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Laser Light Backscattering off an Electron Beam-Plasma System

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Abstract—The case of stimulated backscattering Raman instability that is combined with a two-stream Buneman instability has been analyzed. This problem bears on fabricating a rapidly tunable free electron laser (FEL) for IR countermeasures in the 8–10 μm wavelength range. The analysis is based on a weakly nonlinear theory and assumed a cold nonrelativistic electron beam and an equilibrium plasma to be described with a two-fluid model. The physical mechanism is based on the coherent bunching of the beam-plasma system due to the nonlinear ponderomotive force and the slow beam wave having negative energy. This paper analyzes a novel scattering configuration in which the IR (CO₂) laser at 10.6 μm collides head-on against a nonrelativistic e -beam firing through a puff of laser produced plasma of $n_p \lesssim 10^{16} \text{ cm}^{-3}$, which has been previously produced by the CO₂ laser itself (e.g., producing the critical density 10^{19} cm^{-3} and the reduced quiver velocity $v_{0s}/c = 0.1$ – 0.01 at pulsed 10^{14} – 10^{12} W/cm^2) and becomes underdense subsequently. The upshifted radiation is tunable at an arbitrary Ω_1 determined by the Doppler frequency relationship $\Omega_1 = \Omega_0(1 + \beta_0)/(1 - \beta_0)$, where Ω_0 is the IR pump frequency and $\beta_0 = V_0/c$ (≈ 0.14 for the 8 μm wavelength) is the e -beam parameter of 5.11 keV and A/cm^2 such that $n_b/n_p \approx 10^{-6}$. The synergic backscattering growth rate is the sum of both instabilities, e.g., $[(\sqrt{3}/2)(n_b/2n_p)^{1/3} + (v_{0s}/4c)(\Omega_0/\omega_{0p})^{1/2}]\omega_{0p}$, and may be referred to as the plasmon avalanche instability. Assuming the wide-interval-pulsed (WIP) e -beam can be piecewisely flushed outside the interaction chamber after streaming through a laser produced plasma on each pass, then a useful free electron laser operation will result. Such a device is appropriately called the quasi-free electron laser (QFEL). The effect of axial magnetic field

within the interaction chamber has also been included in the analysis. The advantages of QFEL result from 1) efficient use of the nonrelativistic e -beam generation and two-stream bunching, 2) use of proven CO₂ laser technology, and 3) its rapid tunability.

I. INTRODUCTION

THE present investigation has been motivated by the interest of fabricating a simple and rapidly tunable free electron laser (FEL) in the 8–10 μm wavelength atmospheric window desirable for aircraft countermeasures. This readily suggests itself to use the proven technology of a CO₂ laser at 10.6 μm and a nonrelativistic e -beam for the upshifted backscattering. Therefore, there remains the aspect of rapid tunability which is addressed here and achieved as follows.

The concept of synergic backscattering using beam-plasma two-stream (TS) instability superimposed on the Raman backscattering instability can provide a rapidly tunable output in the 8–10 μm wavelength regime that is also useful for target designators, beam riders, moving target indicators, terrain following, weapon delivery, and imaging. This paper analyzes a novel scattering configuration shown in Fig. 1 in which the IR (CO₂) laser at 10.6 μm wavelength collides head-on against a nonrelativistic e -beam firing through a puff of laser produced plasma, which is previously created by the CO₂ laser itself. Based on the weakly nonlinear theory [1] and a two-fluid model, we present an analysis of the synergic backscattering.

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Fig. 1. Coordinate system for the synergistic backscattering due to coherent bunching. The positive energy plasma wave is mixed with the negative energy beam wave under the nonlinear ponderomotive force of the pump radiation pressure.

No explosive decay instability [2] among the resonant triplet waves is possible provided that negative energy wave [3] of the resonant triplet is constrained to have the highest frequency [4] and a nonzero initial value. Working within the validity of the weakly nonlinear theory, we find that the fastest growth that may be experimentally realizable is the synergistic effect between the stimulated backscattering Raman instability and the two-stream Buneman instability. In contrast to the explosive instability, an avalanche instability occurs when the positive energy plasma wave and the negative energy beam wave have been triggered by the pump ponderomotive force of the nonlinear radiation pressure. If these two instabilities occur simultaneously, the individual growth rates are added exponentially. The growth rate turns out to be the sum of the stimulated backscattering Raman growth rate predicted by the weakly nonlinear theory and the two-stream Buneman growth rate calculated by the linear theory. The analysis can be reduced to each of the well-known cases. It is the self-consistent combination solution of both that we shall present.

A. Physical Mechanism

When the negative energy beam wave meets the positive energy plasma wave, the consequence is a two-stream instability [5]. Negative energy means that the difference of the kinetic energy of the e -beam ($1/2 N_0 m_0 V_0^2$) with and without the presence of the slow longitudinal wave ($\omega < k \cdot V_0$) is negative

$$\begin{aligned} & \frac{1}{2} N_0 m_0 (V_0 - V^L)^2 - \frac{1}{2} N_0 m_0 V_0^2 \\ & = -N_0 m_0 V_0 \cdot V^L + \frac{1}{2} N_0 m_0 V^L \cdot V^L \end{aligned} \quad (1a)$$

or, equivalently, the wave energy spectral density

$$W = \frac{\partial}{\partial \omega} (\omega \epsilon_0 (1 + \epsilon)) \frac{\langle E^2 \rangle}{2} < 0 \quad (1b)$$

becomes negative for the slow wave $\omega/k < V_0$. In the case of a pump induced V^L , the first term can be negative for the beam wave while the second term becomes the pump radiation pressure. Both can coherently bunch the beam and plasma system in an oscillation and therefore synergistically scatter back the IR laser pump.

B. Weakly Nonlinear Fluid Approach

In order to calculate such a synergistic backscattering, one must self-consistently derive the linear and nonlinear response cur-

rents for both the e -beam and the equilibrium plasma,

$$J_b = -e N_0 V_0 \hat{z} + J_b^L + J_b^{NL} \quad (2)$$

$$J_p = J_p^L + J_p^{NL}. \quad (3)$$

This will be done by solving the Maxwell wave equation of the total wave and the total current,

$$\begin{aligned} J &= J_b + J_p = \Sigma_\alpha J_\alpha \\ (c^2 \partial_z^2 - \partial_t^2) E - 4\pi \partial_t J^L &= 4\pi \partial_t J^{NL}. \end{aligned} \quad (4)$$

We assume the two-fluid model for the electron beam and plasma system consisting of two species denoted by $\alpha = b$ and p . Consider a cold nonrelativistic electron beam and plasma system described by the (Lorentz force) momentum equation

$$d_t V_\alpha = -\frac{e}{m_0} (E + V_\alpha \times B) \quad (5)$$

and the charge continuity equation

$$-e \partial_t N_\alpha + \nabla \cdot J_\alpha = 0 \quad (6)$$

where

$$d_t \equiv \frac{d}{dt} = \frac{\partial}{\partial t} + V_{0\alpha} \frac{\partial}{\partial z} \equiv \partial_t + V_{0\alpha} \partial_z, \quad (7)$$

all functions are assumed to be functions of z and t alone and having the zero initial value, the constant electron charge, mass, average density, and average beam speed are respectively denoted by $-e$, m_0 , N_0 and V_0 .

