Sonar Cross Sections of Bodies Partially Insonified by Finite Sound Beams

GUILLERMO C. GAUNAURD, SENIOR MEMBER, IEEE

(Invited Paper)

Abstract—An object that is partially insonified by a collimated sound beam may have a scattering cross section sometimes much larger than when the object is totally covered by the incident beam. We quantitatively study this partial insonification problem here, under the classical method of physical optics. The importance of this study stems from the fact that partial coverage of the target by the beam is the situation most likely to occur in many cases of practical importance. We consider several basic target shapes partially insonified by finite beams. These shapes include the spherical, the infinite and finite cylinder, the flat plate, and the capped sphere. High-frequency approximations of the resulting integrals, obtained by means of the saddle-point method, show the relative importance of the scattering centers located at the beam's specular reflection points, or at the edges of the spots that the beam shines on the scatterers. The physical-optics method is extended to obtain formulas for the bistatic cross sections of partially insonified objects. The results are numerically evaluated and graphically displayed in many pertinent instances and compared to the predictions of approaches, such as the Fresnel-zones method and the Geometrical Theory of Diffraction (GTD). The predictions of the physical-optics method have all the advantages and deficiencies of this method and, with very minor modifications, hold equally well for the partial illumination of objects by beams of electromagnetic radiation.

I. INTRODUCTION

SONAR CROSS SECTIONS are determined analogously, and in many instances, identically, to radar cross sections (RCS). The literature on the determination of these cross sections by various methods is very extensive [1]–[4] (note many references in [4]).

It is surprising that the determination of scattering cross sections of objects which are *partially* insonified by finite sound *beams* has not generated much interest in the past. It has been noticed that, sometimes, objects that are insonified by a collimated finite sound beam emerging from a sonar have larger cross sections than when they are totally "lit." This empirical observation will be analyzed in the present paper by means of the method of physical optics, or Kirchhoff method. Quantitative predictions will be produced to explain that behavior since partial, rather than total, insonification of a target by a beam is a situation much more likely to occur in practice.

We will show how basic findings of the method of Fresnel zones can be improved and corrected by the method of physical optics, and in isolated cases, by the Geometrical

Theory of Diffraction [5], [6] in cases of partial beam insonification of simple target shapes. Comparison of cross-section results for finite and infinite bodies will demonstrate, for targets that have a Rayleigh distance, the importance of this distance in the distinction between near and far scattered fields. High-frequency approximations based on saddle-point evaluations of some integrals will show the relative dominance of the various scattering centers located at the specular reflection points, and at the edges of the spots that the incident beam insonifies on various targets. We have included an extension of the physical-optics method that permits the evaluation of bistatic or differential scattering cross sections of partially insonified bodies. We have illustrated most cases with graphical displays. The physical-optics results presented here suffer from the *deficiencies* of the method, which cannot account for the material composition of targets, the spreading of beams, or their lobe structure. These complexities must be handled by other methods.

II. SONAR CROSS SECTIONS DETERMINED BY THE PHYSICAL-OPTICS METHOD

The sonar or backscattering cross section of any object is

$$\sigma = 4\pi |f(\theta = \pi)|^2 \quad (1)$$

where the scattering function $f(\theta)$, in the backscattered direction $\theta = \pi$, is the quantity that defines the target strength

$$TS = 10 \log |f(\pi)|^2. \quad (2)$$

Kirchhoff showed that, in three-dimensions, the Rayleigh-Kirchhoff integrals can be approximated by the expression

$$f(\pi) = \pm \frac{i}{\lambda} \iint_S (\vec{k} \cdot \vec{n}) \exp(2i\vec{k} \cdot \vec{r}') dA' \quad (3)$$

where S is the insonified portion of the scatterer's surface. For a finite beam insonifying a spot on the target's surface, S is the surface of the spot. The outward normal is $\vec{n}$, the surface element is dA' , $\vec{k}$ is the propagation vector of the incident beam, and the upper (lower) signs correspond to rigid (soft) scatterers. In addition, $i = \sqrt{-1}$ and $\lambda = 2\pi/k$ = the wavelength of the radiation in the incident beam. The Kirchhoff approximation, involved in obtaining (3), requires that the field at the insonified spot be the sum of the incident plus the reflected beams, while the field in the shadow is zero. A third assumption, which is usually understood, is that each surface element of the target is assumed to belong to a plane

Manuscript received November 1, 1984; revised March 22, 1985. This work was supported by the Naval Surface Weapons Center Independent Research Board.

The author is with the Naval Surface Weapons Center, Electromagnetics Branch, White Oak, Silver Spring, MD 20910.

tangent to the element. A brief derivation of (3) is given in Section V. (9) is

A. Partial Insonification of a Sphere at a Round Spot

Let a beam of radius R_0 intersect a sphere of radius a ($R_0 < a$) as shown in Fig. 1(a). It is clear that

$$\vec{k} \cdot \vec{r}' = ka \cos \theta$$

$$\vec{k} \cdot \vec{n} = \cos \theta$$

$$dA' = a^2 \sin \theta d\theta d\phi \quad (4)$$

and (3) reduces to

$$f(\pi) = \pm \frac{i}{\lambda} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=\theta_0}^{\pi} \cos \theta \cdot \exp(2ika \cos \theta) a^2 \sin \theta d\theta \quad (5)$$

where

$$\theta_0 = \sin^{-1}(R_0/a)$$

$$\gamma \equiv \sqrt{1 - R_0^2/a^2} = -\cos \theta_0. \quad (6)$$

Clearly, $0 \leq \gamma \leq 1$. The transformation $\mu = \cos \theta$ changes (5) into the form

$$f(\pi) = \pm ika^2 \int_{\mu=-1}^{-\gamma} \exp[2i\mu ka] \mu d\mu \quad (7)$$

which is immediately integrable. The result is

$$f(\pi) = \pm \frac{a}{2} \left\{ \left[\frac{\sin \beta \gamma}{\beta} - \frac{\sin \beta}{\beta} - \gamma \cos \beta \gamma + \cos \beta \right] + i \left[\frac{\cos \beta \gamma}{\beta} - \frac{\cos \beta}{\beta} + \gamma \sin \beta \gamma - \sin \beta \right] \right\} \quad (8)$$

where $\beta \equiv 2ka$. Substituting (8) into (1) yields

$$\frac{\sigma}{\pi a^2} = \left| \left[1 + \frac{1}{\beta^2} \right] + \left[\gamma^2 + \frac{1}{\beta^2} \right] - 2 \left\{ \left[\gamma + \frac{1}{\beta^2} \cos \beta(1-\gamma) + \frac{(1-\gamma)}{\beta} \sin \beta(1-\gamma) \right] \right\} \right| \quad (9)$$

which is the sonar cross section of the partially insonified sphere. The first bracket is the specular reflection contribution. The second bracket is the contribution from the boundary of the spot. The curly bracket corresponds to interaction terms between the previous two contributions. The parameter γ controls the spot size and it is unity for a very small spot, in which case $\sigma = 0$. A more suitable form for the final result in

$$\frac{\sigma}{\pi a^2} = \left| 1 + \gamma^2 - 2\gamma \cos [2ka(1-\gamma)] - \frac{(1-\gamma)}{ka} \sin [2ka(1-\gamma)] + \frac{\sin^2 [ka(1-\gamma)]}{k^2 a^2} \right|. \quad (10)$$

