r)^2 L f_s \kappa T_0 F}{P_T G_T G_R \lambda^2} \tag{16}$$

This constraint provides a lower limit on the number of samples required to achieve a a bit energy level that ensures an adequate BER at the receiver.

V. Performance of Radar Waveform

The underlying assumption in integrating an in-band message into a radar pulse is that the radar performance will not be significantly impacted by the addition of short duration phase discretes into the LFM signal. Two possible metrics of performance are the signal autocorrelation function and the power spectral density (PSD) plot.

The autocorrelation plots are useful to determine how the radar receiver's matched filter processes the received signal, and the PSD provides a measure of how much energy is outside the the main bandwidth of the pulse. For an LFM signal, which is the use case for this paper, the majority of the spectral energy is in the bandwidth of the LFM signal. However, adding short duration phase discretes causes energy to occur outside the main lobe of the PSD. Since energy must be conserved these sidelobes represent energy that is not available in the main lobe for radar processing. However, a more complete metric that encompasses the autocorrelation and the PSD is the radar ambiguity function.

The ambiguity function is essentially the Fourier Transform of the cross-correlation of the radar pulse with its matched filter. The radar ambiguity function provides a measure of the response of a matched filter to a finite energy signal in the presence of a time delay, τ, and Doppler frequency shift, ν. The periodic ambiguity function is well-known in the literature and can be expressed as:

$$|\chi(\tau, \nu)| = \left| \int_0^T u(t) u^*(t-\tau) e^{2\pi j \nu t} dt \right| \tag{17}$$

where the variables τ and ν represent the time delay and Doppler-shifted frequency of the returned signal, respectively.

The ideal (and hypothetical) ambiguity function has a 'thumbtack' appearance at the origin (e.g. $\tau = 0$ and $\nu = 0$) for a Dirac delta impulse signal. In this situation the radar can provide accurate target position as long as there is no delay or Doppler frequency shift. More realistically, radars need to have some tolerance to time delays and Doppler shifts so the ideal 'thumbtack' response is not necessarily a desired matched filter response.

Figure 3 is the ambiguity function for the basic LFM signal. The plot illustrates good accuracy for determining target position in time but allows some flexibility for frequency error due to Doppler effects. As a comparison, Figure 4 shows the ambiguity function of a LFM signal with an embedded BPSK modulated communication message using phase changes of $\pm$ 90 degrees to encode a '1' or '0'. As Figure 4 clearly shows, while time delay remains the same, Doppler tolerance has greatly diminished.

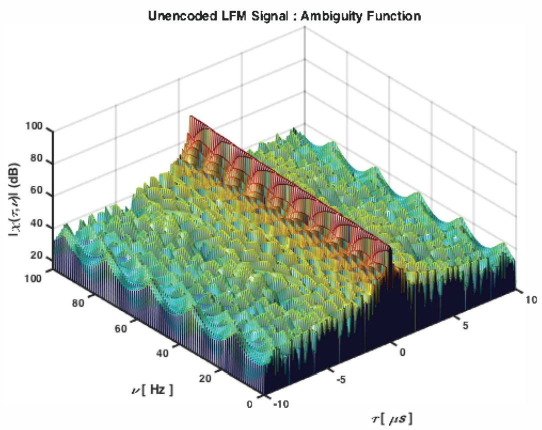

Fig. 3. Ambiguity Function of Unencoded LFM Signal

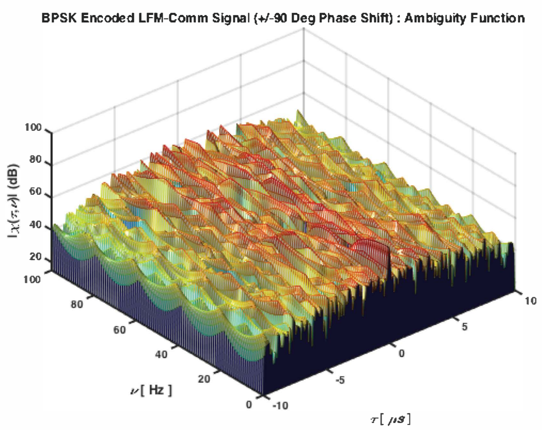

Fig. 4. LFM-Comm Ambiguity Function: BPSK Encoding (+/-90 Degree Phase Shift)