C. Present Contributions

The parametric instability means that the longitudinal and transversal modes are coupled through the nonlinear current which is driven by the applied pump strength. Since the analysis is similar to the parametric instability, it is not new for each special case, but the idea of synergistic backscattering which combines both the Raman and the Buneman instabilities is new. The advantage of a simple model with an elementary analysis is that any realization and variation becomes obvious. A typical variation of the synergistic backscattering is an axial magnetic field in the interaction chamber. This will also be treated in the second part of the papers. For those who have already acquired the knowledge [6]–[10] and only want to know the synergistic backscattering, they may skip the elementary analysis and stop at points of interest written in subsection format within each section.

D. Applications

Since FEL precedent has been pointed out by Kapitza and Dirac [11], Marcuse [12], Pantell *et al.* [13], Madey [14], and Sukhatme and Wolff [15] and FEL experiments done by Elias *et al.* [16], Deacon *et al.* [17] and McDermott *et al.* [18], many people [19]–[27] have been working on various aspects of FEL devices in different laboratories all over the world in order to improve the efficiency, the size, the economics, and the recycling. The present goal is to find the fastest tuning mechanisms in the 8–10 μm regime, because the fast system response to the *e*-beam tuning is very desirable for aircraft countermeasures (CM). The interest is recently revived due to the fact that a simple FEL of 100 W at the 8–10 μm wavelength can be a useful supplement to the family of the color-center, the solid state, and the chemical laser useful for CM. The present analysis seems to indicate that a puff of plasma in the interaction chamber at the instance of tuning may actually enhance the CO₂ laser light backscattering by means of TS bunching.

E. Nomenclature

The conventional laser energy is derived from the potential energy of bounded electrons and the laser power from the decay of the inverse population. Similarly, the energy of FEL is obtained from the kinetic energy of free electrons and the power from the positive slope of the electron energy distribution function (e.g., the left-hand side of Maxwellian distribution). Since the stochastic longitudinal beam or plasma oscillation has been called the quasi-particle plasmon, then the plasmon contributing FEL may be coined by the word quasi-free electron laser (QFEL) both in the sense of quasi-particles and in the sense of the limited regime of their frequency (see the dispersion curve shown in Fig. 2).

II. ELEMENTARY THEORY OF LINEAR RESPONSE AND NONLINEAR COUPLING

The parametric instability for the laser light backscattering from the plasma alone has been studied based on the two-fluid model and Maxwell equations. The present analysis follows the same model but is slightly generalized to include an electron beam firing through the equilibrium plasma. The linear dispersion predicts for the beam-plasma system the two-stream Buneman instability, while the nonlinear dispersion for the beam-plasma and pump system gives rise to the stimulated Raman instability. Thus, the present analysis must have both instabilities included and be reducible to each case.

A. The Lorentz Force Momentum Equation

Both the *e*-beam fluid velocity V_b and the *e*-plasma fluid velocity V_p have the following expansion.

$$V_\alpha = V_{0\alpha} \hat{z} + V_\alpha^L + V_\alpha^{NL} \quad (8a)$$

where the 0th order equals by definition the initial condition

$$V_{0b} = V_0 \quad (8b)$$

$$V_{0p} = 0 \quad (8c)$$

and wavy underscores or boldface letters denote vector quantities, the upper case letters for the macroscopic fluid quantities.

Fig. 2. Dispersion curve. The conservation of momentum $k_1 = k_0 - k_2$ is indicated with wavy underscores denoting vector quantities. It turns out that wave equation (40) determines the magnitude of $k_2 = K_2 \cdot \hat{z}$ to be negative, because the beam-plasma wave satisfying the dispersion relation $\epsilon(\omega_2, k_2) = 1$ given by (38) is identically represented by the complex conjugated inversion $\epsilon(-\omega_2^*, -k_2) = 1$ given by the dotted line in the figure.

The Lorentz force momentum equation (5) predicts the linear response velocity, with respect to the Cartesian coordinate system shown in Fig. 1 and the total time derivative equation (7)

$$m_0 d_t V_{\alpha y}^L = -eE_y - eV_{0\alpha} B_x c^{-1} \quad (9a)$$

$$m_0 d_t V_{\alpha z}^L = -eE_z. \quad (9b)$$

Also from (5) follows the nonlinear coupling between the linear components

$$m_0 d_t V_{\alpha y}^{NL} = -eV_{\alpha z}^L B_x c^{-1} - m_0 V_{\alpha z}^L \partial_z V_{\alpha y}^L \quad (10a)$$

$$m_0 d_t V_{\alpha z}^{NL} = -eV_{\alpha y}^L B_x c^{-1} - m_0 V_{\alpha y}^L \partial_z V_{\alpha z}^L \quad (10b)$$

B. Charge Continuity Equation

The electron number density is similarly expanded

$$N_\alpha = N_{0\alpha} + N_\alpha^L + N_\alpha^{NL} \quad (11)$$

which, up substitution into the charge continuity (6), yields

$$d_t N_\alpha^L = -N_{0\alpha} \partial_z V_{\alpha z}^L \quad (12a)$$

$$d_t N_\alpha^{NL} = -N_{0\alpha} \partial_z V_{\alpha z}^{NL} - \partial_z (N_\alpha^L V_{\alpha z}^L). \quad (12b)$$

Thus, from (12a) and (9b) follows the linear response density

$$m_0 d_t^2 N_\alpha^L = N_{0\alpha} e \partial_z E_z. \quad (13)$$

C. Radiation Pressure

According to the weakly nonlinear theory, the radiation pressure is defined, as in (1),

$$P_\alpha^r = \frac{1}{2} N_{0\alpha} m_0 V_\alpha^L \cdot V_\alpha^L, \quad (14)$$

which can be derived from (9a) and the Faraday law

$$\partial_t B_x = c \partial_z E_y. \quad (15)$$

This is due to the initially homogeneous assumption that the spatial variation of the transversal response (9a) can be separated from the temporal variation of (9a) as follows.