The high-frequency (i.e., $ka \gg 1$) limit of (10) is

$$\frac{\sigma}{\pi a^2} = |1 + \gamma^2 - 2\gamma \cos [\beta(1-\gamma)]| \quad (11)$$

while the case of total insonification is (i.e., $R_0 = a$)

$$\frac{\sigma}{\pi a^2} = 1 - \frac{\sin 2ka}{ka} + \frac{\sin^2 ka}{k^2 a^2} \quad (12)$$

which is a known result [2], [4]. In this case of total insonification (i.e., $\gamma = 0$), two scattering centers contribute to the sonar cross section, the shadow boundary (i.e., the sphere's equator) and the specular reflection point (i.e., the South Pole). At high frequencies (i.e., $ka \gg 1$), the specular reflection contribution is dominant over that of the shadow boundary.

The physical-optics (or Kirchhoff's) method cannot distinguish between rigid and soft targets, and predicts cross sections somewhere in between these two extremes of behavior. The oscillations of the cross section predicted by (10) or (12) are due to the presence of circumferential or creeping waves that revolve around the sphere and interfere with the incident and reflected fields every time they cross around the lit side.

In the case of partial insonification (10), the spot's boundary is a parallel rather than the sphere's equator. Here, the specular reflection contribution is no longer dominant over that of the spot's boundary. Now, both contributions to the cross section have the same importance, and neither one is negligible. This implies that for the right values of γ and ka , the cross section of the spot-insonified sphere can be (up to four times) larger than for the totally insonified sphere. This result has useful practical implications, and explains empirical observations.

The result in (10) for the cross section of a sphere partially insonified at a round spot can also be written in the convenient form

$$\frac{\sigma}{\pi a^2} = \left| \frac{1}{\beta} \{ (1+i\beta)[1 - e^{i\beta(1-\gamma)}] + i\beta(1-\gamma)e^{i\beta(1-\gamma)} \} \right|^2. \quad (13)$$

If this cross section were to be rederived using Keller's Geometrical Theory of Diffraction (GTD) [5], [6], the final result is that of the physical-optics method as given above, plus a correction term that is introduced by a diffraction effect from the spot's edge which is accounted by the GTD. This theory is not to be further mentioned or used in the present paper, but the final result for this case is given below for completeness. It

Fig. 1. (a) Partial insonification of a sphere at a spot by a finite beam of diameter $2R_0 < 2a$. (b) Partial insonification of a plate at a spot of area A at normal incidence. (c) Front and top views of a circular cylinder under partial insonification at off-axis incidence. (d) Isometric view of the partially insonified cylinder at off-axis incidence.

is

$$\frac{\sigma}{\pi a^2} = \left| \frac{e^{-i\beta}}{\beta} \{ (1+i\beta)[1 - e^{-i\beta(1-\gamma)}] + i\beta(1-\gamma)e^{-i\beta(1-\gamma)} \} + \frac{2i}{n} \sin \frac{\pi}{n} (1-\gamma^2)e^{-i\beta\gamma} \right| \left[\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right]^2 \quad (14)$$

where $n = 1.5 + \delta/\pi$, $\delta = \sin^{-1} \gamma$, and $\gamma = \sqrt{1 - R_0^2/a^2}$. We now proceed to other shapes.

B. Beam Insonification of a Flat Plate, at a Spot

From Fig. 1(b), it is clear that $\vec{k} \cdot \vec{r}' = -kz$ and $\vec{k} \cdot \hat{n} = -1$, therefore at normal incidence, and for a square beam of side $2a$, the integral in (3) yields

$$\begin{aligned} f(\pi) &= \pm \frac{i}{\lambda} \int_{-a}^{+a} \int_{-a}^{+a} (-1) \exp(-2ikz') dx' dy' \\ &= \mp \frac{4a^2}{\lambda} \exp(-2ikz') \end{aligned} \quad (15)$$

and the sonar cross-section is found from (1) to be

$$\sigma = 4\pi(A/\lambda)^2 \quad (16)$$

where A is the area of the spot (i.e., $A = 4a^2$). The result in (16) is valid for a spot of any shape. For a circular spot of radius a , then $A = \pi a^2$. If the incidence is at oblique angles (i.e., $\theta \neq 0$, $\phi \neq 0$), then an analogous evaluation, for $ka \gg 1$, yields

$$\begin{aligned} \sigma &= 4\pi \left(\frac{A}{\lambda} \right)^2 \cos^2 \theta \left[\frac{\sin(2ka \sin \theta \sin \phi)}{2ka \sin \theta \sin \phi} \right]^2 \\ &\quad \cdot \left[\frac{\sin(2ka \sin \theta \cos \phi)}{2ka \sin \theta \cos \phi} \right]^2. \end{aligned} \quad (17)$$

This result is the same as that for a finite plate of area A , insonified by an infinite plane wave. It contains contributions of the type $\sin x/x$ from the edge of the beam, and a specular reflection contribution of the type $(A/\lambda)^2$. Note that the value $(A/\lambda)^2$ appears only in the far (Fraunhofer) region, after the reflected beam has had a chance to spread out. In the near field, or in the geometrical acoustics limit, the target strength of the beam-insonified plate is zero (see Section V, Particular Case 4). This would be the case of a high-frequency beam normally incident on a "mirror." Incident and reflected energies are equal at small distances, and $10 \log$ of a ratio of value unity is zero.