Figure 5 depicts the power spectral density (PSD) and autocorrelation of the same mixed LFM-Comm signal employing $\pm$ 90 degree constant phase shift for message encoding. The PSD plot clearly shows the increased bandwidth ceased by the use of large phase angle changes. Further, the autocorrelation plot exhibits fairly high sidelobes beyond the first sidelobe.

The combination of the poor ambiguity function performance (when compared to the baseline LFM ambiguity plot) and the wider bandwidth of the PSD may not be acceptable radar performance. For these reasons we will explore the use of new BPSK modulations with reduced phase magnitude changes.

VI. Numerical Results

To illustrate the feasibility of in-band modulation using reduced magnitude phase discretes, numerical simulations have been performed using a LFM pulse with the following parameters in Table I.

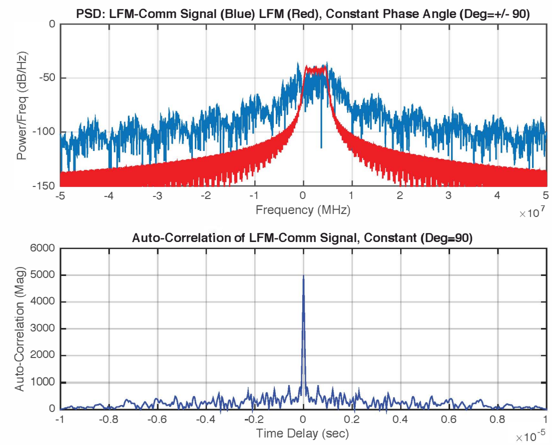

Fig. 5. PSD/Autocorrelation Encoded LFM Pulse ($\phi_\delta = \pm$ 90 Deg)

TABLE I
LFM Pulse Parameters

Pulse Width (PW)	10μsec
Bandwidth (BW)	5 MHz
Center Frequency (f_c)	2.5 MHz
Sampling Frequency (f_s)	5×10^8 Hz
Signal Gain, A	1

The pulsewidth and sample rate provide a digitized signal of 5001 samples that can be used to encode a message sequence. The plots that follow illustrate the autocorrelation, PSD and ambiguity function response, respectively, for the basic unencoded LFM pulse. These provide a baseline for comparison with the LFM pulses that have embedded M-sequence messages.

For the encoded pulse, the carrier is assumed to be a C-band radar with a 10 KW peak power operating at 5 GHz. The overall gain of the transmitter antenna and receiver is assumed to be 2000 with 50dB of attenuation due to transmitter, receiver, measurement, and processing losses. In addition, a BER of 10^{-5} was desired with a maximum operating range of 500 KM (300NM). A series of numerical simulations were done using equation (16) to determine the minimum sample length for various phase discrete values from ± 10 through a traditional BPSK of ± 90 degrees. As expected the most stressing was for ± 10 degrees of phase shift with a minimum sample length of 44.469 samples. At the other extreme, the use of ± 45 or ± 90 degree phase shifts required significantly less bits to achieve the required BER - on the order of 1.

The following plots show a reduced phase angle of 15 degrees ($\phi_\delta = \frac{\pi}{12}$). At 15 degrees the required sample length was 11.19 samples. To ensure adequate BER for unforeseen errors at low phase angles, a 6-bit preferred primitive polynomial was chosen to provide a bit-sample length of 63.

The following primitive polynomial were used to generate two 63 bit binary preferred pair M-sequences with two initial seeds, (1 0 0 0 1 1) and (1 0 0 0 1 1).

$$\mathbf{P}(\mathbf{x}) = x^6 + x^5 + x^3 + x^2 + 1 \tag{18}$$

The chosen primitive polynomial and seeds above were found using an exhaustive search to find a preferred pair that exhibited very low cross-correlation characteristics. The resulting binary (1, 0) bit stream of each M-sequence was then converted to a bipolar (1, -1) sequence. This bipolar sequence constitutes the basis function for a bit. An individual bit is encoded as a '1' by multiplying this basis sequence by ϕ_δ or as '0' by multiplying the basis sequence by $-\phi_\delta$.