$$m_0 \partial_t V_{\alpha y}^L = -eE_y \quad (16a)$$

$$m_0 V_{0\alpha} \partial_z V_{\alpha y}^L = -eV_{0\alpha} B_x c^{-1}, \quad (16b)$$

which is obviously true for the plasma, because $V_{0p} = 0$ which makes the spatial equation (16b) trivial. To prove the separation for the beam, we differentiate the spatial equation (16b) with ∂_t and use the Faraday induction equation (15) to replace $\partial_t B_x$ in favor of $c\partial_z E_y$.

$$m_0 V_{0\alpha} \partial_t \partial_z V_{\alpha y}^L = -e V_{0\alpha} \partial_z E_y, \quad (16c)$$

which becomes identical to the temporal equation (16a) if the assumed zero initial value has been used to integrate (16c) with respect to the z axis. Therefore, we can use the spatial equation (16b) alone to rewrite the nonlinear equations (10a) and (10b) as

$$d_t V_{\alpha y}^{NL} = 0 \quad (17a)$$

$$N_{0\alpha} m_0 d_t V_{\alpha z}^{NL} = -\partial_z P_{\alpha}^r, \quad (17b)$$

which imply, using (12b), the nonlinear density modulation

$$m_0 d_t^2 N_{\alpha}^{NL} = \partial_z^2 P_{\alpha}^r - d_t \partial_z N_{\alpha}^L m_0 V_{\alpha z}^L. \quad (17c)$$

D. Nonlinear Current

Substituting the expansion of the fluid velocity (8a) and the number density (11) into the linear and the nonlinear currents

$$J_{\alpha}^L = -e N_{\alpha}^L V_{0\alpha} - e N_{0\alpha} V_{\alpha}^L \quad (18)$$

$$J_{\alpha}^{NL} = -e N_{0\alpha} V_{\alpha}^{NL} - N_{\alpha}^L V_{\alpha}^L - e N_{\alpha}^{NL} V_{\alpha} \quad (19)$$

one derives the differential equations governing the currents. The differential equation that relates the linear currents with the electric field is

$$\partial_t J_{\alpha y}^L = \frac{1}{4\pi} \omega_{0\alpha}^2 E_y \quad (20)$$

$$d_t^2 J_{\alpha z}^L = \frac{1}{4\pi} \omega_{0\alpha}^2 \partial_t E_z \quad (21)$$

where use is made of (8) and (16a), the intermediate result, obtained from (12a) and (9b),

$$d_t J_{\alpha z}^L = N_{0\alpha} (e^2/m_0) E_z + N_{0\alpha} e V_{0\alpha} \partial_z V_z^L$$

and the plasma oscillation frequency defined as

$$\omega_{0\alpha} = (4\pi N_{0\alpha} e^2/m_0)^{1/2}. \quad (22)$$

Similarly, the differential equation of the nonlinear current is derived as follows.

$$d_t J_{\alpha y}^{NL} = -e d_t N_{\alpha}^L V_{\alpha y}^L \quad (23)$$

$$d_t^2 J_{\alpha z}^{NL} = (e/m_0) \partial_t \partial_z P_{\alpha}^r - e \partial_t d_t N_{\alpha}^L V_{\alpha z}^L. \quad (24)$$

Use is made of (17b), and the following intermediate result,

$$d_t J_{\alpha z}^{NL} = (e/m_0) \partial_z P_{\alpha}^r - e d_t N_{\alpha}^L V_{\alpha z}^L - e V_{0\alpha} d_t N_{\alpha}^{NL},$$

obtained from (17a) and (19), as well as the nonlinear density equation (17c).

III. LINEAR AND NONLINEAR WAVE EQUATIONS

We assume the laser pump to produce a real positive quiver velocity

$$v_{0s} = (e\delta E_0/m_0\omega_0) \quad (25)$$

and also no pump depletion. The transversal pump E_0 , the transversal backscattering E_1 , and the longitudinal beam-plasma oscillation E_2 are defined in the lab frame Fig. 1 with the real wave number k_l ($l = 0, 1, 2$) and the complex frequency ω_l , $l = 1, 2$ as follows.

$$E_0 = \delta E_0 \exp[-i(k_0 z + \omega_0 t)] \quad (26a)$$

$$E_1 = \delta E_1(t) \exp[i(k_1 z - \omega_1 t)] \quad (26b)$$

$$E_2 = \delta E_2(t) \exp[i(k_2 z - \omega_2 t)]. \quad (26c)$$

The total electric field has the real transverse component

$$E_y = E_0 + E_0^* + E_1 + E_1^* \quad (27a)$$

and the real longitudinal component

$$E_z = E_2 + E_2^*. \quad (27b)$$

The real wave vectors are strictly conserved, cf. Fig. 2,

$$k_1 = k_0 - k_2 \quad (28a)$$

while the complex ω has the growth rate Γ determined by the wave equation having a small but complex mismatch,

$$\Delta\omega = \omega_1 + \omega_2 - \omega_0 = \Delta\Omega + i\Gamma. \quad (28b)$$

A. Complex Conductivities

The differential equations (20) and (21) of the linear currents responding to the electric fields equation (26) have the simple solutions in terms of the transversal (I) and the longitudinal (II) conductivities

$$J_{\alpha y}^L = \sigma_{\perp}^{\alpha}(\omega_0) E_0 + \text{c.c.} + \sigma_{\perp}^{\alpha}(\omega_1) E_1 + \text{c.c.} \quad (29a)$$

$$J_{\alpha z}^L = \sigma_{\parallel}^{\alpha}(\omega_2, k_2) E_2 + \text{c.c.} \quad (29b)$$

where c.c. abbreviates the complex conjugate and the conductivities are expressed as

$$\sigma_{\perp}^{\alpha}(\omega) = \omega_{0\alpha}^2 / (-4\pi i\omega) \quad (30a)$$

$$\sigma_{\parallel}^{\alpha}(\omega, k) = \omega_{0\alpha}^2 \left(\frac{\omega}{\omega - k V_{0\alpha}} \right)^2 / (-4\pi i\omega). \quad (30b)$$

Similarly, the linear density equation (12a) has the solution

$$-e N_{\alpha}^L = \sigma_{\parallel}^{\alpha}(\omega_2, k_2) (k_2/\omega_2) E_2 + \text{c.c.} \quad (31a)$$

and the linear velocity equations (18) and (19b) have two components

$$V_{\alpha y}^L = J_{\alpha y}^L / (-e N_{0\alpha}) \quad (31b)$$

$$V_{\alpha z}^L = i \frac{e}{m_0} \frac{E_2}{k_2 V_{0\alpha} - \omega_2} + \text{c.c.} \quad (29c)$$