C. Spot Insonification of a Finite Cylinder

Let a finite beam of radius b be incident obliquely on a circular cylinder of radius a . From (3), the method of physical optics gives

$$f(\pi) = \pm \frac{i}{\lambda} \iint_S (-\cos \phi) \exp(-2ika \cos \phi) a d\phi dz \quad (18)$$

because $\vec{k} \cdot \vec{r}' = -ka \cos \phi$, $\vec{k} \cdot \hat{n} = -\cos \phi$, and $dA = adz d\phi$. The spot S that the circular beam intersects on the cylinder's surface can be defined in terms of angles ϕ_0 and ϕ_1 , which depend on the known oblique incidence angle ϕ' . These angles are

$$\begin{aligned} \phi_0 &= \arcsin \left[\frac{b}{a} \cos \phi' \right] \\ \phi_1 &= \arcsin \left[\frac{b}{a} + \sin \phi' \right] - \phi' \end{aligned} \quad (19)$$

and the spot on the cylinder's surface is then defined by

$$\begin{aligned} |z| &\leq \{b^2 - a^2[\sin(\phi + \phi') - \sin \phi']^2\}^{1/2}, \\ -\phi_0 &\leq \phi \leq \phi_1 \end{aligned} \quad (20)$$

which yields an integral of the form

$$\begin{aligned} f(\pi) &= \mp \frac{2ika}{\pi} \int_{\phi=-\phi_0}^{\phi_1} \{b^2 - a^2[\sin(\phi + \phi') - \sin \phi']^2\}^{1/2} \\ &\quad \cdot \exp(-2ika \cos \phi) \cos \phi d\phi. \end{aligned} \quad (21)$$

At normal incidence, $\phi' = 0$ and then $\phi_0 = \phi_1 = \arcsin(b/a)$. This is the situation depicted in Fig. 1(c) and (d). In this case, (21) reduces to

$$\begin{aligned} f(\pi) &= \mp \frac{2ika}{\pi} \int_{\phi=0}^{\sin^{-1}(b/a)} [b^2 - a^2 \sin^2 \phi]^{1/2} \\ &\quad \cdot \exp(-2ika \cos \phi) \cos \phi d\phi. \end{aligned} \quad (22)$$

This integral can be evaluated exactly when $a = b$ in terms of the Struve H [7] and the Bessel J [8] functions of orders zero and one. The pertinent result is

$$\begin{aligned} &\int_0^{\pi/2} \exp(-iz \cos \phi) \cos^2 \phi d\phi \\ &= \frac{\pi}{2} \left[J_0(z) - \frac{1}{z} J_1(z) \right] - i \left[H_0(z) - \frac{1}{z} H_1(z) \right] \end{aligned} \quad (23)$$

and in this case (20) yields

$$\begin{aligned} f(\pi) &= \mp ika^2 \left\{ \left[J_0(2ka) - \frac{1}{2ka} J_1(2ka) \right] \right. \\ &\quad \left. - i \left[H_0(2ka) - \frac{H_1(2ka)}{2ka} \right] \right\} \\ &\xrightarrow{ka \gg 1} + ika^2 H_0^{(2)}(2ka). \end{aligned} \quad (24)$$

Using the principal asymptotic expansion of the Hankel [7], [8] function of the second-kind $H_0^{(2)}$, the sonar cross section becomes

$$\sigma = 4\pi |f(\pi)|^2 \xrightarrow{ka \gg 1} ka(2a)^2. \quad (25)$$

In other instances (i.e., $a \neq b$ or $\phi' \neq 0$), the integrals in (21) and (22) have to be evaluated numerically for known values of ϕ' . However, to obtain the high-frequency (i.e., $ka \gg 1$)

behavior of these integrals, one can perform an asymptotic evaluation by the method of steepest descents. Applying the basic one-term asymptotic formula (for $\rho \gg 1$) [9]:

$$\int_{-\infty}^{+\infty} A(\phi) \exp [-i\rho\alpha(\phi)] d\phi \cong \sqrt{\frac{2\pi}{i\rho\alpha''(\phi_s)}} A(\phi_s) \exp [-i\rho\alpha(\phi_s)] \quad (26)$$

where ϕ_s is the root of $\alpha'(\phi) = 0$, yields in both cases

$$f(\pi) \xrightarrow{ka \gg 1} b \left(\frac{ka}{\pi} \right)^{1/2} \exp [-i(2ka - 3\pi/4)]. \quad (27)$$

Thus, for (21) and (22), the corresponding sonar cross section is

$$\sigma = 4\pi |f(\pi)|^2 \xrightarrow{ka \gg 1} ka(2b)^2. \quad (28)$$

We note that this result is the same as that of a finite-length cylinder of radius a , length $2b$, insonified by a high-frequency (i.e., $ka \gg 1$) infinite plane wave at beam aspect [10] (i.e., at $\theta = 90^\circ$). This interesting result is due to the fact that in both cases the same first Fresnel zone is generated on the cylinder's surface. We point out that if the analysis in this section is repeated for a square rather than a circular beam of side $2b$, then the saddle-point approximation to the cross section, for $ka \gg 1$, comes out to be the same as in (28). Thus the cross section is independent of spot shape, as long as the same first Fresnel zone is generated by the spot. Fresnel zones are further discussed in Section V.

The result in (28) is a specular reflection contribution, since this is the usual output of the saddle-point method. It can be easily shown that the contribution from the spot's boundary is proportional to $(ka)^{1/2}$, and thus it is negligible for $ka \gg 1$ when compared to the specular reflection contribution, which is of order ka . This behavior is quite different than that of the spot-insonified sphere. There, spot boundary and specular reflection contributions to the cross section were of the same order of magnitude, and neither one could be neglected.

D. Spot Insonification of an Infinite Cylinder

The integral in (3) is slightly modified in this two-dimensional case as follows:

$$f_2(\pi) = \mp \sqrt{\frac{-i}{\lambda}} \iint_S (\hat{k} \cdot \hat{n}) \exp (2i\vec{k} \cdot \vec{r}') dA' \quad (29)$$

where

$$\vec{k} \cdot \vec{r}' = -ka \cos \theta$$

$$\hat{k} \cdot \hat{n} = \cos \theta$$

and

$$dA' = ad\theta.$$

Therefore (29) becomes

$$f_2(\pi) = \mp 2 \sqrt{\frac{k}{2\pi i}} \int_0^{\sin^{-1}(R_0/a)} (-\cos \theta) \cdot \exp (-2ika \cos \theta) a d\theta \quad (30)$$

where R_0 is the width of the beam and a is the radius of the cylinder. The above integral can be evaluated exactly for the case of total insonification (i.e., $R_0 \geq a$) with the help of the known integral

$$\int_0^{\pi/2} \exp (-iz \cos \theta) \cos \theta d\theta = 1 - \frac{\pi}{2} [H_1(z) + iJ_1(z)] \quad (31)$$

and the resulting (two-dimensional) cross section then is

$$\sigma \equiv 2\pi |f_2(\pi)|^2 = ka^2\pi^2 \left\{ \left[\frac{2}{\pi} - H_1(2ka) \right]^2 + J_1^2(2ka) \right\} \quad (32)$$

where H_1 and J_1 are the Struve [7] and Bessel [8] functions of order one, respectively. Using the well-known high-frequency (large argument) asymptotic forms for these functions, we find that for $ka \gg 1$, the cross section reduces to $\sigma \rightarrow \pi a$. For partial insonification (i.e., $R_0 < a$), the integral in (30) must be evaluated numerically. However, for $ka \gg 1$, a high-frequency asymptotic approximation for it can be obtained by the saddle-point method. Application of the one-term asymptotic formula (26) yields

$$\int_0^{\sin^{-1}(R_0/a)} \exp [-i(2ka) \cos \theta] \cos \theta d\theta = \frac{1}{2} \sqrt{\frac{\pi i}{ka}} \exp (-2ika) \quad (33)$$

and, therefore, the high-frequency asymptotic approximation for the cross section is also $\sigma \rightarrow \pi a$ for partial insonification. The contribution from the spot boundary is negligible compared to the specular reflection contribution, just as in the case of the finite cylinder. Both of these cases are different than that of the spot-insonified sphere.