As previously mentioned, the radar pulse in this example consists of 5001 samples. Based on the bit sequence length 79 bits can be encoded for each M-sequence, thus providing 158 bits of information per pulse. In general, the maximum bit throughput per pulse is

$$n_b = \frac{t_p}{f_s n_\lambda} \tag{19}$$

where t_p is the length of the radar pulse, f_s is the digital sample rate, and n_λ is the required bit-sample length to encode a bit.

The effect of using a smaller phase discrete magnitude for encoding the bit stream can be clearly seen in the radar ambiguity plots for channelized BPSK of two bit streams using a 15 degree phase shift and an M-sequence length of 63 samples (Figure 6). In the plot the use of smaller phase discretes preserves the Doppler tolerance of the LFM radar pulse.

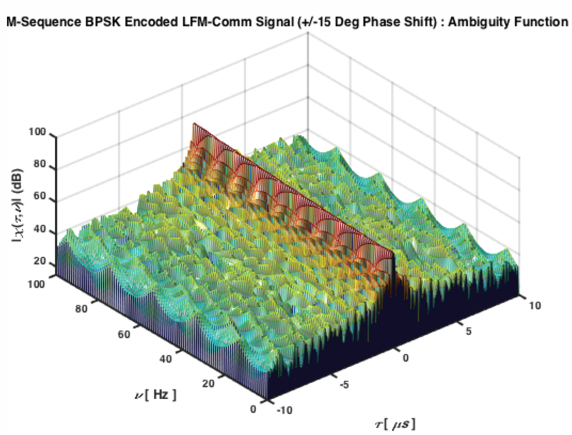

Fig. 6. LFM-Comm Ambiguity Function: BPSK Encoding (+/-15 Degree Phase Shift)

Figure 7 provides a view of the autocorrelation (i.e. zero-Doppler cut of the ambiguity function) and pulse power spectral density (PSD). In the upper figure the PSD of the M-sequence encoded pulse is in blue while the PSD of the unencoded, basic LFM pulse is shown in red. As can be seen while additional energy is distributed in the sidelobes the autocorrelation of the communication pulse is consistent with what one would expect for an LFM matched filter with a high in-band ($\tau = 0$) autocorrelation value and low out of band correlation.

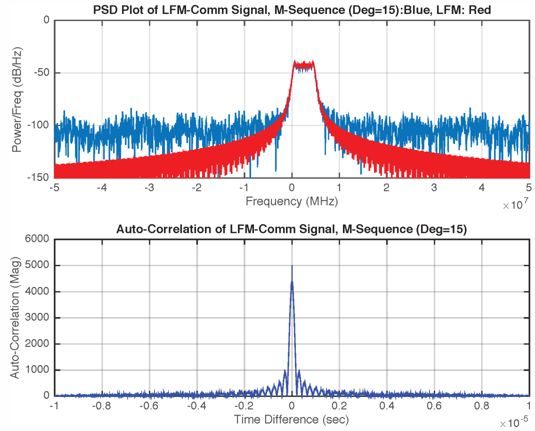

Fig. 7. PSD/Autocorrelation Encoded LFM Pulse ($\phi_\delta = \pm$ 15 Deg)

VII. Conclusion

This paper provides a survey on ongoing research in the use of reduced magnitude phase changes, binary M-sequences and IMOP of radar signals to provide a joint radar-communication waveform design. In this approach in-band modulation of an LFM radar pulse was done using reduced phase angle modulation. In addition, binary M-sequences were used to both improve message recovery at the target receiver and to allow for channelized communications. Although the data rate (bits/pulse) for this approach is low, the communications could allow for administrative functions, low probability of intercept/detection communications, or auxiliary functions such as navigation for a combined radar-navigation signal.

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