Using these linear order solutions, the nonlinear radiation pressure equation (14) has two components

$$P_{\alpha\parallel}^r = \frac{1}{8\pi} \omega_{0\alpha}^2 \left(\frac{iE_2}{k_2 V_{0\alpha} - \omega_2} + \text{c.c.} \right)^2 \quad (30a)$$

$$P_{\alpha\perp}^r = 2\pi\omega_{0\alpha}^2 (\sigma_{\perp}^{\alpha}(\omega_0) E_0 + \text{c.c.} + \sigma_{\perp}^{\alpha}(\omega_1) E_1 + \text{c.c.})^2 \quad (30b)$$

and the nonlinear currents equations (23) and (24) responsible for the backscattering are mainly the following.

$$J_{\alpha y}^{NL} = \sigma_{\perp}^{\alpha}(\omega_0) E_0 \sigma_{\parallel}^{\alpha}(\omega_2^*, k_2) E_2^* \cdot \left(\frac{k_2}{\omega_2^* N_{0\alpha} e} \right) + \text{c.c.} \quad (31a)$$

and

$$J_{\alpha z}^{NL} = \sigma_{\perp}^{\alpha}(\omega_0) E_0 \sigma_{\parallel}^{\alpha}(\omega_0 - \omega_1^*, -k_0 - k_1) E_1^* \cdot \left(\frac{k_0 + k_1}{\omega_1^* N_{0\alpha} e} \right) + \text{c.c.} \quad (31b)$$

where the complex mismatch (28b)

$$\omega_0 - \omega_1^* + (k_0 + k_1) V_{0\alpha} \neq 0$$

guarantees the finite longitudinal conductivity equation (31b).

B. Backscattering Wave Equation

From (4) follows

$$\begin{aligned} [c^2 \partial_z^2 - \partial_t^2 - (\omega_{0b}^2 + \omega_{0p}^2) (-i\omega_1)^{-1} \partial_t] \delta E_1(t) \\ \cdot \exp [i(k_1 z - \omega_1 t)] \\ = \left(\frac{v_{0s}}{c} \right) c k_2 \epsilon(\omega_2^*, k_2) \partial_t \delta E_2^*(t) \exp [-i(k_0 + k_2)z \\ - i(\omega_0 - \omega_2^*)t] \end{aligned} \quad (32)$$

where use is made of (30a) and (31a) and the dielectric constant

$$\epsilon(\omega, k) = \Sigma_{\alpha} \sigma_{\parallel}^{\alpha}(\omega, k) / i\omega = \frac{\omega_{0b}^2}{(\omega - k V_0)^2} + \frac{\omega_{0p}^2}{\omega^2}. \quad (33)$$

Using the chain rule of differentiation, one obtains the linear dispersion equation

$$[\omega_1^2 - c^2 k_1^2 + (\omega_{0b}^2 + \omega_{0p}^2)] \delta E_1(t) = 0$$

i.e.,

$$\omega_1 = \pm (c^2 k_1^2 + \omega_{0b}^2 + \omega_{0p}^2)^{1/2} = \pm \Omega_1(k_1) \quad (34)$$

and the amplitude variation equation

$$\begin{aligned} -[\partial_t^2 - \frac{2i\omega_1 \partial_t}{c} + (\omega_{0b}^2 + \omega_{0p}^2) (-i\omega_1)^{-1} \partial_t] \delta E_1(t) \\ = \left(\frac{v_{0s}}{c} \right) c k_2 \epsilon(\omega_2^*, k_2) \exp [i(\omega_1 + \omega_2^* - \omega_0)t] \\ \cdot \underline{[i(\omega_2^* - \omega_0) + \partial_t] \delta E_2^*(t)} \end{aligned} \quad (35)$$

where use is made of the momentum conservation

$$k_0 \cdot \hat{z} = -k_0 = (k_1 + k_2) \cdot \hat{z} = k_1 + k_2. \quad (36)$$

C. Beam-Plasma Wave Equation

Similarly, from (4) follows

$$\begin{aligned} -(\partial_t^2 + \epsilon(\omega_2, k_2) i\omega_2 \partial_t) \delta E_2(t) \exp (ik_2 z - i\omega_2 t) \\ = \left(\frac{v_{0s}}{c} \right) c (k_0 + k_1) \frac{\omega_0 - \omega_1^*}{\omega_1^*} \\ \cdot \epsilon(\omega_0 - \omega_1^*, -k_0 - k_1) \partial_t \delta E_1^*(t) \\ \cdot \exp [-i(k_0 + k_1)z - i(\omega_0 - \omega_1^*)t] \end{aligned} \quad (37)$$

which upon differentiation yields both the linear dispersion equation

$$[-\omega_2^2 + \omega_2^2 \epsilon(\omega_2, k_2)] \delta E_2(t) = 0$$

i.e., $\epsilon(\omega_2, k_2) = 1$ implies $\omega_2 = \Omega_2(k_2) + i\Gamma_2(k_2)$ satisfying the linear dispersion relation

$$1 - \frac{\omega_{0p}^2}{\omega_2^2} - \frac{\omega_{0b}^2}{(\omega_2 - k_2 \cdot V_0)^2} = 0 \quad (38)$$

and the amplitude variation equation

$$\begin{aligned} -(\partial_t^2 - \frac{2i\omega_2 \partial_t}{c} + i\omega_2 \epsilon(\omega_2, k_2) \partial_t) \delta E_2(t) \\ = \left(\frac{v_{0s}}{c} \right) c (k_0 + k_1) \frac{\omega_0 - \omega_1^*}{\omega_1^*} \\ \cdot \epsilon(\omega_0 - \omega_1^*, -k_0 - k_1) \exp (i(\omega_1^* + \omega_2 - \omega_0)t) \\ \cdot \underline{[i(\omega_1^* - \omega_0) + \partial_t] \delta E_1^*(t)}. \end{aligned} \quad (39)$$

IV. WEAKLY NONLINEAR AMPLITUDE VARIATION

Due to the slow amplitude variation, the scattering amplitude equation (35) should only keep the underlined leading order after assuming the linear dispersion (34), (38)

$$\begin{aligned} \partial_t \delta E_1(t) \approx \frac{v_{0s}}{c} c k_2 \left(\frac{\omega_2^* - \omega_0}{\omega_1^*} \right) \delta E_2^*(t) \\ \cdot \exp [i(\omega_1 + \omega_2^* - \omega_0)t] \epsilon(\omega_2^*, k_2); k_2 < 0 \end{aligned} \quad (40)$$

according to the standard approximation of weakly nonlinear theory, i.e.,

$$\partial_t^2 \delta E_1 \ll \omega_1^2 \delta E_1, \delta E_0 \partial_t \delta E_2^* \ll i(\omega_2^* - \omega_0) \delta E_0 \delta E_2^* \quad (41a,b)$$