E. Total Insonification of a Finite Cylinder

Although this is not a case of partial insonification, it is very worthwhile to outline briefly the procedure in this instance. Let the cylinder's length be $2b$, a its radius, and k the wavenumber of the incident wave. The contribution from the curved side of the cylinder is given (3) by:

$$f(\pi) = \pm \frac{ik}{2\pi} \int_{\phi=-\pi/2}^{\pi/2} d\phi \int_{z=-b}^{+b} (-\sin \theta) \cdot \cos \phi e^{-2ika \sin \theta \cos \phi} e^{-2ikz \cos \theta} a dz \quad (34)$$

Fig. 2. Geometry and coordinates for the insonification of a finite-length cylinder.

where we have used the quantities

$$\begin{aligned}\hat{k} \cdot \hat{n} &= -\sin \theta \cos \phi \\ \vec{r}' &= a \cos \phi \hat{e}_x + a \sin \phi \hat{e}_y + z \hat{e}_z \\ |z| &\leq b \\ -\pi/2 \leq \phi \leq \pi/2 \\ \vec{k} \cdot \vec{r}' &= -k(a \sin \theta \cos \phi + z \cos \theta).\end{aligned}$$

The geometry and the incident wave are illustrated in Fig. 2. The z integral yields

$$\begin{aligned}\int_{-b}^{+b} \exp(-2ikz \cos \theta) dz &= 2b \frac{\sin(2kb \cos \theta)}{2kb \cos \theta} \\ &= 2bj_0(2kb \cos \theta)\end{aligned}\quad (35)$$

and the ϕ integral can be performed by means of (31). Thus

$$\begin{aligned}f(\pi) &= \mp ikab \sin \theta j_0(2kb \cos \theta) \\ &\cdot \left\{ \left[\frac{2}{\pi} - H_1(2ka \sin \theta) \right] - iJ_1(2ka \sin \theta) \right\}\end{aligned}\quad (36)$$

is the exact contribution from the cylinder's curved side. For the high-frequency behavior we use the asymptotic forms

$$\begin{aligned}H_1(z) &\xrightarrow{|z| \rightarrow \infty} \frac{2}{\pi} + Y_1(z) \\ H_1^{(2)}(z) &\xrightarrow{|z| \rightarrow \infty} \sqrt{\frac{2}{\pi z}} \exp[-i(z - 3\pi/4)]\end{aligned}\quad (37)$$

which yield the following high-frequency ($ka \gg 1$) behavior:

$$\begin{aligned}f(\pi) &= \mp \frac{ia}{2 \cos \theta} \sin(2kb \cos \theta) \sqrt{\frac{\sin \theta}{\pi ka}} \\ &\cdot \exp[i(\pi/4 - 2ka \sin \theta)].\end{aligned}\quad (38)$$

The cross section resulting from this analysis is found from (1) to be

$$\begin{aligned}\sigma &= 4\pi b^2 \left[\frac{\sin(2kb \cos \theta)}{2kb \cos \theta} \right]^2 \left[\sqrt{\frac{ka}{\pi}} \sin \theta \right]^2 \\ &\xrightarrow{\theta \rightarrow \pi/2} ka(2b)^2\end{aligned}\quad (39)$$

where the last result is the beam-aspect value that was anticipated before, in (28), after using the method of steepest descents.

The contribution from the top flat surface of the cylinder is

$$\begin{aligned}f(\pi) &= \pm \frac{i}{\lambda} \int_{r'=0}^a \int_{\phi=0}^{2\pi} (-\cos \theta) \\ &\cdot \exp[-2ik(r' \sin \theta \cos \phi + b \cos \theta)] r' dr' d\phi\end{aligned}\quad (40)$$

where we have used the relations

$$\begin{aligned}\hat{k} \cdot \hat{n} &= -\cos \theta \\ \vec{k} &= -k \sin \theta \hat{e}_x - k \cos \theta \hat{e}_y \\ \vec{r}' &= r \sin \alpha \\ \vec{r}' &= (r \sin \alpha) \cos \phi \hat{e}_x + (r \sin \alpha) \sin \phi \hat{e}_y + b \hat{e}_z \\ \vec{k} \cdot \vec{r}' &= -k(r' \sin \theta \cos \phi + b \cos \theta).\end{aligned}\quad (41)$$

If we use the known expressions

$$J_0(z) = \frac{1}{2\pi} \int_0^{2\pi} \exp(-iz \cos \phi) d\phi$$

and

$$\int z J_0(z) dz = z J_1(z)$$

then the integrals in (40) can be evaluated, and the result is

$$f(\pi) = \mp ika \left\{ \frac{a}{2} \cos \theta \exp(-2ikb \cos \theta) \frac{J_1(2ka \sin \theta)}{ka \sin \theta} \right\}\quad (42)$$

which is the exact value of this contribution. Using the known high-frequency asymptotic expansion for $J_1(z)$, the high-frequency ($ka \gg 1$) behavior of the above result is

$$\begin{aligned}f(\pi) &= \mp \frac{ia \cot \theta}{\sqrt{4\pi ka} \sin \theta} \exp(-2ikb \cos \theta) \\ &\cdot \cos(2ka \sin \theta - 3\pi/4).\end{aligned}\quad (43)$$

The complete answer, in closed form, taking into account top and side contributions, respectively, is given by the sum of the two results given in (42) and (36). The high-frequency asymptotic behavior of that complete answer is given by the sum of the contributions given in (43) and (38). The complete

cross section is found by substituting $f(\pi)$ from these complete results into (1).

We note that (42) contains the result for a circular flat disk by setting $b = 0$. Substituting that result into (1) yields the disk's cross section

$$\sigma = \frac{\pi a^2}{\tan^2 \theta} J_1(2ka \sin \theta) \quad (44)$$

which at normal incidence (i.e., $\theta = 0$) reduces to

$$\sigma = \pi a^2 (ka)^2 = 4\pi (A/\lambda)^2 \quad (45)$$

where $A = \pi a^2$ is the area of the disk. This is the result anticipated in the discussion after (16).