neglecting $\partial_t^2 \delta E_1$ and $\delta E_0 \partial_t \delta E_2^*$ due to the second order of smallness in the chain differentiations of (35). Similarly, the longitudinal wave amplitude (39) reduces to the underlined terms

$$\begin{aligned} \partial_t \delta E_2(t) \approx \frac{v_{0s}}{c} c k_2 \frac{(\omega_0 - \omega_1^*)^2}{\omega_2 \omega_1^*} \\ \cdot \epsilon(\omega_0 - \omega_1^*, +k_2) \delta E_1^* \exp [i(\omega_1^* + \omega_2 - \omega_0)t] \end{aligned} \quad (42a)$$

which, by taking the complex conjugate and multiplying both hand sides with $\exp (i[\omega_1 + \omega_2^* - \omega_0^*]t)$ becomes

$$\begin{aligned} \exp [i(\omega_1 + \omega_2^* - \omega_0^*)t] \partial_t \delta E_2^* = \frac{v_{0s}}{c} c k_2 \\ \cdot \frac{(\omega_0^* - \omega_1)^2}{\omega_2^* \omega_1} \epsilon(\omega_0^* - \omega_1, +k_2) \delta E_1 \end{aligned} \quad (43)$$

or, explicitly by using definition (33) and pushing $\exp(i[\ ]t)$ over ∂_t ,

$$\begin{aligned} & [\partial_t - (\Gamma_2 - \Gamma_1 + i\Delta\Omega)] \delta E_2^* \exp[(\Gamma_2 - \Gamma_1)t] \\ & \cdot \exp(i\Delta\Omega t) = \frac{v_{0s}}{c} ck_2 \\ & \cdot \frac{\omega_{0p}^2}{\omega_2^* \omega_1} \left[1 + \left(\frac{n_{0b}}{n_{0p}} \right)^{1/2} \left(1 - \frac{k_2 V_0}{\omega_0^* - \omega_1} \right)^{-2} \right] \delta E_1 \end{aligned} \quad (44a)$$

where

$$\Delta\Omega = \Omega_1 + \Omega_2 - \Omega_0. \quad (44b)$$

To solve (40) and (44) simultaneously, we abbreviate

$$\begin{aligned} X(t) &= \delta E_1(t) \\ Y(t) &= \delta E_2^*(t) e^{(\Gamma_2 - \Gamma_1)t} e^{i\Delta\Omega t} \end{aligned} \quad (45a,b)$$

$$\begin{aligned} A &= \left(\frac{v_{0s}}{c} \right) (ck_2) \left(\frac{\omega_2^* - \omega_0}{\omega_1} \right) \epsilon(\omega_2^*, k_2); \\ B &= \left(\frac{v_{0s}^*}{c} \right) (ck_2) \frac{(\omega_0^* - \omega_1)^2}{\omega_2^* \omega_1} \epsilon(\omega_0^* - \omega_1, +k_2) \\ &= \left(\frac{v_{0s}^*}{c} \right) (ck_2) \frac{\omega_{0p}^2}{\omega_2^* \omega_1} \left[1 + \left(\frac{n_{0b}}{n_{0p}} \right)^{1/2} \left(1 - \frac{k_2 V_0}{\omega_0^* - \omega_1} \right)^{-2} \right] \end{aligned}$$

$$C = (\Gamma_2 - \Gamma_1) + i\Delta\Omega. \quad (46a,b,c)$$

Then (40) and (44) assume the following simple forms.

$$\partial_t X(t) = AY(t) \quad (47)$$

$$\partial_t Y(t) - CY(t) = BX(t) \quad (48)$$

together with the initial conditions

$$X(0) = 0, \quad Y(0) = 0. \quad (49a,b)$$

Thus, the exact solution follows

$$X(t) = 2\alpha \frac{A}{\Gamma_+} e^{1/2 C t} \sinh \left[\frac{1}{2} \sqrt{C^2 + 4AB} t \right] \quad (50)$$

$$Y(t) = 2\alpha e^{1/2 C t} \sinh \left[\frac{1}{2} \sqrt{C^2 + 4AB} t \right]. \quad (51)$$

The additivity of growth rates enters through Γ_+ defined to be the only nonzero root of the characteristic equation

$$\Gamma^2 - CT - AB = 0 \quad (52)$$

$$\Gamma_{\pm} \equiv \frac{1}{2} C \pm \frac{1}{2} \sqrt{C^2 + 4AB} \quad (53)$$

where C abbreviates the two-stream growth rate Γ_2 and the real mismatch $\Delta\Omega = \Omega_1 + \Omega_2 - \Omega_0$ in turning the backscattering Ω_1 from the pump Ω_0 . This nonlinear growth rate is the synergic growth rate plotted in Fig. 3. $(AB)^{1/2}$ is proportional to the Raman backscattering growth rate Γ_{Raman} . When $k_2 V_0 \approx \omega_{0p}$, $\omega_2 \neq 0$, $\omega_2 \neq k_2 V_0$, $\omega_2 \neq k_2 V_0 + \omega_{0p}$, (53) gives

$$\begin{aligned} \omega_2 &= |k_2 V_0| - \omega_{0p} + \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) \left(\frac{n_{0b}}{n_{0p}} \right)^{1/3} \omega_{0p}; \\ \Gamma_2 &= \frac{\sqrt{3}}{2} \left(\frac{n_{0b}}{n_{0p}} \right)^{1/3} \omega_{0p}. \end{aligned} \quad (54a,b)$$

Fig. 3. Synergic growth rate. The result $\Gamma^{NL} = \Gamma_{TS} + \Gamma_{\text{Raman}}$ is normalized by Γ_{Raman} and is plotted with respect to the e-beam parameter V_0/c .

When $n_{0p} = 0$, $C = 0$,

$$\Gamma_{\pm} = \Gamma_{\text{Raman}} = \frac{|v_{0s}|}{c} |ck_2| \left(\frac{|\Omega_2|}{\Omega_1} \right)^{1/2} \quad (55)$$

where $|\Omega_2| \approx \omega_{0p}$ and $|ck_2| \approx 2\Omega_1 \approx 4\Omega_0$.

V. EXACT SOLUTION

There are various methods used to obtain the solutions (50), (51) and perhaps the straightforward one has the following two steps.

1) The necessary condition for simultaneous solutions is obtained by cross-differentiating (47), (48)

$$(\partial_t^2 - C\partial_t - AB) \begin{pmatrix} X \\ Y \end{pmatrix} = 0 \quad (56)$$

which yields the compatibility (52) by seeking the solution $\exp(\Gamma t)$ for (56). Clearly, the cross-differentiation has enlarged the range of admissible solutions to include four unknown constants: a , b , α , and β .

$$X(t) = ae^{\Gamma_+ t} + be^{\Gamma_- t} \quad (57)$$

$$Y(t) = \alpha e^{\Gamma_+ t} + \beta e^{\Gamma_- t}. \quad (58)$$

2) The sufficient condition follows by substituting (57) and (58) into each individual equation

$$a\Gamma_+ e^{\Gamma_+ t} + b\Gamma_- e^{\Gamma_- t} = A\alpha e^{\Gamma_+ t} + A\beta e^{\Gamma_- t} \quad (59a)$$

$$\alpha(\Gamma_+ - C)e^{\Gamma_+ t} + \beta(\Gamma_- - C)e^{\Gamma_- t} = Bae^{\Gamma_+ t} + Bbe^{\Gamma_- t} \quad (59b)$$

which gives

$$a = \frac{A}{\Gamma_+} \alpha; \quad b\Gamma_- = A\beta \quad (60a,b)$$

$$\alpha(\Gamma_+ - C) = Ba; \quad \beta(\Gamma_- - C) = Bb. \quad (61a,b)$$

Since both B and $\Gamma_{\pm} - C$ can be zero sometimes according to (46) except $\Gamma_+ \neq 0$, then except for (60a) we cannot divide either side of (60b), (61a,b) with zero in solving α , β in terms of a , b or vice versa. This fact allows us to accommodate two initial conditions (49a,b)

$$\beta = -\alpha; \quad b = -\alpha \frac{A}{\Gamma_+} \quad (62a,b)$$

and thus gives the exact solutions (50), (51).

The arbitrary constant α is due to the arbitrary phase and amplitude at the onset of backscattering and can be adjusted to maximize the onset of backscattering amplitude. Explicitly, the scattering amplitude and longitudinal amplitude are given as follows.

$$\delta E_1 = \frac{2\alpha}{\Gamma_+} \frac{v_{0s}}{c} c k_2 \frac{\Omega_2}{\Omega_1} \left(1 - \frac{\Omega_0 + i\Gamma_2}{\Omega_2}\right) e^{1/2\Gamma_2 t} e^{i/2\Delta\Omega t} \sinh\left(\left[\Gamma_+ - \frac{1}{2}(\Gamma_2 + i\Delta\Omega)\right] t\right) \quad (63)$$

$$\delta E_2^* = 2\alpha e^{-1/2\Gamma_2 t} e^{-i/2\Delta\Omega t} \sinh\left(\left[\Gamma_+ - \frac{1}{2}(\Gamma_2 + i\Delta\Omega)\right] t\right). \quad (64)$$

Unlike the avalanche electric breakdown which limits the spot size of picosecond laser Bloembergen pointed out in 1972, the avalanche instability is due to the synergistic effect of the laser backscattering off the counter e -beam through a plasma. The growth rate of the stimulated Raman scattering is combined with the growth rate of the e -beam plasma two-stream instability to give the growth rate of the avalanche instability.

VI. RIGHT-HAND CIRCULARLY POLARIZED PUMP AND UNIFORM MAGNETIC FIELD

We would like to obtain a higher backscattering frequency. This can be achieved by a uniform magnetic field B_0 along the z -axis inside the interaction chamber. We consider the right-hand circularly polarized (RHCP) pump with respect to the magnetic field axis along the e -beam direction. We use the complex orthogonal base vectors defined as follows.

$$\hat{e}_\pm = \frac{1}{\sqrt{2}} (\hat{e}_x \pm i\hat{e}_y), \quad (65)$$

such that the unit dyadic is

$$1 = \hat{e}_+^* \hat{e}_+ + \hat{e}_-^* \hat{e}_- + \hat{e}_z \hat{e}_z = \hat{e}_x \hat{e}_x + \hat{e}_y \hat{e}_y + \hat{e}_z \hat{e}_z \quad (66)$$

and vector product rules follow respectively for the upper and the lower signs

$$\hat{e}_\pm \times \hat{e}_\mp = \mp i \hat{e}_z, \quad (67a)$$

$$\hat{e}_z \times \hat{e}_\pm = \mp i \hat{e}_\pm. \quad (67b)$$

Note that when we form the decomposition, the subscripted sign is just opposite to the algebra sign of Cartesian components,

$$J \cdot 1 = J_+ \hat{e}_+ + J_- \hat{e}_- + J_z \hat{e}_z \quad (68a)$$

where

$$J_+ = \frac{1}{\sqrt{2}} (J_x - iJ_y), \quad (68b)$$

$$J_+^* = J_-. \quad (68c)$$

If we define the cyclotron frequency

$$\Omega_0 = \frac{eB_0}{m_0 c} \quad (69)$$

then we can show that the time scale factor $\exp(i\Omega_0 t)$ accounts for the effect of uniform magnetic field B_0 exerted on the initially equilibrium system. This becomes obvious

from the Faraday law.

$$\partial_t B + c \hat{e}_z \partial_z \times E = 0, \quad (70a)$$

$$\partial_t B_\pm \mp ic \partial_z E_\pm = 0, \quad (70b)$$

and the Lorentz force equation.

$$m_0(d_t - \Omega_0 \hat{e}_z \times) V^L = -e \left(E + \frac{V_0}{c} \hat{e}_z \times B \right), \quad (71a)$$

$$m_0(d_t - \Omega_0 \hat{e}_z \times) B^{NL} = -\frac{e}{c} V^L \times B - m_0 V_z^L \partial_z V^L, \quad (71b)$$

which becomes

$$m_0(d_t \pm i\Omega_0) V_\pm^L = -e \left(E_\pm \mp i \frac{V_0}{c} B_\pm \right), \quad (72a)$$

$$m_0 d_t V_z^L = -e E_z. \quad (72b)$$

Using the integrating factor $\exp(i\Omega_0 t)$ and eliminating B_+ ,

$$m_0 \partial_t d_t \exp(i\Omega_0 t) V_+^L = -e d_t E_+ \exp(i\Omega_0 t), \quad (73)$$

we conclude that the linear response is satisfied in time and space separately

$$m_0 \partial_t V_+^L \exp(i\Omega_0 t) = -e E_+ \exp(i\Omega_0 t) \quad (74a)$$

$$m_0 V_0 \partial_z V_+^L \exp(i\Omega_0 t) = i e \frac{V_0}{c} B_+ \exp(i\Omega_0 t) \quad (74b)$$

provided that the time scale factor has been used to account for the effect of B_0 . Therefore, we introduce the following helicity transformation which can transform all equations into a set identical to the set that has no external magnetic field in the interaction chamber.