All the above results are based on the physical-optics, or Kirchhoff, method, which is an approximate method based on the Kirchhoff approximations. Thus the "exact" solutions mentioned here are exact only within the context of the Kirchhoff method. The truly exact solution for a beam partially insonifying a rigid or soft cylinder is not available. In fact, such exact solutions are only possible for the ellipsoid and its degenerate cases.

III. DIFFERENTIAL OR BISTATIC SCATTERING CROSS SECTIONS OF PARTIALLY INSONIFIED OBJECTS

The scattered field of a convex object is given by the sum of two contributions so that $\psi_s = I_1 + I_2$, where

$$I_1 = \frac{1}{4\pi} \iint_{S_1} \left[\frac{\partial \psi_i}{\partial n_0} \frac{e^{ikR}}{R} - \psi_i \frac{\partial}{\partial n_0} \left(\frac{e^{ikR}}{R} \right) \right] dS_0$$

$$I_2 = \frac{1}{4\pi} \iint_{S_2} \left[\frac{\partial \psi_i}{\partial n_0} \frac{e^{ikR}}{R} + \psi_i \frac{\partial}{\partial n_0} \left(\frac{e^{ikR}}{R} \right) \right] dS_0 \quad (46)$$

and $R = |\vec{r} - \vec{r}_0|$, S_1 is the noninsonified portion of the body, S_2 is the spot insonified on the body, and $\psi_i = Ae^{ikz_0}$ is the incident plane wave.

1) For a sphere partially insonified, as shown in Fig. 1(a), in the far field ($r \gg r_0$), I_1 can be integrated immediately to yield the diffraction pattern of a disk, i.e.,

$$I_1 = \frac{Ae^{ikr}}{r} \frac{ik}{2} (1 + \cos \theta) R_0^2 \frac{J_1(kR \cos \theta)}{kR_0 \sin \theta} \quad (47)$$

The determination of I_2 is accomplished by a saddle-point evaluation of the integral after a transformation to a new (primed) set of coordinates centered at the specular reflection point. In these coordinates θ_0' , ϕ_0' , one has

$$I_2 = \frac{Ae^{ikr}}{r} \frac{i\lambda a}{2} \int_{\sin^{-1} R_0/a}^{\theta_0' = \beta(\phi_0')} g(\theta_0') \exp [i\lambda h(\theta_0')] d\theta_0' \quad (48)$$

where

$$h(\theta_0') = \cos \theta_0'$$

$$g(\theta_0') = \sin \theta_0' \cos \theta_0'$$

$$\lambda = 2ka \sin (\theta/2) \quad (49)$$

and the upper limit is the equation of the spot edge in the new coordinates, which need not be specifically known. The final result is

$$I_2 = \frac{Ae^{ikr}}{r} \frac{i\lambda a}{2} \left[-\frac{i}{\lambda} (e^{-i\lambda} - \gamma e^{-i\lambda\gamma}) - \frac{1}{\lambda^2} (e^{-i\lambda} - e^{-i\lambda\gamma}) \right] \quad (50)$$

The total value of the scattering amplitude $f(\theta)$, which is the coefficient of Ae^{ikr}/r , from these two contributions I_1 and I_2 , is

$$f(\theta) = \frac{a}{2} \left[i \frac{R_0}{a} \cot \frac{\theta}{2} J_1 \left(\frac{R_0}{a} ka \sin \theta \right) + e^{-i\lambda} - \gamma e^{-i\lambda\gamma} - \frac{i}{\lambda} (e^{-i\lambda} - e^{-i\lambda\gamma}) \right] \quad (51)$$

This yields a normalized bistatic cross section of the form

$$\frac{4}{a^2} \frac{d\sigma}{d\theta} = \left| \frac{2}{a} f(\theta) \right|^2 = \left| i \frac{R_0}{a} \cot \frac{\theta}{2} J_1 \left(\frac{R_0}{a} ka \sin \theta \right) + e^{-i\lambda} - \gamma e^{-i\lambda\gamma} - \frac{i}{\lambda} (e^{-i\lambda} - e^{-i\lambda\gamma}) \right|^2 \quad (52)$$

where λ and γ were given above. In the backscattered direction $\theta = \pi$, this result reduces to that given in (10) since

$$\frac{4}{a^2} \left(\frac{d\sigma}{d\theta} \right)_{\theta=\pi} = \frac{\sigma}{\pi a^2} \quad (53)$$

2) For a partially insonified sphere with an elevated cap, the geometry is shown and defined in Fig. 3. The cap is impenetrable and centered within the spot. The radii of the spot and the cap are R_0 and R_2 , respectively, and h is the cap thickness. I_1 has the same value given above, and I_2 can be evaluated by the same procedure described above to yield ($R_0 > R_2$):

$$I_2 = \frac{Ae^{ikr}}{r} \frac{a}{2} \left\{ \gamma_2 e^{-i\lambda\gamma_2} - \gamma e^{-i\lambda\gamma} - \frac{i}{\lambda} (e^{-i\lambda\gamma_2} - e^{-i\lambda\gamma}) + \left(1 + \frac{h}{a} \right) \left[e^{-i\lambda'} - \gamma' e^{-i\lambda'\gamma'} - \frac{i}{\lambda'} (e^{-i\lambda'} - e^{-i\lambda'\gamma'}) \right] \right\} \quad (54)$$

where λ and γ are as before and

$$\lambda' = 2k(a+h) \sin \frac{\theta}{2}$$

$$\gamma' = \left[1 - \left(\frac{R_2}{a+h} \right)^2 \right]^{1/2} = 1 - \frac{b}{a+h}$$

and

$$\gamma_2 = \left(1 - \frac{R_2^2}{a^2} \right)^{1/2}$$

$$= \left\{ 1 - \left(\frac{a+h}{a} \right)^2 \left[1 - \left(1 - \frac{b}{a+h} \right)^2 \right] \right\}^{1/2} \quad (55)$$

Fig. 3. A circular sound spot of radius R_0 insonified on a sphere of radius a , with a cap of thickness h and radius $R_2 < R_0$.

Clearly, γ controls the spot size, while γ_2 and γ' control the size and thickness of the cap.