$$\mathcal{V}_+^L = V_+^L \exp(i\Omega_0 t) \quad (75a)$$

$$\mathcal{V}_+^{NL} = V_+^{NL} \exp(i\Omega_0 t) \quad (75b)$$

$$E_+ = (E_0 + E_R + E_L^*) \exp(i\Omega_0 t) \quad (75c)$$

$$B_+ = -i(B_0 + B_R + B_L^*) \exp(i\Omega_0 t) \quad (75d)$$

where the scattering field has subscripts R and L denoting respectively RHCP and LHCP analogously. For example, the linear and nonlinear currents become

$$J^L = -e N_0 \mathcal{V}_\pm^L \quad (76a)$$

$$J_\pm^{NL} = -e N^L \mathcal{V}_\pm^L - e N_0 \mathcal{V}_\pm^{NL}. \quad (76b)$$

Consequently, we have found

$$J_+^L = \sigma_\perp(\omega_0 - \Omega_0) E_0 + \sigma_\perp(\omega_R - \Omega_0) E_R + \sigma_\perp(\omega_L^* + \Omega_0) E_L^* \quad (77)$$

where the transversal conductivity has been affected by the cyclotron frequency Ω_0

$$\sigma_\perp(\omega - \Omega_0) = i \frac{e^2 N_0}{m_0(\omega - \Omega_0)}. \quad (78)$$

Similarly, we have obtained for the incident RHCP and the

backscatter RHCP

$$J_z^{NL} = \frac{k_0 + k_R}{N_0 e (\omega_R^* - \omega_0)} \sigma_{\perp}(\omega_0 - \Omega_0) \sigma_{\parallel}(-(\omega_R - \omega_0), -(k_0 + k_R))^* \delta E_0 \delta E_R^* \cdot \exp[-i(k_0 + k_R)z + i(\omega_R^* - \omega_0)t] + \text{c.c.} \quad (79a)$$

and

$$J_+^{NL} = -\frac{k_{\parallel}}{e N_0 \omega_{\parallel}} \sigma_{\parallel}(\omega_{\parallel}, k_{\parallel}) \sigma_{\perp}(\omega_0 - \Omega_0)^* E_z E_0^* + \text{c.c.} \quad (79b)$$

where the longitudinal conductivity $\sigma_{\parallel}$ is not affected by the axial magnetic field.

Thus, the wave equation for the RHCP wave is derived as

$$-\partial_t^2 E_z - c^2 \partial_z^2 E_z + (\omega_{0b}^2 + \omega_{0p}^2) (-i(\omega_R - \Omega_0))^{-1} \partial_t \delta E_R(t) \cdot \exp(ik_R z - \omega_R t) = \left(\frac{V_{0s}}{c} \right) c k_{\parallel} \epsilon(\omega_{\parallel}^*, k_{\parallel}) \partial_t \delta E_z^*(t) \exp[-i(k_0 + k_{\parallel})z - i(\omega_0 - \omega_{\parallel}^*)t] \quad (80)$$

where the pump quiver velocity under the external magnetic field is defined as the upper case V_{0s} ,

$$V_{0s} = \frac{e \delta E_0}{m_0 (\Omega_0 - \omega_0)} \quad (81)$$

and is real and positive assuming no pump depletion.

It follows the linear dispersion

$$k_R = \frac{\omega_R}{c} \left(1 - \frac{\omega_{0b}^2 + \omega_{0p}^2}{\omega_R (\omega_R - \Omega_0)} \right)^{1/2} \quad (82)$$

and the amplitude variation

$$\omega_R \left[1 - \frac{c^2 k_R^2}{\omega_R^2} \right] \partial_t \delta E_R(t) \approx \left(\frac{V_{0s}}{c} \right) \cdot (c k_{\parallel}) (\omega_{\parallel}^* - \omega_0) \epsilon(\omega_{\parallel}^*, k_{\parallel}) \delta E_z^*(t) \exp[i(\omega_R + \omega_{\parallel}^* - \omega_0)t]. \quad (83)$$

Similarly, the longitudinal wave equation is

$$-\partial_t^2 E_z - i \omega_{\parallel} \epsilon(\omega_{\parallel}, k_{\parallel}) \partial_t E_z = (k_0 + k_R) \left(\frac{\omega_R^* - \omega_0}{\omega_R^* - \Omega_0} \right) V_{0s} \epsilon(\omega_0 - \omega_R^*, -(k_0 + k_R)) \partial_t \delta E_R^*(t) \exp[-i(k_0 + k_R)z - i(\omega_0 - \omega_R^*)t] \quad (84)$$

and from which follows the linear dispersion

$$1 = \epsilon(\omega_{\parallel}, k_{\parallel}) \quad (85)$$

and the amplitude version

$$\omega_{\parallel} \partial_t \delta E_z(t) = c(k_0 + k_R) \frac{V_{0s}}{c} (\omega_R^* - \omega_0) \left(\frac{\omega_R^* - \omega_0}{\omega_R^* - \Omega_0} \right) \epsilon(\omega_0 - \omega_R^*, -(k_0 + k_R)) \partial_t \delta E_R^*(t) \cdot \exp[i(\omega_R^* - \omega_0 + \omega_{\parallel})t]. \quad (86)$$

Fig. 4. Right-hand circularly polarized pump with respect to the uniform axial magnetic field.

Thus, the effect of the external magnetic field $B_0 \hat{z}$ enters through the pump quiver velocity equation (81) and the Doppler shifted denominator $(\omega_R^* - \Omega_0)^{-1}$. The higher backscattering frequency is schematically indicated in Fig. 4.

VII. DISCUSSION AND CONCLUSION

The advantage of using a fluid model is to be able pedagogically to introduce a new concept, the synergic backscattering, with a familiar analysis, the weakly nonlinear theory, for which an axial guiding magnetic field can be, and has been, readily included in Section VI. The disadvantage is obviously that any kinematic and temperature effect, for example, the hot plasma ($\bar{V}_{TP}/V_{0b} \lesssim 0.1$) stabilization of the two-stream instability by means of Landau damping, cannot be systematically treated. A thorough investigation of QFEL must take into account the electron beam-plasma distributions, the temperature effect, in order to determine the saturation levels and efficiencies of FEL [28], which is undertaken for a nontrivial proposition, and beyond the scope of the model. Nevertheless, we obtain a self-consistent estimation based on the fluid model. From a weakly nonlinear amplitude variation, Section IV, it follows that the backscattering complex amplitude $\delta E_1(t)$ and the beam-plasma wave amplitude $\delta E_2(t)$ are intimately related as follows.

$$\delta E_1(t) \frac{\Gamma_{NL}}{\omega_{0p}} \approx \delta E_2^*(t) \frac{v_{0s}}{c}, \quad (87)$$

in case of no mismatch and pump depletion, $A \approx (v_{0s}/c) \omega_{0p}$. The nonlinear synergic backscattering growth rate is bounded by the following formula.