The normalized bistatic cross section for the partially insonified capped sphere is then obtained from (47) and (54), viz.,

$$\begin{aligned} \frac{4}{a^2} \frac{d\sigma}{d\theta} = & \left| i(1-\gamma^2)^{1/2} \cot \frac{\theta}{2} J_1[(1-\gamma^2)^{1/2}ka \sin \theta] \right. \\ & + \gamma_2 e^{-i\lambda\gamma_2} - \gamma e^{-i\lambda\gamma} - \frac{i}{\lambda} (e^{-i\lambda\gamma_2} - e^{-i\lambda\gamma}) \\ & + \left(1 + \frac{h}{a}\right) \left[e^{i\lambda'} - \gamma' e^{-i\lambda'\gamma'} \right. \\ & \left. \left. - \frac{i}{\lambda'} (e^{-i\lambda'} - e^{-i\lambda'\gamma'}) \right] \right|^2 \end{aligned} \quad (56)$$

which can be particularized to the backscattering direction $\theta = \pi$ by means of (53). The final result is

$$\begin{aligned} \frac{\sigma}{\pi a^2} = & \left| \gamma_2 e^{-i\beta\gamma_2} - \gamma e^{i\beta\gamma} - \frac{i}{\beta} (e^{-i\beta\gamma_2} - e^{-i\beta\gamma}) \right. \\ & \left. + m \left[e^{-i\beta m} - \gamma' e^{-i\beta m\gamma'} - \frac{i}{\beta m} (e^{-i\beta m} - e^{-i\beta m\gamma'}) \right] \right|^2 \end{aligned} \quad (57)$$

where $\beta = 2ka$, and $m = 1 + (h/a)$. This result is the same one would have obtained by means of (3).

IV. COMMENTS ON THE NEAR AND FAR FIELDS OF THE CYLINDER

The question of why we have studied the case of an infinite cylinder, when only finite-length cylinders can be realized in practice, has not yet been fully addressed. We have seen that the finite cylinder partially insonified at a spot behaves quite differently from the sphere or any other three-dimensional convex body. The cylinder has a "Rayleigh distance." So, if a sound wave of wavelength λ is incident upon a cylinder of length L and we observe the scattered echo at a range r , this distance will have to be at least of value L^2/λ (i.e., the Rayleigh distance) for the observation to be a far-field observation and compare favorably with (39). This follows because the scattering function f , the cross section σ , and the whole concept of target strength have been developed for the far field. If one is constrained to operate at small wavelengths (i.e., say 0.01 cm), then one must be very far away from a finite-length cylinder to be able to make far-field observations of its returned echoes. Since this is hardly ever possible in the laboratory, one makes the measurements anyway, at smaller distances, with the understanding that this near-field data cannot be compared to the far-field results of the finite cylinder. What can it be compared to then? It soon becomes clear that it must be compared to the far-field result of the *infinite* cylinder. Cylinder waves scattered by a cylinder have a Hankel function behavior in the near field of the finite cylinder (i.e., within its Rayleigh distance), and also in the *near* and *far* fields of the infinite cylinder. The far-field behavior of the finite cylinder is spherical, and of the usual form e^{ikr}/r . Therefore the near field of the finite cylinder is essentially the same as the far-field behavior of the infinite cylinder. We note that the three-dimensional target strength of the finite cylinder (i.e., $TS = aL^2/2\lambda$) is a factor L^2/λ larger than the two-dimensional target strength of an infinite cylinder (i.e., $TS = a/2$), and this factor is precisely the Rayleigh distance.

V. THE FRESNEL-ZONE METHOD TO DETERMINE SCATTERING CROSS SECTIONS

If a train of wavefronts separated one from the next by a distance $\lambda/4$ is incident on an immovable impenetrable object, then the projected areas produced by the intersections of these wavefronts with the target's surface are called Fresnel zones. If the projector emitting the waves is at infinity, the wavefronts will be plane as shown in Fig. 4. Each zone reflects an echo to an observation point 0, which is the sum of the contributions of secondary sources on the target's surface lying within that zone. Since the n th zone is $\lambda/4$ units farther away from 0 than the $(n-1)$ th zone, the sound reflected from the n th zone and received at 0 has traveled $\lambda/2$ units more than sound reflected from the $(n-1)$ th zone. The contributions received at 0 are totally out of phase for all pairs of adjacent zones. The contribution from the n th zone does not completely cancel that from the $(n-1)$ th zone, because the amplitudes of the secondary wavelets are not exactly equal. However, the

of the arc $\overline{QT}$, thus $\overline{PQ} = \overline{PT} = a$. It is not hard to show that

$$\begin{aligned}\overline{ST} &= [(r+a)^2 + a^2 - 2a(r+a) \cos \theta]^{1/2} \\ &\cong r + \frac{2a}{r} (r+a) \sin^2 \frac{\theta}{2} \\ \overline{T0} &\cong R + \frac{2a}{R} (R+a) \sin^2 \frac{\theta}{2}\end{aligned}\quad (61)$$

and the substitution into the definition of the first Fresnel zone, viz.,

$$\overline{SQ} + \overline{Q0} + \lambda/2 = \overline{ST} + \overline{T0}$$

yields

$$\sin \frac{\theta}{2} = \frac{1}{2a} \left[\left(\frac{1}{r} + \frac{1}{R} + \frac{2}{a} \right) / \lambda \right]^{-1/2}. \quad (62)$$

It is clear that $\overline{TN} = a \sin \theta \cong 2a \sin \theta/2$. Since the projection of the first Fresnel zone on a plane normal to the wave train is approximately an ellipse, then its semi-axes are

$$\begin{aligned}\overline{TN}_1 &= \left[\left(\frac{1}{r} + \frac{1}{R} + \frac{2}{a_1} \right) / \lambda \right]^{-1/2} \\ \overline{TN}_2 &= \left(\frac{1}{r} + \frac{1}{R} + \frac{2}{a_2} \right) / \lambda \right]^{-1/2}\end{aligned}$$

where a_1, a_2 are the principal radii of curvature at the center of the first zone. Since $A = \pi \overline{TN}_1 \cdot \overline{TN}_2$, it follows that

$$\psi_R = \frac{C}{R} \left[\left(\frac{1}{r} + \frac{1}{R} + \frac{2}{a_1} \right) \left(\frac{1}{r} + \frac{1}{R} + \frac{2}{a_2} \right) \right]^{-1/2} \quad (63)$$

where C is the amplitude of the velocity potential in the incident wave and r, R are the distances of the target from the source and receiver, respectively. The target strength TS is usually defined for a receiver 1 yd away from the target, thus

$$\begin{aligned}TS &= 20 \log \left| \frac{\psi_R}{C} \right|_{R=1} \\ &= 10 \log \left[\frac{1/R^2}{\left(\frac{1}{R} + \frac{1}{r} + \frac{2}{a_1} \right) \left(\frac{1}{R} + \frac{1}{r} + \frac{2}{a_2} \right)} \right]_{R=1}\end{aligned}\quad (64)$$

which is a formula containing many useful particular cases.

A. Particular Cases

1) Any convex surface of radii a_1, a_2 (for $r, R \gg a_1, a_2 > \lambda$):

$$\psi_R = \frac{C}{R} \sqrt{\frac{a_1 a_2}{4}}$$

or

$$TS = 10 \log \frac{a_1 a_2}{4}.$$

2) A sphere ($a_1 = a_2 = a$) at large distances ($r, R \gg a$):

$$\psi_R = \frac{Ca}{2R}$$

or

$$TS = 10 \log \frac{a^2}{4}.$$

3) An infinite plane ($a_1 = a_2 = \infty$):

$$\psi_R = \frac{C}{1 + \frac{R}{r}}$$

or

$$TS = 10 \log \frac{1}{\left(1 + \frac{1}{r}\right)^2}.$$

4) Infinite plane if the incident waves are plane ($r = \infty$):

$$\psi_R = C$$

or

$$TS = 0.$$

5) Infinite plane if incident waves are spherical, originating at the receiver ($r = R$):

$$\psi_R = \frac{C}{2}$$

or

$$TS = -6 \text{ dB}$$

6) Infinite cylinder ($a_1 = a, a_2 = \infty$), when incident waves are plane ($r = \infty$) and receiver is at large distances ($R \gg a$):

$$\psi_R = \frac{C}{R} \sqrt{\frac{aR}{2}}$$

or

$$TS = 10 \log \frac{a}{2}.$$

7) Infinite cylinder under spherical waves, source and receiver coinciding ($r = R$), but still at large distances ($r = R \gg a$):

$$\psi_R = \frac{C}{2} \sqrt{\frac{a}{R}}$$

or

$$TS = 10 \log \frac{a}{4}.$$

All the results by the first Fresnel-zone analysis coincide with those of geometrical acoustics, since both approaches just give the specular return. Had the contributions from *all* the zones been included, we would have found results identical to those given by the physical-optics method, since the starting integral formula is the same in both cases (i.e., (3) or (59)).

A useful rule of thumb to distinguish near from far fields is as follows. "If sound waves fall upon and are scattered by an

Fig. 6. Normalized sonar cross section of a circular spot insonified on a sphere by a finite beam versus ka for various (i.e., seven) values of the spot-size parameter γ .

object, and three-dimensional spreading does *not* immediately occur, then there is some characteristic (Rayleigh) distance past which one must be to be in the far-field of the object. If three-dimensional (spherical) spreading occurs immediately as soon as the wave is scattered, and this is always the case for a convex body, then there is no Rayleigh distance, and one reaches the far field of the object when the range is many times the acoustic wavelength (i.e., for $kr \gg 1$). It is only in this latter case that high-frequency or far-field approximations are equivalent. It is also in situations like this latter one that a condition such as $r = L^2/\lambda$ can be shown to be an *identity* rather than a true restriction.

VI. NUMERICAL RESULTS AND DISCUSSION

Equation (10) or (13) is shown in Fig. 6 plotted versus ka , in the range $0 \leq ka \leq 10$, for various values of the spot-size parameter γ . Fig. 7 shows plots of the same (10) for the same seven values of γ , but now ka varies in the range $1 \leq ka \leq 400$, and the scale is logarithmic. Observation of this figure shows that for large ka values, the normalized cross section $\sigma/\pi a^2$ oscillates very rapidly about the mean value $1 + \gamma^2$ within the bounds $(1 - \gamma)^2$ and $(1 + \gamma)^2$. This behavior can be easily extracted from the analysis of (11). The case $\gamma = 0$, corresponding to total insonification, is the plot of the known result in (12). Fig. 8 displays the plot of (10), now versus γ (for $0 \leq \gamma \leq 1$), for various values of ka . This graph supplements the information in Figs. 6 and 7 and displays it in another useful way. As ka increases, the cross section oscillates rapidly within the envelopes to the curves shown in Fig. 8. It is sometimes convenient to display the cross section versus the ratio R_0/a , for various values of ka . This is shown in Fig. 9. The upper (lower) envelopes have expressions $(1 \pm \sqrt{1 - R_0^2/a^2})^2$ and the mean value of these two extremes is

$2 - R_0^2/a^2$. This plot is very useful when comparing to experimental results, since R_0 is just the spot radius. For comparison purposes we have generated the companion figure to Fig. 7, but containing the additional correction term of the GTD shown in (14), in Fig. 10. Fig. 10 then displays (14) versus ka ($0 \leq ka \leq 400$) in a logarithmic scale for the same seven spot sizes used before. This GTD prediction shows larger cross-sectional oscillations than those predicted by the physical-optics method, for all spot sizes due to an additional diffractive contribution from the spot's edge, inherent to that theory. The differences between the physical-optics (Kirchhoff) method and the GTD are illustrated quite vividly in Fig. 11, which displays quantities easily obtainable from (13) (solid line) and (14) (dashed line). The deep diffractive dips of the GTD prediction are hardly noticeable in the physical-optics method. We note that Fig. 11 is for a totally insonified sphere under both Kirchhoff and GTD methods. The case of partially insonified sphere is contrasted, in the same fashion, in Fig. 12. This figure displays plots of the sonar cross section from (13) and (14) normalized to λ^2 , and plots them versus kd for various values of ka . Here d is the height of the spherical segment that the finite beam insonifies on the sphere. The solid lines are the result of the physical-optics method, while the dashed lines contain the additional GTD correction shown in (14).

The spot insonification of a sphere with an elevated cap is shown in Fig. 13. The beam radius is larger than that of the cap. Fig. 3 describes the geometry of the sphere, the beam, and the cap. The cross section σ is normalized to λ^2 , and it is plotted versus kb for various values of ka (viz., 1, 5, 10, and 16) as predicted by the physical-optics method in (57). The entire plot is calculated for the single kh value of $\pi/2$. Note that b is the cap depth, as shown in Fig. 3. Fig. 14 contains

Fig. 7. Normalized sonar cross section of a sphere partially insonified at a spot of radius $R_0 (< a)$ plotted versus ka in the range $0 \leq ka \leq 400$ for various values of γ . This is the physical-optics result given in (10).

Fig. 8. Normalized sonar cross section of a spot insonified on a sphere by a finite beam versus γ ($0 \leq \gamma \leq 1$) for various values of ka .

Fig. 9. Normalized sonar cross section of a spot insonified on a sphere by a finite beam versus the ratio R_0/a for various values of ka . The envelopes bound the rapid oscillations of the cross section.

Fig. 10. Plot of the sonar cross section of a spot insonified on a sphere by a finite beam versus ka for various values of γ . This is a plot of (14), which contains an additional correction term accounting for diffraction from the spot's edge as predicted by the GTD.