$$\frac{\Gamma_{NL}}{\omega_{0p}} \lesssim \frac{\sqrt{3}}{2} \left(\frac{n_b}{2n_p} \right)^{1/3} + \frac{v_{0s}}{4c} \left(\frac{\Omega_0}{\Omega_{0p}} \right)^{1/2} \quad (88)$$

which is the sum of the growth rates of the two-stream instability [31]–[39] and the stimulated Raman backscattering [29], [30]. Furthermore, if we allow pump dynamics, then the known saturation mechanism of electron trapping, namely, conservation of action density, Galeev and Sagdeev (according to Manheimer *et al.* [29]) or Manley-Rowe relations (according to Dawson *et al.* [28]) say

$$\frac{v_{0s}}{c} > \frac{\omega_{0p}}{c k_2} \left(\frac{\omega}{\Omega_0} \right)^{1/2} \quad (89)$$

where ω stands for ω_{0p} for the space charge dominant regime or Ω_1 for the ponderomotive force dominant regime. In any case, we can rewrite the saturation condition equation (89) in terms of the reduced Raman growth rate as follows.

$$\frac{\Gamma_{\text{Raman}}}{\omega_{0p}} = \frac{v_{0s}}{4c} \left(\frac{\Omega_0}{\omega_{0p}} \right)^{1/2} > \frac{\omega_{0p}}{4ck_2} \left(\frac{\omega}{\omega_{0p}} \right)^{1/2} \quad (90)$$

which is greater than 1 percent for the present case $\Omega_2 \approx \omega_{0p}$, $ck_2 \approx 2\Omega_1 \approx 4\Omega_0$. These equations (87)–(90) can provide the needed guideline for further analyses, and may be adequate for us to decide whether or not the scheme is feasible in the present day technology. Numerical values for laser and e -beam are given as follows. The CO_2 laser flux I is available at 10^{14} – 10^{12} W/cm² which can produce for $\lambda_0 = 10.6$ μm the reduced quiver velocity, $v_{0s}/c = 0.1$ – 0.01 using

$$v_{0s} = eE_{\text{max}}/m_0\Omega_0 = 25.6\sqrt{I} \lambda_0 \text{ cm/s}. \quad (91)$$

Since we can use the pulsed CO_2 laser at $\Omega_0 = 1.8 \times 10^{14}$ rad/s to produce the plasma in the interaction chamber (using $\omega_{0p} = 5.64 \times 10^4 \times \sqrt{n_p}$ rad/s, the critical density would be $n_p \approx 10^{19}$ cm⁻³), then the electron plasma density is subsequently reduced in the interaction chamber to $n_p = 10^{16}$ cm⁻³ and becomes underdense with respect to the laser. Since the Doppler frequency relationship

$$\Omega_1 = \Omega_0(1 + \beta_0)/(1 - \beta_0), \quad (92)$$

i.e.,

$$\Omega_1 \approx \Omega_0 4\gamma_0^2$$

determines $\beta_0 = V_{0b}/c = 1/7 = 0.14$ for the shortest wavelength $\lambda_1 = 8$ μm , then the e -beam kinetic energy, $(\gamma_0 - 1)m_0c^2 = 0.01 \times 0.511$ MeV = 5.11 keV, is nonrelativistic. Since the current density for $n_b \approx 10^{10}$ cm⁻³ is

$$J = n_b e V_{0b} = 10^{10} \text{ cm}^{-3} \times 1.6 \times 10^{-19} \text{ C} \times 10^9 \text{ cm/s} \\ = 1 \text{ A} \cdot \text{cm}^{-2} \quad (94)$$

then Child's law for nonrelativistic and space-charge limited current density

$$J = 2.34 \times 10^3 (\text{voltage})^{3/2} (d_{\text{plate}})^{-2} \text{ A} \cdot \text{cm}^{-2} \quad (95)$$

predicts a reasonable micropervance K , defined J (voltage)^{-3/2} $\times \pi a^2$, used for the Pierce electron gun. Thus, we have arrived at a reasonable beam-plasma density ratio.

$$\frac{n_b}{n_p} \approx 10^{-6}. \quad (96)$$

Thus, the reduced synergic growth rate, Γ_{NL}/ω_{0p} , is about 1 percent according to (88). Therefore, the growth of backscattering matches simultaneously with the growth of the beam-plasma wave according to (87), and, therefore, the plasma will not become nonlinear first to destroy the desired process. There is enough time for the growth until the saturation occurred at 1 percent, according to (90) and (89).

After giving numerical examples, a few words of caution are in order. It is well known that the fluid model of the beam-plasma system fails when the cold beam condition and the collisional quenching were not satisfied.

$$\frac{\bar{V}_{Tb}}{V_{0b}} \ll \left(\frac{n_b}{n_p} \right)^{1/3} \gtrsim \frac{\gamma_{\text{collision}}}{\omega_{0p}}. \quad (97)$$

However, it is incorrect to assume that the two-stream instability is stabilized when the left-hand side inequality is reversed for the hot beam. The instability persists as long as more of the fast electrons are decelerated by giving energies to the wave while less of the slow electrons are accelerated by receiving energies from the wave. These synchronous electrons happen precisely at the positive slope of electron beam distribution function, the inverse population needed for FEL that has been identified in Section I-E. As a matter of fact, the hot-beam, two-stream growth rate Γ_2 is merely reduced by a factor, according to Briggs [31].

$$\frac{2\Gamma_2}{\omega_{0p}} = \pi \frac{\omega_{0b}^2}{\omega_{0p}^2} V_{0b}^2 \frac{df(v)}{dv} \bigg|_{v=\omega/k} \quad (98)$$

which is $[(V_{0b}/\bar{V}_{Tb})(n_b/n_p)^{1/3}]^2$ less than unity, in case of the shifted Maxwell-Boltzmann distribution of the half width $\bar{V}_{Tb}$.

The present scheme is different from FEL mainly in the energy extraction mechanism. Since the energy of 1 μm wavelength is 1 eV, the energy increment from 10.6 to 8 μm is from 0.1 to 0.125 eV. This is a trivial 25 percent compared to that of FEL $4\gamma_0^2$. If a 100 W CO_2 laser were upshifted equivalently by means of a "moving plasma mirror" (QFEL), which is made of a TS bunched beam-plasma system having the thickness of less than 1 cm/pulse and assuming a 1 percent reflection efficiency, then such a compact QFEL laser has a tunable output of 1.25 W at the atmospheric propagation window. The size of QFEL is that of the power supply less than 1 M³. Nevertheless, an experiment of QFEL is challenging for at least two aspects, namely, producing a 5 keV cold e -beam operating in the wide-interval-pulse (WIP) mode as well as measuring 10^{-12} s TS oscillation for characterizing the convective versus the absolute instability of the "moving plasma mirror."

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