Fig. 11. Contrast between the predictions of (13) and (14) for a spot on a sphere versus ka . Here the sphere is totally insensitized, and the ordinates show the sonar cross section normalized to λ^2 .

Fig. 12. Further contrast between the normalized sonar cross section σ/λ^2 of a spot on a sphere versus kd for various values of ka . Solid lines give the physical-optics prediction while dashed lines give the GTD prediction.

Fig. 14. Normalized sonar cross section (in decibels) versus the acoustical depth kd of a cap placed on a sphere illuminated by a beam. The sphere has various acoustic sizes (i.e., $ka = 1, 5, \dots$) and the cap is of thickness $kh = \pi$ (From (57)).

Fig. 13. Normalized sonar cross section (in decibels) versus the acoustical depth kb of a cap placed on a sphere illuminated by a beam. The sphere has various acoustic sizes (i.e., $ka = 1, 5, \dots$) and the cap is of thickness $kh = \pi/2$ (From (57)).

Fig. 15. Plots of the normalized differential scattering cross section versus angle θ at a fixed value $ka = 16$, and of the normalized backscattering (i.e., sonar, $\theta = \pi$) cross section $\sigma/\pi a^2$ versus $ka (1 \leq ka \leq 400)$ for a sphere with a cap attached of thickness h , partially insonified at a spot of larger size than the cap. Various spot and cap sizes are considered (from (56) and (57)).

plots analogous to those in Fig. 13, but they are now computed for the fixed kh value of π . These plots show that the addition of a thin cap can enhance or reduce the spot-insonified sphere's cross section by over an order of magnitude.

Bistatic cross sections for a capped sphere partially insonified by a finite beam, as predicted by the physical-optics method in (56), are shown in Fig. 15. This figure also shows monostatic cross sections from (57). The bistatic calculations are shown for $ka = 16$ at all angles θ , while the monostatic calculations are for $\theta = \pi$, and for ka in the range $1 \leq ka \leq 400$. It is all for a sphere with a cap attached of thickness h , upon which a finite beam shines a spot larger than the cap. The results of these figures extend work computed and displayed in an earlier publication [13] of ours, where we used the Kirchhoff method to study partial insonification of spheres.

Fig. 16 shows the normalized sonar cross section of a rigid cylinder of radius a versus ka , as predicted by the physical-optics method in (32). The physical-optics or Kirchhoff prediction is compared to the known [1] exact solution (solid line) to show that the agreement is relatively good in amplitude, but poor in phase.

The present analysis serves to quantitatively predict, within the context of the physical-optics or Kirchhoff method, the

monostatic or bistatic cross-sectional enhancements or reductions present at suitable frequencies or angles when a smooth convex object is partially insonified by a collimated sound beam. All cases analyzed have shown that partial insonification of (simple) targets can enhance their cross sections appreciably, sometimes over an order of magnitude. The physical-optics (Kirchhoff) method cannot account for the lobe structure of the beam, the possible energy leakage from it, or the material composition of the targets. We have compared some of the findings to those of the GTD, or to those of the Geometrical Acoustics Method (Fresnel zones). The physical-optics method yields good simple predictions for frequencies that are high, but not as high as those of the other mentioned methods. The physical-optics method, presented here for sound beams, holds equally well for light beams with very minor modifications in terminology. To account for beam spreading and target compositions, more general [11], [12] approaches are required.

ACKNOWLEDGMENT

The author acknowledges the computational assistance of J. Barlow.

Fig. 16. Normalized sonar cross section of a rigid cylinder versus ka as given by the Kirchhoff or physical-optics method in (32) (dashed line), compared to the known exact solution (solid line), to show the relative agreement.

REFERENCES

- [1] J. Bowman *et al.*, *Electromagnetic and Acoustic Scattering by Simple Shapes*. New York: Wiley Interscience, 1969.
- [2] J. W. Crispin and K. Siegel, *Methods of Radar Cross-Section Analysis*. New York: Academic, 1968.
- [3] G. T. Ruck, Ed., *Radar Cross-Section Handbook*, vols. 1 and 2. New York: Plenum, 1970.
- [4] J. W. Crispin and A. Moffett, "Radar cross-section estimation for (a) simple, (b) complex shapes," *Proc. IEEE*, vol. 53, no. 8, (a) pp. 833-848, (b) 973-982, 1965.
- [5] J. B. Keller, "The geometrical theory of diffraction," *J. Optical Soc. Amer.*, vol. 52, pp. 116-130, 1962.
- [6] J. B. Keller, "Asymptotic solutions of some diffraction problems," *Commun. Pure Appl. Math.*, vol. 9, pp. 207-265, 1956.
- [7] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. Washington, DC: NBS, 1964, chs. 12 and 9.
- [8] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*. New York: Academic, 1965, arts. 8.4 and 8.5.
- [9] L. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. Englewood Cliffs, NJ: Prentice Hall, 1973, art. 4.2.
- [10] R. J. Urick, *Principles of Underwater Sound for Engineers*. New York: McGraw-Hill, 1967, ch. 9.
- [11] G. Gaunard and H. Überall, "Acoustics of finite beams," *J. Acoust. Soc. Amer.*, vol. 63, pp. 5-16, 1978.
- [12] L. Felsen and G. Deschamps, Eds., *Proc. IEEE* vol. 62, pp. 1409-1616, 1974.
- [13] G. Gaunard, "Monostatic and bistatic cross sections of a large (capped) sphere insonified at a circular spot," *J. Acoust. Soc. Amer.*,

vol. 61, pp. 1121-1132, 1977.

Guillermo C. Gaunard (M'82-SM'83) was born July 19, 1940 in Havana, Cuba. He received the Ph.D. degree in acoustics from Catholic University, Washington, DC, in 1971.

He has been a Research Physicist at the Research Department of the Naval Surface Weapons Center, White Oak, MD, since 1971, specializing in acoustics and electromagnetic radiation and scattering. He is currently Group Leader in the Electromagnetics Branch of that institution, where he carries on and supervises a research program on the basic

interaction of electromagnetic, acoustic, and elastic waves with materials and structures. Since 1978 he has occasionally lectured at Catholic University and recently at the University of Maryland, College Park. At last count, he had authored 70 scientific journal articles and 30 government technical reports. He has given over a 100 technical presentations before Professional Societies or at Congresses/Symposia. He holds three patents and has authored three review chapters on his own work.

Dr. Gaunard is a member of many professional and honor societies including the NY Academy of Sciences, the American Physical Society, the Philosophical Society of Washington, the American Academy of Mechanics, the International Union of Mathematical Physics, Sigma Xi, Tau Beta Pi, etc. He is a Fellow of the Acoustical Society of America and the Washington Academy of Sciences. His professional biography appears in *American Men and Women of Science* and a dozen other similar books.